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AN

Intire SYSTEM

C OF B

ARITHMETIC:

CONTAINING

I. *Vulgar.*	IV. *Sexagesimal.*	VII. *Lineal.*
II. *Decimal.*	V. *Political.*	VIII. *Instrumental.*
III. *Duodecimal.*	VI. *Logarithmical.*	IX. *Algebraical.*

With the Arithmetic of Negatives, and Approximation or Converging Series.

The Whole intermix'd with *Rules New*, *Curious*, and *Useful*, mostly Accounted for in the *PREFACE*.

The *Algebraic Part* is rendered more *Plain* and *Easy*, than hath been done, by Instructive Rules and Examples *Literally* and *Numerally*, *in a Method New*: Solving Equations, Simple, Quadratic, Cubic, &c. several ways.

And in the proper Places of this Work are *An Accurate Table of Logarithms* to 10000, and Rules to find those to 100000000, and Natural Numbers to such Logarithms; with *the full Use of the Table* in Multiplication, Division, Involution, Evolution, and in the Solution of *all Cases of Compound Interest*, of which there are 24 Large and Exquisite Tables, (and one for the Valuation of Church or College-Leases of their Land) as also those of *Simple Interest and Discount*, with a new Method of finding the later and the present Worth of Money for Days.

Also Ample Definitions and Explanations of Numbers, Quantities and Terms used in all Parts of Arithmetic, in Alphabetical Order; rendring the Whole more Intelligible, and Easier-Learned.

With an APPENDIX, shewing the Mensuration of more Superficies and Solids than any Book wrote purposely on that Subject has exhibited.

Necessary for all who who would in a short Time, and with little Study, acquire a competent Knowledge of Numbers and Species, or would make any considerable Progress in the Mathematicks.

By *EDWARD HATTON*, Gent,

The Third Edition.

LONDON:

Printed for RICHARD MANBY on *Ludgate-Hill.* MDCCLIII.

To the LEARNED

Dr JOHN KEILL, M.D.

Fellow of the Royal Society, and Professor of Astronomy in the University of Oxford.

GREATNESS and Goodness (you know, Sir) compleat the Rational Specie: By Greatness I mean true Magnanimity embellished with Learning and Science; by Goodness I suppose such an one as above, finished with an humble condescending communicative Temper or Disposition: And by how much any one excels in the former, by so much he is to be admired for the later Endowments. I need not tell the World how much the Qualifications and Acquirements of the first kind are your's, Sir: Your Learned Treatises of Philosophy, Astronomy, *&c.* in another Language, are sufficient Indications of that: And for the other, I have experienced your Condescending, Generous, and Diffusive Goodness, not only in your giving me an Opportunity of being known to you; but by the many Favours in so short a time received from you: Which propitious Providence having so ordered as to be the Consequence of my writing the following Book, I could neither justly nor naturally think of Dedicating it to any but Yourself.

AND altho' I am far from having intended this Treatise for the few of your Superior and Universal Genius; yet I

am not without Hopes, but that upon an impartial and particular Perusal, even such as You may find some things Novel and Acceptable. And as for those who have not, but are desirous to acquire, the Knowledge of Arts Mathematical, (a Study so exceeding Pleasant and Useful) I have endeavoured to guide them at least to the Entrance of that large and unfrequented Path, in which you and some others of your most Illustriously Learned Royal Society have made so vast a Progress.

THE Preface enumerates some of those many Things which I take to be New, and not contained in any other Treatise of Arithmetic in our own Language: And for what is in others, I know you are a much better Judge than,

SIR,

Your Obliged and most Humble Servant,

Edw. Hatton.

TO THE

READER.

HAVING *for many Years past spent my Leisure Hours in Mathematical Studies, and not meeting with any Treatise teaching in a Succinct Order, and Good Method, all the Parts of Arithmetic; and having made Emendations and Improvements in most, and investigated and discovered things new in many others: I supposed the Publication thereof would be accordingly received by the Studious in this Art. But to be a little more particular, I have, by way of* Introduction, *inserted* very Useful *and* Copious Definitions of Numbers, *and* Explications of Terms *Arithmetical, which will admit of the Whole being easier understood and sooner learnt. I have next given the* most large *and* plain Numeration-Table; *and* Tables and Rules *to assist the less knowing* in Addition without Pricking *and* Carrying; *and how to add Money, Weight, Measure,* &c. *by one Rule: also* Tables *of* Weight, Measure, *&c. in a more accurate Method, and more various than others: The* Reason *of the Way of Working in* Substraction *and* Multiplication; *and in Multiplying by several Digits, I have given the Reason why the Products of all from the first are put one place more towards the left hand: Also a large Table for Beginners, and a new brief one of my own Contrivance for the more Skilled. In* Division *I have shewed* four several

 Ways,

Ways, *with the Reason of the Method of Operation, and how to find all the even Parts of any Numbers, which is applied in finding the Aliquot Parts of a Pound* Sterling, *&c. And besides the plain and best Way of* Extracting the Square and Cube Roots of Numbers, *I have given Rules how to do that of the* Biquadrate *by an Example* altogether New, *and must be esteemed Curious, especially to those who are not acquainted with Algebraic Canons. In* Progression *you have several things not to be found in other Books of this Subject; as Rules to find the Total of a Series produced by different Factors, and of the Changes to be rung on any Number of Bells,* &c. *with the* Reason *of the* Abstruse Rules *for* finding the Totals of Series's *whether the Ratio be Arithmetical or Geometrical: And have recommended this Part of Arithmetic to the Learner's Perusal, and not to be passed over, as is too common both in Books and Schools. And whereas others on this Subject give but four, or at most five kinds of* Rules of Proportion, *I have in this Treatise* exhibited Twelve. *The* Rules of Practice are far more Numerous, Brief, and Methodical, *than any done by another Hand: And here are many things* more than common *in* Fellowship, *as three several Ways of answering Questions,* &c. *as also in* Alligation, Barter, *and* Equation of Payments.

In Decimal Arithmetic (*besides the Hint given of a New Specie thereof*) *you have* so many things truly Novel and Curious, *especially in Reduction and Multiplication, as would be too prolix to enumerate; where you will find some Answers exhibited by short and accurate Methods quite contrary to the General Rules given for Addition and Substraction*; i. e. *By Adding and Substracting one and the same Number, as Units to Tens place,* &c. *and the contrary.*

And in the Use of Decimals, besides the Way of Answering any Question by Decimals as exactly as by Vulgar Fractions (*which to me is wholly new*) *you have* the only Genuine Tables of Discount *that I know extant, with a* New *as well as more* Brief *and Easy Method of* Computing *both* Discount *and* Present Worth *of any Sum for Days,* (*so much practised by Traders*) *the Foundation of which you will find in* Algebra, *Chap.* 10. *I have next shewed the* Arithmetic of Duodecimals *and* Sexagesimals; *the former used in Mensuration of Superficies and Solids, the later* (*chiefly*) *in Astronomy, whence called by some Astronomicals: and these I have done in a new plain Method, and for the more difficult Cases. As to* Political Arithmetic, *I have said enough to shew its Nature and Manner of Process to discover what is required, illustrating the same by Examples, and have referred to others who have purposely and wholly treated thereon. In* Logarithms *I have* omitted nothing that I could find necessary *or deficient in other Authors; and believe I may with Veracity affirm, That no Book of*

of Arithmetic affords so ample and plenary Instructions on that Head: And have endeavoured to render that part which has put many to a Ne plus ultra (*as being somewhat less Intelligible, and not easily retained in Memory*) *I mean the* Addition and Substraction of the Logarithms of Decimals, plain *to any of a moderate Capacity; and have enlarged in a familiar way on the* Use of Logarithms in Multiplication, Division, Involution, Evolution *or the Extraction of the Roots of all Powers; and in the* Solution of all the Cases of Compound Interest: *As also how Operations in Vulgar Fractions are performed by* Logarithms, whereof I have given you a large Table *actually and by Inspection to* 10000 *and made it subservient for Natural Numbers, tho' so great as* 100000000, *by help of a Table of Proportional Parts* (*there also inserted*) *or without it; and have taken the elaborate Pains to write both Tables all over with my own Hand, that I might the better answer for their Accuracy: In doing whereof, I have corrected* 33 *Errors in one of our last published Tables to* 10000; *and comparing the same also with Mr* Norwood's *3d Edition of his Trigonometry, I have rectified his in* 182 *several places: so that I hope mine will be found very Correct. In* Lineal Arithmetic *I have performed my* Operations three several ways, *and have shewed how* to make the Lines of Chords *for the Mensuration of Arcs of Circles; the* Line of half Tangents *for Diameters, or those Great Circles represented thereby in Projections of the Sphere; and the* making the Line of Numbers, *commonly called* Gunter's Line, *to any Radius from a Table of Logarithms; and this both on a right Line and on the Ambit of a Circle: which I would not omit, in regard I found many of the Instrument-Makers that I discoursed on this matter, to be ignorant of it. What I have* advanced in Instrumental Arithmetic, *will be found not only New and Pleasant, but very Useful, particularly the new and plain Way of Working by* Neper's Bones; *but especially the Use of my* New Circular Instrument *contrived by me, whereby* the Reduction of Coin, Weight, and Measure to Decimals, and the contrary, are speedily, easily, and accurately performed; *not by guess, (as some Scales only have them) but actually and explicitly, as I have shewn by various Examples. And Multiplication, Division, and Extraction of Roots,* &c. *may be done by the two Lines of Numbers made to turn round one within the other, and are placed next the Limb or Edge of the Instrument, the Dimension whereof being so large as* 33 *Inches, I doubt not but it will be allow'd as the best and most compleat extant for all Arithmetical Uses. In* Algebra *I have previously given many* more Numerous and Useful Definitions *relating to this Analytical Art, than any Treatise of Arithmetic affords; and have throughout that profound Science illustrated the Nature and* demonstrated the Truth of the Symbolical Operations

tions by Numeral Examples, *intermixing several things New, and my own Invention: particularly the Way of finding* the Unciæ of Powers *to the 15th, and the Powers of a Binomial to the 10th, as also the* Uncia of any Term or Member of any Power *without the Knowledge of those of any previous Power. I have also shewed the* Reason of the Process in Extracting the Roots *from the respective Powers of a Binomial; and have likewise made the* Algorithm of Surds *very perspicuous by Examples in* Numbers as well as Species, *and have done the like in all the Rules of Algebra; and I have shewed not only the* Solution of Simple Equations *and those by* various Positions, *but* Quadratical, *and* Cubical; *each three different ways: In all which, as well as in* Approximation or Converging Series, *I have purposely designed to render the* Manner of Solution intelligible *to a mean Capacity, that being the principal thing in which the Learned Authors on this Subject have been deficient.* The Arithmetic of Negatives *I have fully accounted for, as will appear if to what is said under* Negative Arithmetic, *in the Alphabetical Explanation at the beginning of Algebra, you add what is in* Sect. 3. *of* Chap. 7. *in* Sect. 2, 3, 4, 5. *of* Chap. 10. *and under the last of my Examples of Converging Series, near the Close of this Treatise.*

But notwithstanding the particular Regard of my former Labours, and especially of this, to promote a sort of Learning so Useful to the Public; yet so numerous are the capacious and prejudiced Readers of this our envious Age, that it would be Vanity in me to hope to escape their Censures. But if my Endeavours prove acceptable to the two Classes of Readers for whom they were chiefly intended, i. e. *the Candid and Industrious Teacher and the Diligent and Studious Learner; I shall esteem it a good Step towards an ample Compensation for the uncommon Care and Trouble of this Performance, thus dedicated to the Service of the Public; and shall the less regard the Carps and rash Judications of pragmatical and ungrateful Dispositions, who fancy there is nothing in Arithmetic beyond what they have acquired the Knowledge of: or if they learn any thing from a Book of this kind, will be the last in paying their Acknowledgment, and probably the first in decrying the Work, because it will instruct some to know more than they would have them, or (as we say) make others as wise as themselves. However, I would not be discouraged by the ill Conduct of such from imitating the most Perfect and Benign Pattern, who is kind to the Unthankful and the Evil: for whatever some Authors may have proposed in being so, I have disclosed my Thoughts this way, much more to promote the Proficiency of the Reader, than to indicate the Science of the Author.*

And for the Encouragement of the Impartial and Ingenuous Student, who laying aside Pride and Prejudice, designs only Improvement in the Knowledge

of

of what is contained in the subsequent Pages, I can assure him, that I have faithfully adapted this Work to the Capacity of the less Acute, and have offered to his Perusal many things which he will not find elsewhere; of which I have given some Instances, as above, though you have many more mentioned in the Index, which will be found Exuperations.

I have not in this Treatise (as is common where there are many Parts of Mathematics, &c. *in one Volume) only touched on each kind; for you will find* Vulgar Arithmetic, Decimals, Logarithms, *and* Algebra, *as* copiously insisted on, *as in almost any Treatise wrote only upon some one of those Species: Nor can any thing be expected to make the whole more truly agreeable to the Title, which is a piece of Justice that all Authors owe to the Public, and cannot be denied but to have been fully observed by me, as I am not without Vouchers from good Hands, to confirm the Truth of, especially in my* Merchant's Magazine, *and my* Index to Interest.

And for the Satisfaction of such as are cautious of buying the first Impression of a Book, because, say they, *there may be Additions to future Impressions; I do hereby promise, that as I know of no Necessity for it, so I have no Thoughts of doing any thing farther on this Subject.*

I have, besides what I promised in my Proposal to Subscribers, added an Appendix, which contains the best Way of measuring a greater Variety of Superficies and Solids, than any Book, tho' wrote purposely on that Subject, exhibits.

In fine, I am so sensible of the Care and Assiduity used to finish this Body of Arithmetic, *so as to render it in some degree* Compleat, *that I hope I need not doubt of its being acceptable to the World; which hath already favoured me, by a kind Reception of my former Endeavours in this Way, altho' I had not bestowed near that Thought, which (in Gratitude) I have done on this Work, in order to present it, as near as I could, in Perfection.*

For besides all that is abovesaid, I have not only comprised in this Treatise, the most material Tables, and other things in my Index to Interest *(which shall never be but here re-printed) but have added Tables of Simple Interest at* 4 per Cent. *of Discount at* 4 *and* 5 per Cent. *and* 16 *Tables of Compound Interest at* 3, 4, 5, *and* 8 per Cent. *And the said Index (before the making these considerable Additions to it) was approved of, and recommended as the most copious, easy, and useful, Book that hath been written on that Subject, by the judicious Persons following; besides many others, as I have their own Hands by me to demonstrate.*

S. Shepherd

S. Shepherd *Esq*; *Deputy-Governour*,
Sir W. Chapman *Bart.*
Sir Samuel Clarke,
Sir John Blunt *Bart.*
then Directors
of the South-Sea *Company*.

Sir G. Cook, *first Prothonotary of the Court of* Common-Pleas.

The Reverend Mr Whiston, *sometime Professor of Mathematics in the University of* Cambridge.

Nath. Pigot, *of the* Middle Temple *Esq*;

P. Lacy, *of the* Inner Temple *Esq*;

Mr If. Jackson, *of the* Middle Temple, *Sub-Treasurer.*

Mr J. Frewen, *of* Lincolns-Inn.

Mr Humphry Ditton, *Master of the Mathematical School at* Christ's-Hospital.

Mr S. Waters, *Accomptant General*
Mr P. Sergeant,
Mr J. Fletcher,
Accomptants
to the East-India *Comp.*

AN

An INDEX,

Referring to the Pages, Chapters, and Sections, which are placed on each Folio.

Carpenters

Decimals

Interest

Number

Questions

Substraction

Triplicate

THE

THE INTRODUCTION:

Containing an Alphabetical Definition of all kind of Numbers, and the Terms of Art used in Common Arithmetic explained; which the Reader may have recourse to, as he finds occasion.

Note, *That the like for Algebra is inserted immediately before that Species of Arithmetic.* Vid. *Sect.* 1. of *Chap.* X.

A*Bundant Numbers,*] Are ſuch as having their Aliquot Parts added together, the Sum exceeds the Numbers of which they are Parts: as 36 is an abundant Number, becauſe its Aliquot Parts, 1, 2, 3, 4, 6, 9, 12, 18, added, makes 55, which is more than the 36.

Abſtract Numbers.] Such Numbers in general, as have no Denomination annexed to them.

Abſolute Numbers.] Such in reality as they appear to be, as 2, &c. is 2 Units contrary to 2 leſs than nothing (as in Algebra is explained) and to negative Indices of Logarithms: alſo it is that known Number in an Equation, which ſolely poſſeſſeth one ſide. See *Algebra.*

Aliquot Parts,] Are the even Parts of any Number, when there is no Remainer in the Diviſion. See at the End of *Diviſion.*

Aliquant Parts.] Such as will not divide another Number aſſigned without a Surplus or Reſt: as 7 is an aliquant Part of 18 or 23, &c.

Amiable Numbers,] Are 2 ſuch, as that the Sum of the Aliquot Parts of one, will make up the other Number alternately.

Arithmetic] In all its Parts (or in the moſt comprehenſive Senſe) is the Art of ſolving Queſtions by Figures, Lines, Inſtruments, or Symbals, as by the ſeveral kinds following doth appear.

Arithmetical Complement.] See *Sect.* 4. of *Chap.* VII. and *Complement* under *C.*

Arithmetical Progreſſion.] See *Progreſſion,* Chap. II. Sect. 2.

Articles,] In this Science, are Numbers diviſible by 10 without Remainer; as 10, 20, 30, 100, 120, 130, &c. 2000, 4000, &c. the ſame with round Numbers or Decades.

Artificial Numbers,] Are Logarithms. See *Chap.* VII.

Antecedent Numbers.] The firſt named of two that are compared together: As if it be ſaid that ſome Numbers are in proportion as 3 is to 7; here 3 is the Antecedent, and 7 the Conſequent.

Binarys.] Two Places of any Number, (as pointed for the Square Root.)

Biquadrate.] See *Powers* under *P*.

Broken Numbers.] Such as are commonly called Fractions, which ſee.

Cardinal Numbers,] The 9 Digits.

Central Numbers.] Such as have a Digit placed in the middle between a like Number of Reverſe, Circulating, &c. Numbers, as 3457345, 12521, 78678, &c.

Characteriſtic] Of any Logarithm, is the Index, See *Chap.* VII.

Circular Numbers,] Are ſuch as being ſquared, cubed, &c. the Figure in Units place of the Power, is the ſame with that of the Root; as 6 times 6 is 36, 5 times 5 is 25, &c. which are ſometimes called Spherical Numbers.

Circulating Numbers,] According to ſome, are thoſe Digits which are regularly repeated, whether the Number be of one and the ſame Digit repeated, as 222, 7777, &c. or of many as 494949, 372372, &c. And when theſe fall out in reducing Money, Weight, Meaſure, &c. to Decimals, you gain many Decimal Places with the leſs trouble.

Commenſurate Numbers.] Such as one Number will juſtly meaſure; as 6 and 15 are meaſured by 3, &c. And the Fraction $\frac{2}{5}$ is of like Value with $\frac{6}{15}$, becauſe the Terms are commenſurate by 3.

Compleat Numbers.] See *Perfect Numbers.*

Complement-Arithmetical,] Is the Remainer when any Logarithm is deducted from 10.

Compoſed Numbers.] The ſame as Compoſit Numbers; which ſee.

Compound Fractions.] Fractions of Fractions, as $\frac{1}{3}$ of $\frac{4}{7}$, &c. See *Chap.* II. *Sect.* 1.

Compoſit Numbers.] Such as are meaſured by ſome other beſides an Unit; as 297 is compoſed of 99 by 3, either of which meaſures 297.

Common Multiples.] See *Multiples* following.

Conſequent Numbers.] The ſecond of any two in a Proportion. See *Antecedent.*

Contract Numbers.] The ſame as Concrete Numbers, (but is no proper Term.)

Concrete Numbers.] Such as have a Denomination annexed, as 5 *Lib.* 7 Yards, &c.

Cube

Cube Numbers.] The Product of Square Numbers by their Roots; as 8 is the Cube of 2, 27 of 3, being Products of 4 by 2, and of 9 by 3.

Cubed Square and *Cubed Cube Numbers.*] See *Powers* under *P.*

Cube Root.] See *Roots* under *R.*

Decimal Fractions.] Those whose Denominators are 10, or some Power of 10.

Decades.] The same with Articles, as above.

Deficient Numbers.] Those whose Aliquot Parts (added together) are less than the Numbers of which they are Parts; as 26 is a deficient Number, because its Parts 1, 2, and 13, make but 16.

Differences, or Common Differences, as relating to Logarithms.] See the 3*d* and 4*th* general Heads under *Sect.* 4. of *Chap.* VII.

Digits,] In Arithmetic, are Numbers of 1 Place, as 1 to 9 inclusive.

Direct Proportion,] Is when the 4*th* N°. (sought) is the Quote arising by dividing the Product of the 2*d* and 3*d* Number by the 1*st*.

Dividends.] Numbers which are to be divided. See *Division*, *Sect.* 5. *Chap.* I.

Divisors.] Numbers of Parts into which another Number is to be divided. See *Chap.* I. *Sect.* 5.

Dividual.] Separable or Divisible. Also that Part of a Dividend which is immediately under your Operation.

Diminutive Numbers,] Are the same as Deficient Numbers, which are explained above.

D.] From *Denarius* (a Penny): The Mark put over a Column of Pence in Books of Accounts.

Feet. Inch. 12ths

Duodecimals.] Fractions whose Denominators are 12, as 7 : 9 : 7 is 7 foot, 9 inches, (or 9 twelfths of a foot) and 7 twelfths of an inch, &c. Which are often marked by Measurers thus;
F. ′. ″.   F. ′. ″. ‴.
7 : 9 : 7; and 48 : 11 : 3 : 10; and read the first, 7 foot, 9 primes, and 7 seconds; the second is, 48 foot, 11 primes, 3 seconds, and 10 thirds. See *Operations* thereby, *Chap.* IV.

Duplicate, *Triplicate*, *Sesquiplicate*, &c. Proportion.] See *Sect.* 3. of *Chap.* II.

Equimultiples.] See *Multiples.*

Evenly even Numbers,] Are those which even Numbers will measure by other even ones; as 48 is measured by 8, 6 times.

Evenly odd Numbers,] Are those which even Numbers measure by odd ones; as 40, which 8 doth measure by 5.

Evolution] The Extraction of Roots of Numbers, *&c.*

Exponents,] Are Numbers (smaller Characters than those they stand next) which are placed near the upper part of a Root toward the right-hand, and shew what Power of that Root is expressed; as 3^2 is the Square of 3 or 9; 3^4 is the Biquadrate of 3 or 81. Also 4^3 is the third Power or Cube of 4, which is 64; the Exponents being 2, 4, and 3, standing higher than the others or Roots. So likewise 15^5 is the Sursolid or fifth Power of 15; 7^6 is the squared Cube or sixth Power of 7; 20^7 is the second Sursolid or seventh Power of 20, which is 1280000000. See *Powers* here and in Algebra.

Exclusions.] Such Numbers in a Question, as being excluded, renders it less perplexed and the easier resolved; as might be shewn by several Examples.

Factors.] Both the Numbers, which for the most part in Multiplication are called the Multiplicand and Multiplicator, (or Multiplier.)

Figurate Numbers.] Such as do represent some Geometrical Figure, either Superficial or Solid. See those Numbers and Linear.

Fractions.] Any Part or Number of Parts of an Unit *ad infinitum:* as $\frac{2}{3}$ is Vulgar; $\frac{1}{10}$, *&c.* Decimal; and $\frac{1}{12}$, *&c.* Duodecimal. See *Chap.* II, III. and *Sect.* 10. of *Chap.* III.

Geometrical Progression.] See *Progression.*

Harmonical Proportionals.] See *Musical,* and *Chap.* II. *Sect.* 3. *Head.* 11.

Homogeneal Numbers,] Or Homogeneous, are those of the same nature; as the Indices of 2 Logarithms, if they are both affirmative, (without a Mark under) are Homogeneous; or they are so, if they are both negative, or have a Mark under thus, $\underline{3}$ and $\underline{2}$, *&c.*

Homologous.] Numbers or Terms in a proportion are said to be Homologous, when there is a Similitude between the Antecedents and Consequents; for as 3 to 9, so 7 . 21: here 3 is homologous to 7, as 9 to 21.

Heterogeneal Numbers.] Those which are not of the same kind; as when one is negative, the other affirmative, as 3 and $\underline{2}$; contrary to homogeneal. Also mix'd Numbers, composed of a Whole and a Fraction, as $2\frac{1}{3}$: and in Surds, they are such as have different Radical Signs, as $\sqrt{}^3$ and $\sqrt{}^4$. Vid. *Algebra*, Chap. X.

Improper Fractions.] Such whose Numerator is equal to or exceeds the Denominator. See Chap. II. *Incommen-*

Incommensurate Numbers.] Two ſuch, as no one Number (except an Unit) will meaſure, contrary to commenſurate; as 7 and 23, *&c.* cannot be meaſured by any one Number.

Indices.] Numbers which ſhew the Power of a Root; and in Logarithms they ſhew the Number of Places which the natural Numbers belonging to any Logarithm conſiſts of, being the ſame with Characteriſticks. See *Exponents*, and alſo Chap. IV.

Ineffable Numbers,] Are Surds, or irrational Numbers; which laſt ſee: Or (according to the Import of the Word) Numbers not to be expreſſed.

Incompoſits.] Numbers which no other but an Unit will meaſure, being contrary to Compoſit, and are the ſame with prime Numbers, as 7, 11, 23, 29, 31, and hundreds more.

Intire or whole Numbers.] Any Number of Units or Ones; (but Fractions are one or more Parts of an Unit) ſo that *one* is the middle between infinite Units and infinite Parts: and tho' it is by ſome not allowed to be a Number, yet ſince it is the Foundation of all Numbers, whether whole or broken, (for 2, 3, 4, *&c.* are ſo many *ones*, and $\frac{2}{7}$, $\frac{1}{7}$, $\frac{4}{7}$, *&c.* are ſo many Parts of one) and ſince 1 is as properly half the Number of things which are but 2 in all, as 2 is half the Number which are four in all; it follows, that ſince 1 is not a Fraction, (or Part of 1, which would be a Contradiction to affirm) it muſt be a Whole, *i. e.* a whole Unit, or the Number 1, as properly as 2 is 2 Units, or the Number 2, 3 the Number 3, 4 the Number 4, *&c.* For Unity is as properly an Individual (or inſeparable from Number) as theſe or any other. And if we take a Series of Numbers, as 1, 2, 3, 4, 5, here 1 in all reſpects is a Number according to its Nature, as any of the other Digits are agreeable to the ſame common Nature of Number, and 1 is one Term or Place in that Rank as well as the others: 'Tis true, 1 does not divide nor multiply, becauſe it would be abſurd to ſay that a Man had 1 Pound, Yard, Ounce, or the like, divided between himſelf; or that becauſe I ſay, 1 Horſe, 1 Field, *&c.* I ſhould exclude the Number 1, becauſe it does not make more or leſs than that Horſe or Field, and ſo be contrary to the common Nature of all other Numbers. And as it multiplies and divides as much as it ought to do, making every thing once itſelf, and giving the Whole to one Perſon where no one is to have a ſhare with him; ſo in Addition, *&c.* it does as all other Numbers do, *i. e.* augments or diminiſhes any other Number ſo much as itſelf amounts to: For

if

if I add 1 to 5, it makes 6; that is, so much more than 5, as is the Number 1 added; as 2 added to 5, makes 7, which is itself (or N° 2.) more than the 5, *&c.* Besides, *one* in all Merchantile Affairs, as well as in the Eye of our *Laws*, and in common Conversation, is a N°. And 'tis far from being a parallel to a Point in Geometry, 10000 of which make no Magnitude, but twice one is what all allow to be *a Number*. And whereas some great Men, as *Euclid*, &c. have defined Number to be a Multitude of Units; it is highly probable they meant no more than this, That as Unity is but 1, and there are Millions of Millions, nay, infinite other Numbers; therefore, say they, that Number (in the general way of speaking, or for the most part without comparison) is a Multitude or Aggregate of Units.

Integers,] Are whole Numbers, compared with their Parts; as 1 Penny is an Integer composed of 4 Farthings, its Parts into which it is divided; 1 Shilling is an Integer compared to Pence or Farthings, of which it is composed. But an absolute Integer is an Unit of the highest Denomination of any Specie of Matter or Thing: as 1 Pound *Sterling*, &c. 1 Ton, 1 Circle, *&c.* and 2, 3, *&c.* of the like, are so many absolute Integers.

Inverse Proportion. See *Reverse*.

Irrational Numbers, or *Surds*,] Are such whose Roots cannot be accurately extracted, as being no Figurate Numbers: But then they are to be considered as Surds or otherways, when compared with the Power they are of. Thus 16 is a Surd, or irrational Number, if supposed the 3d Power of some other; but if you suppose it only the 2d Power, it is not Surd, but compleatly the Square of 4: and the like may be observed of others. And when we meet with Surd Numbers, (or those that are Rational, which are to be wrought with Surds) we mark them thus, $\sqrt[2]{7}$, $\sqrt{7}$ is the way of expressing the Square Root of 7; $\sqrt[3]{27}$ is the Expression of the Cube Root of 27, which is equal to 3, *&c.* But more of this under *Powers*, &c. in the Algebraic Part.

Lib. or *L.* (from *Libra* a Pound-weight, or 20 *s.*) The Mark put over a Column of Pounds.

Lineal (or *Linear*) *Numbers*.] Such as represent or are the Dimensions of a Line, Root, or Side of a Geometrical Figure. Thus if a Figure be an exact (or Geometrical) Square, containing 100 Foot, 100 is the Superficial Number, and the Side or Linear Number

is

is 10: If it be a long Square (or Parallelogram) of the Content of 40 Yards, the Linear Numbers (or Sides) are 4 and 10, or 8 and 5.

Lineal Arithmetic,] Is that Science performed by Lines.

Logarithms,] Are Artificial Numbers, of great use in Mathematicks; the Invention of Lord *Neper*, Baron of *Merchiston*. See Chap. VII.

Mixt Numbers.] Whole Numbers and Fractions, (Vulgar or Decimal) which are placed together: as $13\frac{7}{8}$, 29.75, *&c.* Or any Number composed of Digits, or Digits and Cyphers that are not next the right-hand.

Multiples,] Are Numbers produced by the Multiplication of some known or assign'd Number: as 40 is a Multiple of 8 or 5, because either of those will divide 40 without Remainer, and 20 is a Multiple of 4 and 5.

Multiples, or *Equimultiples,*] Are Numbers multiplied by one and the same Number: as 12 and 28 are Equimultiples of 3 and 7, for 12.28 : : 3.7.

Multiples, or *Common Multiples,*] Are when one and the same Number is produced by different aliquot Parts: as 40 is a common Multiple of 2, 4, and 5, because any of these divide it without Remainer, and because these multiplied one in another produce 40.

Multiplicator.] The same as Multiplier. See Chap. I. Sect. 4.

Musical Proportionals.] The same as Harmonical. See Chap. II. Sect. 3. Head. 11.

Negative Arithmetic.] See (N) near the beginning of Algebra, in which this kind is chiefly used.

Number, or *whole Number,*] Is one or more Units, expressible by the 9 Digits and 0, *ad Infinitum*; of which there are near 50 several Kinds mentioned in this Introduction. See *Intire*, &c. or a broken Number, is one or more Parts of an Unit.

Numerator of a Fraction.] The Number of Parts contained in it. See Chap. II.

Ob. (from *Obolus* a Halfpenny) The Mark put for two Farthings in old Writings.

Perfect Numbers.] Those which are equal to the Sum of their aliquot Parts; as 28 is equal to the Sum of its Parts 1, 2, 4, 7, 14, which is 28; and 1, 2, and 3, are 6.

Parts Proportional.] See 3d and 4th General Heads, §. 4. of Chap. VII.

Plain Numbers,] Are those which may be produced by multiplying some Number in another, as 6 the Product of 3 by 2, 8 the Product.

duct of 4 by 2, &c. (but 7, 11, 13, &c. are no such.) These are the same with superficial Numbers, because the Content of a plain superficial Figure is produced by multiplying two Numbers, which are supposed its Length and Breadth together.

Positive Numbers.] The same with Affirmative. See *Absolute Numbers*, above.

Powers of Numbers,] Are the Root or first Power; the Square or second Power, produced by multiplying the Root in itself, &c. as follows.

	Roots, or 1st Power.	Square, or 2d Power.	Cube, or 3d Power.	Biquadrate, or 4th Power.	Sursolid, or 5th Power.	Squared Cube, or 6th Power.	The second Sursolid, or 7th Power.	The Squared Biquadrate, or 8th Power.	Cubed Cube, or 9th Power.	Squared Sursolid, or 10th Power.
1st Example	2	4	8	16	32	64	128.	256	512	1024
2d Example	3	9	27	81	243	729	2187	6561	19683	59049

Each Power being produced by multiplying the next before it towards the left-hand by the Root.

And that these Terms of the Powers are really what they are called; if the 5th Power or Sursolid (in the 1st Example 32, in the 2d 243) be squared or multiplied in itself, it produces the squared Sursolid or 10th Power, for 32 by 32 gives 1024; or 243 by itself makes 59049. And the like may be observed of the rest: for a more full Account of which, see the Word *Powers* in the beginning of Algebra, Chap. VII.

Promiscuous Numbers.] Composed of Digits without any limited Order, as 7291, 3924, and thousands of others, as under *Mixt Numbers.*

Proper

Proper Fractions.] See *Chap.* II. *Sect.* 1.

Proportion.] The ſame with Ratio; its various kinds. See the beginning of *Sect.* 3. of *Chap.* II. of which ſome are *Subduple*, *Subtriple*, *Submultiple*; which Words ſee: It is marked with Points thus, 3. 4 : : 6. 8. if direct.

Prime Numbers.] Such as no other but Uuity will juſtly divide. See *Incompoſits.*

Proportional Parts.] See *Parts Proportional.*

Progreſſion.] Numbers in Arithmetical and Geometrical Progreſſion. See *Chap.* II. *Sect.* 2.

Q. (from *Quadrans* a Farthing) the Mark put over a Column of Farthings.

Quotient.] See *Diviſion*, *Sect.* 5. *Chap.* I.

Roots of Numbers.] The firſt Powers. See *Powers* above, and *Linear* N°

Ratio,] Is the Reaſon or Proportion that one Number bears to another. Or more particularly it is direct, Simple Ratio, Duplicate Ratio, Triplicate, *&c.* See *Chap.* II. *Sect* 3. Or Ratio in Progreſſion is the Rate by which the Terms increaſe or decreaſe.

Rectangle.] The ſame as Product, which is the Sum produced by multiplying 2 together; as 2 Times 4 is 8 = the Product or Rectangle.

Reverſe Proportion,] Is when the *4th* N°. (ſought) is the Quotient ariſing from the Diviſion of the Product of the *1ſt*, and *2d*, by the *3d* Number.

Reverting Numbers.] Such as are compoſed of a like Number of ſucceſſive Digits aſcending and deſcending, as 345543, 7887, *&c.*

Round Numbers.] The ſame with Articles and Decads, which ſee.

Seſquiplicate Proportion.] See *Head* 12. of *Sect.* 3. of *Chap.* II.

Similar Numbers,] Are either *1ſt*, Plain, which are ſuch as form like or ſimilar plain Figures; as a Plain of 21 formed by the Rectangle of 7 by 3, is ſimilar to another of 84 formed by the Sides or linear Numbers 14 and 6; for as 6 to 14, ſo 3 to 7. Or, *2dly*, Similar ſolid Numbers are ſuch as form like or ſimilar Solids (as Parallelopipedons, or 2 or 3 Dice joined to the end of each other) of different, but proportional Dimenſions.

S. (i. e. *Solidus*) the Mark placed over Shillings in a Column of Money.

Spheric Numbers.] The ſame with Circular Numbers, which ſee.

Subduple Proportion.] Numbers are in ſubduple Ratio, when the Antecedent contains the Conſequent (or the contrary) twice; as 5 to 10, or 40 to 20, and the like.

Subtriple.] Numbers are in a ſubtriple Proportion, when the Conſequent is 3 times the Antecedent, or the contrary; as 9 to 27, or 90 to 30, *&c.*

Submultiple.] Numbers are in a ſubmultiple Ratio, when the Conſequent contains the Antecedent, or the contrary, above three times without Remainer.

Square Numbers.] All ſuch as are produced by multiplying any Number by itſelf, becauſe that Product repreſents a Figure of four equal Sides.

Surd Numbers.] The ſame with Irrational, which ſee.

Sexageſimal Numbers,] Are ſuch Fractions as have 60 for their Denominator, uſed in meaſuring Time, and the Motion of the Celeſtial Spheres, Geography, and Navigation: But they put not down the Denominator in theſe, no more than in Duodecimals, becauſe it never alters, being always 60, as thoſe are 12. Thus $\overset{\circ}{51} : \overset{\prime}{30} : \overset{\prime\prime}{45} : \overset{\prime\prime\prime}{17} : \overset{\prime\prime\prime\prime}{21} : \overset{\prime\prime\prime\prime\prime}{59}$ are read 51 Degrees, 30 Minutes, (or 60*th* Parts of a Degree) 45 Seconds, (or 60*th* Parts of a Minute) 17 Thirds, (or 60*th* Parts of a Second) 21 Fourths, (or 60*th* Parts of a Third) and 59 Fifths, or 60*th* Parts of a Fourth, *&c.*

Of Operations in this kind of Fraction, the Work of Addition is eaſy: Adding up a Column of Minutes, Seconds, *&c.* and dividing by 60, putting the Remainer under the Line, and carrying the Quote to the next Column toward the left hand, *&c.* borrowing 60 always in Subſtraction. And for multiplying Degrees, Minutes, Seconds, Thirds, *&c.* theſe may be done decimally, by reducing the lower Denominations into the Decimal of the higheſt, and then working as is ſhewed in Decimals, *Chap.* III. And by the ſame Rule and Diviſion of Decimals, is the beſt way to divide theſe. But ſee *Chap.* V.

Succeſſive Numbers.] Such as are compoſed of Digits which ſtand in a natural Succeſſion aſcending and Deſcending, as 34567, or 9876.

Tabular Numbers.] Such as are found in Tables, as of Logarithms, Intereſt, *&c.*

Tarif Numbers.] The ſame as Tabular Numbers. But the Term is moſt uſed in Tables of Cuſtoms, and Series of a Diviſor by the 9 Digits.

Terms.] The two Parts of a Vulgar Fraction are called its Terms; and ſo are the ſeveral diſtinct Numbers in an Arithmetical and Geometrical Progreſſion.

Ternary

Ternary Numbers.] Such as have 3 places: Or even 3 places of any Number are so many Ternarys, as used in extracting the Cube Root.

Triplicate Proportion.] See *Proportion*, or *Sect.* 3. of *Chap.* II. *Head.* 10.

Unit, or *Unity.*] The Number 1. See *Intire Numbers*, above.

Unevenly odd Numbers.] Those odd Numbers produced by the Multiplication of 2 odd Numbers together; as 63, which is 7 times 9; 45, which is 9 times 5, *&c.*

CHAP. I.

Treateth of the several Parts of single Arithmetic (called Common Arithmetic in whole Numbers) containing Numeration, Addition, Substraction, Multiplication, Division, and the Extraction of Roots, which are called Simple or Single Rules of Arithmetic, as being Fundamental Parts or Principles, by one or more of which, all Operations by Numbers are performed.

SECT. I. *Of Numeration of Intire Numbers.*

BY this first Part we are taught how to read or write down any Number proposed, by assigning a proper and natural Denomination for the Place of each Digit in any Line, (tho' composed of never so many of those Digits) and then by observing what Digit is there placed.

By a Digit we understand any single Figure possessing but one place, as 1 to 9 inclusive.

There is also a Cypher, (or o) which standing alone, or next the left hand of any single Figure or Number of Figures in whole Numbers is of no value; it only serving to fill up places, in order to augment the value of the simple Figure or Figures, which are placed to the left hand thereof; which places might as well be supplied by a Point (.) or Hyphen (-) thus, if that had obtained a Custom, as the (o) hath.

By what is said it appears, that after we know the Character of the 9 Digits, (which almost every Child learneth so soon as it can read) there is nothing remaining to make one able to read any Num-

ber, but to consider by what Name the several Places in any Number or Rank of Digits is called: and that will appear, by what follows, very obvious.

The Denomination of each particular Place of any Number not exceeding 9 Places, or that of Hundreds of Millions.	*Examples how the Course or order of the Places augments toward the left hand infinitely.*	*How each of the 9 Numbers are to be read in one Line or Number.*
	9th place. 8th place. 7th place. 6th place. 5th place. 4th place. 3d place. 2d place. 1st place.	
One Hundred Millions	100000000	
Ten Millions - - - -	10000000	
One Million - - - -	1000000	One Hund. and 11 Mill. / One Hund. and 11 Thous. / One Hund. and Eleven
One Hundr. Thousand	100000	
Ten Thousand - - -	10000	1 1 1 1 1 1 1 1 1
One Thousand - - -	1000	
An Hundred - - -	100	
Ten - - - - - -	10	
One - - - - - -	1	
Two Hundred Millions	200000000	Milli. / Thouf. / Units.
Twenty Millions - -	20000000	
Two Millions - - -	2000000	2 Hund. 20 2 / 2 Hund. 20 2 / 2 Hund. 20 2
Two Hundr. Thousand	200000	
Twenty Thousand - -	20000	2 2 2 2 2 2 2 2 2
Two Thousand - -	2000	
Two Hundred - -	200	
Twenty - - - - -	20	Millions. / Thousand. / Units.
Two - - - - - -	2	
Three Hundr. Millions	300000000	
Thirty Millions - -	30000000	
Three Millions - -	3000000	3 Hund. 30 3 / 3 Hund. 30 3 / 3 Hund. 30 3
Three Hundr. Thous.	300000	
Thirty Thousand - -	30000	3 3 3 3 3 3 3 3 3
Three Thousand - -	3000	
Three Hundred - -	300	
Thirty - - - - -	30	
Three - - - - - -	3	

From

From the Scheme above it may be observed, *viz.* 1*st.* From the middle Column it is plain that every place towards the left hand in any Number is 10 times the Value or Number of the place next it towards to right hand; as 1, 2, or 3, in the 2d, 3d, or 4th places becomes 10, 100, 1000; 20, 200, 2000; and 30, 300, 3000, *&c.*

2*dly.* The Column next the left hand gives you the Name that any Number in any place (not exceeding the 9th) is to be called by: As the first or uppermost in each of the three Examples above, is the Hundreds of Millions place; and consequently the Numbers are in Value 1, 2, and 3 hundred Millions, because the Digits there placed are 1, 2, and 3. But instead thereof, if 4, 5, 6, *&c.* were put in those places, the Numbers must be 4 hundred Millions, 5 hundred Millions, or 6 hundred Millions, *&c.* the Value or Name depending on the place that any Digit possesseth in any Number.

3*dly.* From the third Column it is evident, that in reading a Number there is a Contraction or Abbreviation of the Words in the first Column to be used. Thus (in the third Example) we do not read the three first Lines 3 hundred Millions, 30 Millions, and 3 Millions; but do only name the word Millions after the last of the three Figures. Thus 3 hundred thirty three Millions, and so of the Thousands, we say 3 hundred thirty three Thousand, *&c.* 333.

By these Rules, any Number, tho' never so great, may be read, as is farther exemplified by the following Table of this Art of Numbering; which is much more copious and ample, than any Arithmetic that I know of exhibits.

The

The Numeration TABLE.

Column heads (Places), from left to right:

- Quinquillions: Hundred of M. | X of Thousand | Thousands | Hundreds | Tens | 5 Units
- Quartillions: Hund. of Thouf. | X of Thousands | Thousands | Hundreds | Tens | 4 Units
- Trillions: Hund. of Thouf. | Tens of Thouf. | Thousands | Hundreds | Tens | 3 Units
- Billions: Hund. of Thouf. | Tens of Thouf. | Thousands | Hundreds | Tens | 2 Units
- Millions: Hund. of Thouf. | Tens of Thouf. | Thousands | Hundreds | Tens | 1 Units
- Hundreds of Thousands. | Tens of Thousands. | Thousands. | Hundreds. | Tens. | Units.

Places each Number or Line of Figures confifteth of.		Number of Lines reckoned upward.
36	103456789876543234567898765432345678	34
	290760004300001325732100000073212437	33
35	31000000423203044322000000321570000	32
34	2000007234003400040000005764300421	31
33	340000023450000425550000003214263	30
32	45600005213456000321570000421652	29
31	3210000450040600012351000013272	28
30	423000004321764321000234002917	27
29	21234567890000000000123456789	26
28	3212345678000000000012345678	25
27	432123456700000000001234567	24
26	54321234560000000000123456	23
25	6543212345000000000012345	22
24	765432123400000000001234	21
23	87654321230000000000123	20
22	9876543212000000000012	19
21	123456789100000000001	18
20	23456789002300000000	17
19	3000045678934000021	16
18	400005678923450432	15
17	50000678912345433	14
16	6000078901234364	13
15	700008900123445	12
14	80000900012346	11
13	9000087654321	10
12	123456789012	9
11	20034567893	8
10	3000456789	7
9	400056789	6
8	60007890	5
7	7000889	4
6	800091	3
5	98765	2
4	4321	1

1. It may not be improper to inform the Reader, that we make use of the words Billions, Trillions, *&c.* to prevent the Confusion that would arise by the often repeating Millions of Millions, *&c.* And if it be enquired how much a Trillion (or the like) is; I look for the Units of Trillions at the top of the Table, and casting my Eye downward in that Line till I come to the lower end of it, I then look directly toward the left hand, and find it a Number consisting of 19 places, *viz.* 18 Cyphers and (1) next the left hand in the 19th place, and is found by involving (or multiplying) a Million in itself, and that Product by a Million. So that a Trillion is as much as to say, one Million of Millions of Millions. So likewise a Quartillion is 1000000 multiplied three times in itself, (called its Biquadrate) and consists of 25 places: A Quinquillion is 1000000 multiplied 4 times in itself, and consists of 31 places, (called the Surfolid or fifth Power of 1000000, of which Powers I shall speak in a proper place.) All which appears plain by the foregoing Table: for Units of Quinquillions, casting your Eye downward to the end of that Column or Series, and thence directly towards your left hand, you find the Figure 31, which shews it consists of 31 places, *viz.* 30 Cyphers and (1) next the left hand. And thus may you find what number of places any Number in the whole Table consists of.

2. Or if you would know the Value of a Number consisting of any number of places, it is but looking for the places in the Column of places, and casting your Eye directly to the right hand, till you come to a Figure, which tracing upward in the same Series or Row, you have the Name or Value of the Number, consisting of any of the places given.

3. By the Table it appears also, that there is no more difficulty in reading a Number of 36, *&c.* places of Figures, than in a Number consisting of 6 Places. For when any large Number is required to be read, begin at Tens place, and tell 6 towards the left hand, over which make a 1: then continue to tell 6 more the same way, and over that make a 2: and so forward 6 more, and make a 3, (which will fall over the 19th place): So that having marked thus your whole Number given as in the Table above, I read, for example, the 34 Number in the Table thus.

103 Thousand, 456 Quinquillions,
789 Thousand, 876 Quartillions,
543 Thousand, 234 Trillions,
567 Thousand, 898 Billions,
765 Thousand, 432 Millions,
345 Thousand, 678.

And the 33 in the Table thus;—290760 Quinquillions,
4300 Quartillions,
1325 Trillions,
732100 Billions,
73 Millions,
212437

In ſhort, the Digit being known, and the place it poſſeſſeth, we have nothing to do but firſt to mention the Digit, and then the Place. As 9 in Units place is 9 Units, 8 is 8 Units, *&c.* 9 in Tens place is 9 Tens or Ninety; 9 in Hundreds place is 9 Hundred, *&c.* So 365 is 3 in Hundreds place; 6 in Tens place, and 5 in Units place; or 3 Hundred, 60, and 5: And the like of any other Numbers.

The narrow Column toward the right hand of the reſt, may ſerve to refer to the Table upon occaſion. As the Minutes ſince the Creation to *Anno* 1716 incluſive, are 3006387360, which conſiſting of ten places, is numbered as the 7th Line in the Table, with reſpect to the Names of the places: So likewiſe it being computed (in the Uſe of Multiplication) that the Number of Diamonds of a quarter of an Inch ſquare that would pave our Globe, ſuppoſing it even and all Earth, are 12813844858011648000; which conſiſting of 20 places, is numbered or read as the 17th in the Table. And the Value of thoſe Diamonds being *l.* 1281384485801164800000, which conſiſts of 22 places, it muſt be read as the 19th in the Table above.

Thus it appears, that the Table is abundantly large enough to afford an Example how the greateſt Number that can ariſe from almoſt any Subject, may be numbered: and, in truth, I have made it ſo large to oblige ſome curious Perſons by way of Speculation, more than for any abſolute Neceſſity there is for it.

There is likewiſe another Way of expreſſing Numbers by Letters, uſed by the *Roman* Numeriſts, and in Accounts in our antient Records, *&c.* and may be termed Literal Numbers, or the Numeral Letters; which are theſe:

I I

I	1	XXV	25	CC	200
II	2	XXX	30	CCC	300
III	3	XXXV	35	CCCC or CD	400
IV	4	XL	40		
V	5	XLV	45	CDXC	490
VI	6	L	50	D	500
VII	7	LV	55	DC	600
VIII	8	LX	60	DCC	700
IX	9	LXX	70	DCCC	800
X	10	LXXX	80	CM, or DCCCC	900
XI	11	XC	90		
XII	12	C	100	M, or CIƆ	1000
XV	15	CX	110		
XX	20	CXLV	145	CIƆD	1500

CIƆDCCXX = 1720.

SECT. II. *Of Addition of Intire Numbers.*

1. ADDITION is a Rule whereby ſeveral (even tho' never ſo many) Sums are aggregated and contracted into one Sum, called the Total.

2. If the Numbers to be added be of one Denomination, or be abſtract Numbers, it is properly called Single Addition; and in this caſe let the Name of the thing you add be what it will, you muſt for every 10 in Units place carry 1 to the Tens place, and for every 10 in that place, carry 1 to the Hundreds; for every 10 in the Hundreds place, 1 to the Thouſands place, and ſo forward, if you have never ſo many places; becauſe in Numeration it has been ſhewn, that 10 in Units is 1 in Tens place, &c.

EXAMPLES.

L. Sterling.	*lb.* Weight.	Ells.	Hundr. Weight.	Barrels.
17642				
3769	9367		9763	1764
8458	14239	9762	872	86
4764	4763	2997	939	9
13	8324	3862	26	732
327	8	8764	147	876
97864	0934	329	9183	384
132837	37635	25714	20930	3851 Sums

	l.	*s.*	*d.*
	7463	19	7½
	421	—	10
	833	4	6¾
	1008	17	—
	94	12	11¼
	9	8	4
	363	10	10½
	1371	—	— ¼
	432	13	4
	92	—	11¾
	140	10	9¼
	832	—	10
	721	19	8
	9367	3	11
	431	10	10¾
	210	11	6
	182	11	11
	32	16	8¼
	1003	—	7
	147	1	11
	82	—	10
	312	15	9½
	1714	3	11
Total	27168	17	8¾

3. If the Numbers to be added are Contract, or have several Denominations, which depend on the Value of the same thing, or on the same Sum received and paid, *&c.* as the Column of Pounds, Shillings, Pence, and Farthings in the Margin; you must begin with the Farthings, and for every 4 of them carry 1 to the Pence, for every 12 Pence carry 1 to the Sillings, for every 20 Shillings carry 1 to the Pounds, *&c.* But because I would not have you point at every 12 in the Pence place, which fullies and fouls Books that are kept clean; I have therefore inserted the following short Table, that you may see how many Shillings is in any Column of Pence that can well be supposed to come to hand; which may serve till you have learned how to divide by 12: for then you need not the Table.

The TABLE used in Addition of Pence.

d.	*s.*	*d.*	*s.*	*d.*	*s.*	*d.*	*s.*	*d.*	*s.*
12=	1	60=	5	108=	9	156=	13	204=	17
24=	2	72=	6	120=	10	168=	14	216=	18
36=	3	84=	7	132=	11	180=	15	228=	19
48=	4	96=	8	144=	12	192=	16	240=	20

Note,

Note, That ¼ is 1 Farthing, or one 4*th* part of a Penny (or other thing) ½ is one half of a Penny, *&c.* ~~and ¾ is 3 fourth parts~~ of a Penny or other thing that is placed before it.

And in consequence you find the Sum of the Farthings in the last Example to be 19 (or 4 Pence 3 Farthings): Put down the ¾ (as you see in the Sum) and carry the 4 *d.* to the Units place of Pence, saying 4 and 1 is 5, and 9 is 14, and 1 is 15, and so on, you'll find when you come to the top of the Series 78 Pence. Put down the 8 on a piece of (by or) waste Paper, and carry the 7 to the Tens place of Pence, saying 7 and 1 is 8, *&c.* till you come to 18, (at the top of the Column) which put down to the 8, and it makes 188 Pence; which by the little Table is 15 *s.* and 8 over: Therefore put down the 8 Pence, and carry the 15 *s.* to the Units place of Shillings, saying 15 and 3 is 18, and 5 is 23, *&c.* to the top of the Column, when the Sum is 77. Put down the 7, and carry 7 to the Tens place, saying 7 and 1 is 8, and 1 is 9, *&c.* which at the top you find 19. Put down the odd 1, (which makes the 7 17 *s.*) and carry half of 18 or 9 to the Pounds, which sum up as by the Example of 1 Denomination, and you find the Sum total 27168 *l.* 17 *s.* 8 *d.* ¾.

4. Hence it may be inferred, that any Numbers may be added, tho' of divers Denominations, as tho' they were of one Denomination or Name. So the Sum of the several Columns of Money foregoing, is 27159 *l.* 182 *s.* 184 *d.* 19 *qrs.* or *l.* 27168 : 17 : 8¾. for the 19 Farthings any one knows is 4 *d.* ¾, 184 Pence by the little Table is 15 *s.* 4 *d.* and 4 *d.* in the Farthings is 15 *s.* 8 *d.* 182 *s.* is *l.* 9 : 2 : — and the 15 *s.* in the Pence is *l.* 9 : 17 : — Put down the 17, and add the 9 *l.* to 27159, makes the Total *l.* 27168 : 17 : 8¾.

This is much the best way; done without pricking and carrying from one Denomination to another, which one is apt to forget when he has a mind to run a Sum over a second time.

	l.	*s.*	*d.*	*qrs.*
	27159 :	182 :	184 :	19
Add			4 :	3
		15 :	4 :	—
	9 :	2 :	—	
	l. 27168 :	17 :	8 :	3

And by one and the same Method you add all kind of Denominations of Weight, Measure, Money, *&c.* And for reducing the Sums properly, (as that above is done) you will easily see how to do that by the little Table following of the Quarters of Hundreds, *&c.* in the Sum of the Pounds, *&c.*

TABLE 1. *Of Averdupois Weight.*

Ton	Hund.	qr.C.	Pounds	Ounces	Drams
1	20	80	2240	35840	573440
	1	4	112	1792	28672
		1	28	448	7168
			1	16	256
				1	16

TABLE 2. *Of English Coin.*

Farth.	Penny	Groat	Shilling	d.	Noble	Angel	Mark	Pound
4	1							
16	4	1						
48	12	3	1					
320	80	20	6 :	8	1			
480	120	30	10		$1\frac{1}{2}$	1		
640	160	40	13 :	4	2	$1\frac{1}{3}$	1	
960	240	60	20		3	2	$1\frac{1}{2}$	1

These Tables are put into the most easy and useful Method that I could think of, and are more especially adapted for reducing *Averdupois Weight (by the first Table) and English Coin (by the second Table)*, from any one Denomination to another, with one Multiplication or Division.

For the more easy adding this or any other Example of Averdupois greater Weight, I have inserted the following little *Table*; which shews the *Quarters* of Hundreds in a Column of Pounds.

A Table

A Table of Quarters of Hundreds in a Column of Pounds.

℔	qr.C.	℔	qr.C.	℔	qr.C.
28	1	140	5	252	9
56	2	168	6	280	10
84	3	196	7	308	11
112	4	224	8	336	12

An Example of adding Averdupois greater Weight.

	Ton.	C.	qr.	℔
	47	3	2	27
	192	19	—	12
	43	18	3	24
	973	—	2	14
	9	15	1	15
	47	18	3	—
	55	9	2	23
	34	16	2	25
	84	19	3	17
	179	16	2	12
	84	8	1	25
	73	18	3	24
	234	17	—	12
Sum	2054	176	24	230
			8	6
Ad.		6	0	—
	8	16	—	
	2063	4	0	6

In the Example last above, having added each single Series or Row of Figures (as by the Rules for one Denomination foregoing) I find the Sum 2054 Ton, 176 Hundred, 24 Quarters, and 230 Pounds.

Then (as in the Example before) I must reduce this Sum properly, which is done thus: By the little Table last inserted, 'tis plain that in 230 ℔ there are 8 qrs. and 6 ℔ In the 24 Quarters are 6 Hundred; and in the 176 Hundred (by taking half except Units place, and adding the odd 10 to the Units) are 8 Ton, 16 C. the Sum of which is Ton 2063 : 4 : 0 : 6, done without pricking, or any Charge to the Memory. But for such as can do Division, the last little Table will not be of such necessary Use as it is to others.

A Table shewing the Ounces in a Column of Drams; or ℔ in that of Ounces.

16=1	80=5	144=9
32=2	96=6	160=10
48=3	112=7	176=11
64=4	128=8	192=12

The next Example is done thus:

Having summed up the Units, and then the Tens place of Drams, I find the Sum 191; the Ounces in like manner 177, and the Pounds 3234.

And by the little Table above (which any one may make or enlarge by adding 16 to itself, &c.) I find that the next Number to and less

less than 191, is 176, against which stands 11 Ounces; and 191 is 15 Drams more, therefore I put 11 *Oun.* 15 *dr.* down as you see. And the Sum of the Ounces being 177, I look for that in the Table, and find the next and less Number thereto to be 176; against which is 11. So that I put down ℔ 11: 1 *Oun.* as you see in the Margin, and as in the foregoing Examples; which 11 added to the Pounds, and the 1 to the 11 Ounces, &c. the Sum reduced by the little Table is ℔ 3245 : 12 : 15.

An Example of adding Averdupois lesser Weight.

℔	Oun.	dr.	
764	15	15	
73	13	12	
92	10	13	
173	—	7	
39	14	11	
91	12	10	
475	9	8	
39	15	12	
99	11	11	
732	4	14	
74	14	9	
86	11	13	
37	7	10	
75	13	14	
32	10	8	
124	4	3	
53	11	14	
176	4	7	
℔ 3234	177	191	
	11	15	Add.
11	1		
℔ 3245	12	15	Sum.

And thus I have given you sundry Examples and Rules, altogether new, for adding large Numbers of several Denominations, as tho' they were but one Denomination, without carrying any thing from one Name or Denomination to another: which new Method I take to be every way preferable to the common, especially where there are large Examples to be added of several Denominations. I shall give you farther Tables, whereby little Tables may be made for Addition; or where from Substraction and Reduction of any thing may be performed.

TABLE 3. Wine-Measure.

Ton	Pipes	Hogsh.	Tiertian	Gallons	Quarts	Solid Inches.
1	2	4	6	252	1008	58212
	1	2	3	126	504	29106
		1	1½	63	252	14553
			1	42	168	9702
				1	4	231
					1	57¾

TABLE 4.

TABLE 4. *Beer-Measure.*

Pints	Quart	Pottle	Gallon	Firkin	Kilderkin	Barrel
2	1					
4	2	1				
8	4	2	1			
72	36	18	9	1		
144	72	36	18	2	1	
288	144	72	36	4	2	1

Note, Ale-Measure hath 32 Gallons to the Barrel.

TABLE 5. *Dry Measure.* (36 Bushels to the Chaldron of Coals.)

Last	Weys	Quarters	Bushels	Pecks	Gallons	Pottles	Quarts	Pints
1	2	10	80	320	640	1280	2560	5120
	1	5	40	160	320	640	1280	2560
		1	8	32	64	128	256	512
			1	4	8	16	32	64
				1	2	4	8	16
					1	2	4	8
						1	2	4
							1	2

TABLE 6. *Long-Measure.*

Barley-Corns.	Inch	Foot	Yard	Pole	Furlong	Mile
3	1					
36	12	1				
108	36	3	1			
594	198	$16\frac{1}{2}$	$5\frac{1}{2}$	1		
23760	7920	660	220	40	1	
190080	63360	5280	1760	320	8	1

TABLE 7.

TABLE 7. Square-Measure.

Square Mile	Square Acres	Square Rods	Square Poles	Square Yards	Square Feet	Square Inches
1	640	2560	102400	3097600	27878400	4014489600
	1	4	160	4840	43560	6272640
		1	40	1210	10890	1568160
			1	$30\frac{1}{4}$	$272\frac{1}{4}$	39204
				1	9	1296
					1	144

TABLE 8. Of Solid Measure.

Solid Inches	Feet Solid	Solid Yard	Solid Pole	Rod	Acre	Mile
1728	1					
46656	27	1				
7762329	$4492\frac{1}{8}$	$166\frac{375}{1000}$	1			
1963885176	$1136507\frac{625}{1000}$	$42092\frac{875}{1000}$	253	1		
157110081408	9092061	336743	2024	8	1	
254358061056000	147197952000	5451776000	32770584	129528	16191	1

TABLE 9. Of Troy-Weight.

Pounds ℔	Ounces ℥	Pennywt. *dw.*	Grains *gr.*
1	12	240	5760
	1	20	480
		1	24

TABLE 10. Apothecaries Weight.

Grains *gr.*	Scruple ℈	Dram ʒ	Ounce ℥
20	1		
60	3	1	
480	24	8	1

TABLE

TABLE 11. *Of Time.*

Year	Weeks	Days	Hours	Minutes	Seconds
1	$52\frac{2}{71}$	$365\frac{1}{4}$	8766	525960	31557600
	1	7	168	10080	604800
		1	24	1440	86400
			1	60	3600
				1	60

TABLE 12. *Of Motion.*

Seconds	Minute	Degree	Sine	Quadrant	Circle
60	1				
3600	60	1			
108000	1800	30	1		
324000	5400	90	3	1	
1296000	21600	360	12	4	1

The Tables foregoing are ſo eaſy, as to need little Explanation: I have put them in two Methods, a ſmall matter differing. Table 1. which is of Averdupois Weight, is to be thus read; 1 Ton is 20 Hundred, 80 Quarters of C. 2240 Pounds, 35840 Ounces or 573440 Drams: Alſo 1 C. is 4 Quarters, 112 ℔. 1792 Ounces, *&c.* reading from the right towards the left hand, all which is uſeful in Reduction.

And the ſame is to be obſerved in the ſecond ſort of Table, only here you read the contrary Way, from the right hand toward the left: As 1 Pound Sterling is $1\frac{1}{2}$ Marks, 2 Angels, 3 Nobles, 20 Shillings, 60 Groats, 240 Pence, 960 Farthings. One Mark is $1\frac{1}{3}$ (or 1 Angel and 1 Third) 2 Nobles 13 *s.* 4 *d.* 40 Groats, 160 Pence, *&c.* And the Numbers above ſhew that 4 Farthings is 1 Penny, 4 Pence 1 Groat, 3 Groats 1 *s.* 6 *s* 8 *d.* = 1 Noble, *&c.*

As to the Proof of Addition, it is either performed by Addition or Subſtraction.

Thus the Sum of the 3 upper Lines added to the Sum of the 2 lower, is equal to the Sum Total: For the Sum of any Parts added to the rest of the Parts, must give the Sum Total of all the Parts which make the Whole.

Or the Sum of the two lowermost deducted from the Sum Total, leaves the Sum of the greater, or of the three uppermost Numbers.

	9876
	5432
	1234
	567
	89
Sum Total =	17198

And thus I hope I have laid down such plain and copious Rules, *&c.* as will be useful to the young Reader in adding Money, Weight, Measure, *&c.*

Sum of the 3 uppermost	16542	
Sum of the 2 lower Lines =	656	
Sum Total, or Sum of the Sums —— ——	17198	Proof.
The lesser deduct	656	
Leaves the greater =	16542	Proof.

When large Sums of Money are (in Records, *&c.*) required to be expressed Literally, or in Numeral Letters, they may be wrote, *&c.* thus;

l.		s.		d.	
MDCCXXXVI	:	VI	:	IX	ob
M^{10}DCCCCXXIX	:	X	:	II	
M^{8}CCXCIX	:	XI	:	IX	
M^{20}DLXXXVII	:	X	:	IX	ob
M^{510}CLII	:	VI	:	VIII	
M^{19}DCCXLV	:	XIX	:	VI	ob
M^{60}CCXXXI	:	XI	:	IV	
M^{631}DCLXXXII	:	XVII	:	——	ob
viz. 631682	:	17	:	——	½
Sums Total					

or thus, more easy for Addition thereof.

l. M. C. X.		s.		d.	
I, VII, III, VI	:	VI	:	IX	ob
X, IX, II, IX	:	X	:	II	
VIII, II, IX, IX	:	XI	:	IX	
XX, V, VIII, VII	:	X	:	IX	ob
DX, I, V, II	:	VI	:	VIII	
XIX, VII, IV, V	:	XIX	:	VI	ob
LX, II, III, I	:	XI	:	IV	
DCXXXI, VI, VIII, II	:	XVII	:	——	ob
or 631682	:	17	:	——	½
Sums Total					

To add the first Column 'tis easy till you come to the Pounds, and there add together 1st all under 10, to each Ten there add all Tens

Tens under 100; to those add what is under 1000, and the Thousands carry'd to the rest.

Note, The M, C, and X are Thousands, Hundreds, and Tens, the Titles of the Columns they stand over, as to place.

SECT. III. *Substraction of Intire Numbers.*

THIS is a Rule whereby the Excess or Difference between two Numbers is discovered; that is, we find out by this part of Arithmetic how much one of two Numbers given, is greater or less than the other. Thus if 15 be compared with 20, we find it lesser by 5.

Hence it appears, that in Substraction there are always 2 Numbers given, from whence a third is discovered.

Of the 2 Numbers given, the greater I call the *Compound Number*, (as being composed of the lesser Number and the Difference between those given.) The lesser I call the *Subtrahend*, it being that which is to be taken from or drawn out of the greater. And when that is done, there ariseth the third Number, which may be termed the Remainer, Excess, or Difference between the Numbers given, and is the Number sought for in this Part of the Art of Numbering.

As in Addition, so in Substraction the Numbers given are either of one simple Denomination, or else they are one or both of several Denominations or Names; for the Numbers given are Abstract or Concrete.

I shall next illustrate what is said, by examples of one and more Denominations in the common Way of Working; and 2*dly*, in another.

Examples of one Denomination, or of Abstract Numbers.

Example 1.	*Example* 2.	*Example* 3.
Lent *l.* 71032	Borrowed *l.* 19721	Received *l.* 9211
Received *l.* 9735	Paid *l.* 8957	Disbursed *l.* 1973
Remainer *l.* 61297	Remainer *l.* 10764	Rem. = *l.* 7238

Examples of several Denominations, (or of Contract Numbers.)

Example 4.	*Example* 5.
Gained *l.* 1192 : 12 : 3	Debtor *l.* 10132 : 10 : 4¼
Lost *l.* 287 : 14 : 7	Creditor *l.* 975 : 15 : ¼
Remaineth gain'd *l.* 904 : 17 : 8	Remaineth Dr *l.* 9156 : 15 : 3½

Example 6.

Received as an Agent or Factor, to be applied and disbursed as *per* Order of my Employer—	*l.* 9000 : —: —
Paid *A. B.* for Goods bought —— *l.* 2072 : 13 : 4	
For a Ship with Rigging ——— 936 : 19 : 2	
For Custom to the King ——— 97 : 15 : 3¼	
For Insurance ——— ——— 72 : 3 : 6	
For Freight ——— —— ——— 100 : 11 : 9¾	
In all ——— ———	3280 : 3 : 1
Which Sum of the Payments being deducted from the *l.* 9000, the Remainer (in the Factor's hands) is ——— ——— ——— ———	*l.* 5719 : 16 : 11

Example 7.

	Weight			Value		
	C.	*qr.*	℔	*l.*	*s.*	*d.*
Bought of *A. B.* Sugars	193	3	25	386	19	4
of *C. D.* more	323	3	17	532	18	6¼
Bought in all	517	3	14	*l.* 919	17	10¼

	Weight				Value		
	C.	*qr.*	℔		*l.*	*s.*	*d.*
Sold *E. F.*	25	1	14	for	59	17	6
to *G. H.*	126	3	25	at	315	12	10
to *J. K.*	94	2	14	at	235	17	6¾
to *L. M.*	87	3	20	at	180	13	4
Sold in all ——— ———	334	3	17	at	792	1	2¾
Remains unsold *C.*	182	3	25		127	16	7½
Which unsold Goods cost me ——— ———					364	18	10¼
From which last Sum deducting the next above it, the Remainer is gained (already) by this Account ——— ——— ———					237	2	2¾

Directions for performing the foregoing Operations.

IN the first Example you are to take the lower Line from the upper, or the lesser Number 9735 from 71032. Begin at Units place, and say 5 from 2 cannot be, but 5 from 10, which you must borrow, and add to 2; that is, 5 from 12, and there remains 7. Then having put the 7 under the Line, say 1 borrowed and 3, (the Figure in Tens place of the Subtrahend) is 4, from 3 (in the compound

pound Number) cannot be; but from 13 (borrowing 10 as before) leaves 9. Which put down, as you see, and say 1 borrowed and 7 (the Figure in Hundreds place of the Subtrahend) is 8 from 10 borrowed (because you cannot take 8 from 0) and there rests 2. One borrowed and 9 is 10 from 1 (in the Thousands place of the compound Number) cannot be, but 10 from 11 leaves 1, which put under, as you see. And lastly, 1 borrowed from 7 in the upper Line, and the Remainer is 6. So the whole Sum remaining is 61297. And thus any Operation of one Denomination is performed.

Then for several Denominations I shall instance, 1*st*, in the fifth Example. You must always begin with the least Name, here Farthings; and say 3 Farthings from 1 cannot be, but borrowing 1 Penny or 4 Farthings, add to the 1 Farthing makes 5; so 3 from 5, and there rests 2 Farthings: put under as in the Example, and say 1 Penny borrowed from 4 Pence in the upper Line, and there rests 3 *d.* Then say 15 *s.* from 10 *s.* cannot be; but 15 from 20 *s.* which you borrow, and the 10, *viz.* from 30, and there resteth 15. Then say 1 borrowed and 5 is 6, from 12, rests 6 Pounds, *&c.* as in the former Examples.

The sixth Example is performed by first adding together the Sums laid out, and then deducting that Sum, which is *l.* 3280 : 3 : 1 from the 9000. Thus 1 *d.* from 12 you borrow, leaves 11 *d.* 1 you borrowed and 3 *s.* is 4 from 20 *s.* which borrow, and the rest is 16; which put under and deduct the 1 borrowed from 10, *&c.* as before.

Lastly, in the seventh Example, you are to deduct the Hundreds, Quarters, and Pounds sold from those bought: to perform which, you must say 17 Pounds from 14 *lb.* cannot be, but 17 from 28 *lb.* which you borrow (that being a Quarter of a Hundred, which is the next Denomination) leaves 11; which added to the 14 *lb*, makes the 25 which you see. Then 1 Quarter of a Hundred borrowed and 3 is 4, from 7 (that is, 4 Quarters or 1 Hundred borrowed and added to the 3 Quarters standing in the upper Line) and the rest is 3 Quarters; which put in the Remainer, saying 1 (that is, 1 Hundred weight) borrowed and 4 (in the Sum of the Sales) is 5, from 7 in the Hundreds bought, leaves 2; which put down, and so proceed, as in substracting one Denomination: and the quantity of Sugar remaining unsold you find to be *C.* 182 : 3 : 25. And the Value of what is sold deducted from the whole Value bought, the Remainer is *l.* 127 : 16 : 7½ (which is the Sum only

wanting

wanting to make up the Cost of all that was bought.) Which Sum remaining deducted from the prime Value of what Sugar remains unsold, *viz.* 182 *C.* 3 *qr.* 25 *lb.* which costs *l.* 364 : 18 : $10\frac{1}{4}$ the remaining Cash is *l.* 237 : 2 : $2\frac{1}{4}$ which is got by this Trading, thus far.

Note, Tho' the Subtrahend, as here *l.* 127 : 16 : $7\frac{1}{2}$ stands over the Compound Number *l.* 364 : 18 : $10\frac{1}{4}$, it is as easily deducted as if the lesser Number stood under the greater, as is most usual.

By the Rules and Examples above, I suppose any one may perform Substraction after the common way.

But the Work may be effected without saying *such a Figure from such a one cannot be*, and 1 *borrowed*, &c. as in the Margin.

		10	10	10	10		
		4	6	1	2	3	10
From	1	5	7	2	3	4	2
Take		8	9	5	6	5	7
Rests =		6	7	6	6	8	5

For where any Digit in the compound Number is less than that standing under it in the Subtrahend, mark the next toward the left-hand with a Point, and put over it a Digit, which is 1 less, as here over 4 in the Number given put 3; over 3, 2, &c.

Then where the Figures put over the pricked ones (or those given) are still less than those respectively in the Subtrahend, place 10 partly over, but a little toward the left hand; then you have the very Numbers at large, from whence Substraction is to be made. Thus 7 from 12 rests 5; 5 from 13 rests 8; 6 from 12 rests 6; 5 from 11 leaves 6; 9 from 16 leaves 7; and 8 from 14 leaves 6.

Now I have only inserted the Figures over those pointed, and the Tens to explain the Rule; but 'tis certain, the Work may be as easily performed mentally, by supposing every Figure as aforesaid less by the 1 (supposed 10) which is borrowed from the next towards the left hand of the Number given to make Substraction from. For whenever you borrow 10, that 10 is presumed to be taken from the next Figure towards the left hand; and of consequence therefore that is 1 less, (1 in any Series being 10 in another next it toward the right hand.) Thus in the Example, 7 from 12 rests 5; where the one Ten borrowed makes the Figure in Tens place a Unit less, *i. e.* the 4 is but 3. And when you say 5 from 13 rests 8; this 1 borrowed is actually taken from the next Digit 3, and leaves it but a 2, and so 2 becomes but a 1, 7 a 6, and 5 a 4, which is all supposed as you perform the Work, without putting down any thing but the Remainer.

So

So alſo in divers Denominations, the Unit borrowed being ſuppoſed one of the next ſuperior Denomination, muſt leave that a Unit leſs, as is explained in the Margin. Where 2 *s.* 6 *d.* 1 *qr.* in the upper of the two Lines given, being leſs than 17 *s.* 9 *d.* $\frac{3}{4}$ in the lower Line; therefore 4 Farthings taken from the 6 *d.* makes the $\frac{1}{4}$ $\frac{5}{4}$, and 5 *d.* reſteth; 12 *d.* borrowed from the 2 *s.* and put over the 5 *d.* makes 17 *d.* and 1 *s.* reſteth; 20 *s.* taken from the 3 *l.* makes the ſaid 1 *s.* to be 21 *s.* and 2 *l.* reſteth, &c. as in one Denomination; then I have it plain to my ſight, that $\frac{3}{4}$ from $\frac{5}{4}$ leaves $\frac{2}{4}$ or $\frac{1}{2}$; 9 *d.* from 12 and 5 or 17 *d.* leaves 8; 17 *s.* from 21 leaves 4 *s*; 7 *l.* from 12 *l.* leaves 5; 9 *l.* from 10 leaves 1 *l.* and 3 *l.* from 6 *l.* leaves 3, without borrowing and paying. Which Method with Uſe will make the Work eaſier and ſhorter than the common Way; I mean, when only the two Sums given are wrote down, and the reſt performed in the Mind, as Example.

	l.	*s.*	*d.*
	10;10	20	12
	6,0, 2	1	$5\frac{5}{4}$
Acquired or earned	7 1 3	2	$6\frac{1}{4}$
Expended =	3 9 7	17	$9\frac{3}{4}$
Saved, or put in Bank	3 1 5	4	$8\frac{2}{4}$ or $\frac{1}{2}$

This is done without the leaſt mentioning, borrowing or paying, and with more Eaſe and Expedition; and may ſerve alſo to ſhew the Nature and Reaſon of the common Way of Working.

	l.	*s.*	*d.*
The Rent-Roll of my Eſtate (ſuppoſe) is *per annum* ——	1765	12	3
Repairs, Taxes, &c. deduct.	279	15	9
My clear Eſtate *per ann.* =	1485	16	6

The Proof of Subſtraction is either *by Addition* or *Subſtraction*: For in the laſt foregoing Example, the Remainer *l.* 1485 : 16 : 6 being added to the Subtrahend, *l.* 279 : 15 : 9, the Sum is the compound Number given, or *l.* 1765 : 12 : 3; which proves the Operation to be right.

Or by Subſtraction. If from the *l.* 1765 : 12 : 3 you take the Remainer *l.* 1485 : 16 : 6, there will then remain *l.* 279 : 15 : 9=the Subtrahend given.

SECT.

SECT. IV. *Multiplication of Intire Numbers.*

MULTIPLICATION is a Rule which ſhews the Number of Units produced by making one aſſigned Number any Multiplicity of times itſelf.

Or 'tis that Part of Arithmetic of ſuch admirable Extent, that no Number or Quantity can be ſo great, but another greater may be diſcovered: For by it not only the Number of ſingle Sands that would compoſe an Heap as big as our Earth, but even one to extend to the ſtarry Firmament are within the limits of this part of the Numeric Art to compute.

It is an Abridgment of the Work of Addition, performing that in a Minute, that Addition would require ſome Hours to do.

Thus if 8766 were to be multiplied by 20000, (or to know how many Hours are in 20000 Years) 'tis only to double 8766, which is 17532, and to that place the four Cyphers toward the right hand will make it 175320000 the Anſwer, (which is done in leſs than half a Minute.) But how long it would require to put down 8766, 20000 times; or 20000, 8766 times; and then add all thoſe Numbers together; I leave the Reader to judge, and conſequently of the Excellency of this Rule, which is juſtly called Multiplication.

As in the laſt Section, ſo in this Part there are two Numbers given to find a third: To inſtance, as above, 8766 and 20000 are given, and 175320000 is the Number produced by or reſulting from the Multiplication.

Of theſe 3 Numbers 1 (commonly the greater of the 2 given) is called the Multiplicand, the other of thoſe given is the Multiplier, and both together they are called Factors; and the Number ariſing therefrom after Multiplication, is called the Product, or (in Geometrical Operations) the Rectangle.

The Relation theſe three Numbers have to each other, is, That the Product containeth either of the Factors ſo often as the other Factor contains a Unit: and conſequently

As 175320000. to 8766 : : ſo 20000. to 1. Or

As 175320000. to 20000 : : 8766. 1. *&c.* But this by the by, till we arrive at the Rules of Proportion.

The whole Buſineſs of Multiplication conſiſts in theſe three things.

1. In knowing mentally and readily what any two Digits multiplied together produce.

2. In

2. In the right placing those Products. And,

3. In collecting those particular Products into one general Product.

Hence it follows, that a Table of the Products of one Digit by another be composed, which, before you can proceed farther in this most pleasant and useful Science, must be learned by heart: thus

The first Column is what is absolutely necessary to get by heart.

The second Column for the most part, that is, in multiplying by 10 and 11, does not charge the Memory: for to multiply any Number by 10, is only to place a Cypher to the right hand of the Multiplicand, (for 1, tho' a Number, as augmenting and diminishing another by Addition and Substraction, yet it neither multiplies nor divides.)

And to multiply any Digit by 11, is only to repeat the Digit as in the Example. And I have added the Multiplication of the Digits by 12, because that being got by heart, saves the trouble of making a whole Line, and of adding two Lines together: for when you have that by heart, you can multiply or divide by 12 (a Number much used, as being the Pence in a Shilling, the Inches in a Foot, *&c.*) as by one single Digit.

The Multiplication Table.

3 times	3 is=9	10 times	2= 20
	4=12		3= 30
	5=15		4= 40
	6=18		5= 50
	7=21		6= 60
	8=24		7= 70
	9=27		8= 80
4 times	4=16		9= 90
	5=20	11 times	2= 22
	6=24		3= 33
	7=28		4= 44
	8=32		5= 55
	9=36		6= 66
5 times	5=25		7= 77
	6=30		8= 88
	7=35		9= 99
	8=40	12 times	2= 24
	9=45		3= 36
6 times	6=36		4= 48
	7=42		5= 60
	8=48		6= 72
	9=54		7= 84
7 times	7=49		8= 96
	8=56		9=108
	9=63		10=120
8 times	8=64		11=132
	9=72		12=144
9 times	9=81		

The wonderful Productions arising from the placing Numbers, are many and surprizing: I shall here insert an Instance, where, by placing only 9 Figures to the best advantage about 5 Digits given, constitutes all the Parts of the common Table that are absolutely necessary to be inserted for the more apprehensive part of Readers.

A most brief Multiplication Table, viz.

Two of these given to be multiplied, or one to be squared.

Two of these multiply, standing against the Numbers given.	1	9	40	Two of these add, respecting the given Numbers in the middle.
	2	8	30	
	3	7	20	
	4	6	10	
	5	5	0	

The large Table is so plain, that it needs no Explanation, as shewing that 8 times 8 is 64, 8 times 9 is 72, and the like of the rest.

And this Table is very near as easy; for you must always look for the Digits to be multiplied, in the middle Column: As suppose I would know what 8 times 6 makes; against 8 in the middle Column is 30 towards the right hand, and against 6 in the middle is 10 in the right-hand Column, the Sum of which is 40; and those standing against the others, in the left-hand Column, are 2 and 4, which multiplied together, make 8: so that 8 times 6 is 48.

And to square any of the middle Numbers, as suppose 7, double 20, which stands against it on one side, and multiply that by itself, which stands against 7 on the left-hand side; so 20 doubled is 40, and 3 squared makes 9: so that 7 times 7 is 49, &c.

So much for the Tables: I shall now proceed to shew how the Operation of Multiplication is performed. And,

2. The Application of the Rules in solving Questions, which may serve, instead of Reduction descending, to shew the Use of Multiplication.

Proposition

Proposition 1. To multiply a Number by another, which consists but of one place.

Example 1.	*Example* 2.
Multiplicand 79533	Multiply—— 175945
Multiplier 8	by 9
Product 636264	Answer = 1583505

Prop. 2. To multiply any Number by another consisting of two, three, or more places. A shorter way for Example 1. See *Multiplication of Decimals.*

Example 1.	*Example* 2.	*Example* 3.
Multiply 74963	47625	76472
by . . . 75	357	2475
374815	333375	382360
524741	238125	535304
	142875	305888
		152944
Product = 5622225	Product = 17002125	Product = 189268200

Prop. 3. To multiply when one or more Cyphers are in the Multiplier, but not at the end.

Example 1.	*Example* 2.
Multiply 57234	97532
by 5203	32004
171702	390128
114468	195064
286170	292596
297788502	3121414128

Prop. 4. How to do when one or more Cyphers are next the right hand of one or both of the Factors.

Example 1.	*Example* 2.	*Example* 3.
9769 } Factors.	7399 } Factors.	34000 } Factors.
1000 }	400 }	2300 }
9769000 = Product	= 2959600 = Product.	= 102
		68
		Product = 78200000

Rules for performing the Operations above.

In Example 1.] Say 8 times 3 (the Units place of the Multiplicand) is 24; put the 4 under the Line, and carry the 2 Tens to the next Product, saying 8 times 3 is 24, and 2 carried is 26; put 6 in Tens Place of the Product, and carry 2 as before, saying 8 times 5 is 40, and 2 carried is 42; put the 2 under the Line as you see, and carry 4 (Tens) saying 8 times 9 is 72, and 4 carried is 76: put the 6 under, and carry the 7, saying 8 times 7 is 56, and 7 carried is 63, which (being the Produce of the last Figure) put all down, and you see that 8 times 79533 is 636264. And so much for Directions to multiply and carry the Tens from one particular Product to another; the Reason of which, and of placing the several Products a degree towards the left-hand, I shall shew by and by.

In Example 2. *of Prop.* 2.] You find by the Rules above, that 7 times 47625 is 333375; and then you multiply 47625 by 5 (the next Figure in the multiplier) saying 5 times 5 is 25: place that 5 under Tens place of the former Product, and proceed as before; and when you come to the 3, put 5 of the Product 3 times 5, under the Tens place of the last Line or Product; and so continue, if you had never so many Figures in the Multiplier, as you see in the other Examples.

In Proposition 3.] You have Examples, that when one or more Cyphers are in the middle of the Multiplier, you must (in beginning to multiply by the Digit next the Cyphers towards the left-hand thereof) place the first Figure of the Product not under Tens place of the former Product, as before; but put it so many places extraordinary towards the left-hand as there are Cyphers: the Examples thorowly explain the Meaning.

In Prop. 4.] It is shewn, that whenever Cyphers are to the right-hand of either or both Factors, you need only to multiply by the significant Figures or Digits, and then place all the Cyphers in one or both the Factors towards the right-hand of that Product; and when 10, 100, 1000, &c. is the Multiplier, you need only to put the Cyphers towards the right-hand of the Multiplicand, and that is your Product, as by the first Example of this Proposition.

The

The Reason of the Method in the Process of the Work of Multiplication, by several Digits in the Multiplier.

I shall give an Instance in the Work of the second Example of the second Proposition foregoing; and shall explain this by three Examples depending on each other, the second of which, supposing the Cyphers left out, is an Abbreviation of the first; and the third Example shews how much the Number of Figures in the second Example is lessened by carrying the Tens from the Product of every two Digits to that of the next, &c. and not putting the whole down, which reduceth 15 Numbers into 3.

```
      Example 1. | Example 2.    Example 3.
          47625  |     47625         47625
            357  |       357           357
      ---------- | ---------     ---------
             35  |     35
            140  |     14 | 0
           4200  |     42 | 00
          49000  |     49 | 000
         280000  |     28 | 0000
                 |        |---------333375=Sum
            250  |     25 | 0
           1000  |     10 | 00
          30000  |     30 | 000
         350000  |     35 | 0000
        2000000  |     20 | 00000
                 |        |---------238125=Sum
           1500  |     15 | 00
           6000  |      6 | 000
         180000  |     18 | 0000
        2100000  |     21 | 00000
       12000000  |  12 | 000000
                 |     |-----------142875=Sum
      ---------- | ----------------------
  Sum=17002125   | 17002125 | 17002125 Sum Total
  or Product     |                     or Product
```

In the first of these Examples you have the whole Work of the Multiplication as the Figures stand, the whole being put down without Carrying or Abridgment; as 7 times 5 is 35, 7 times 20 is 140,

140, 7 times 600 is 4200, &c. Then 50 times 5, 20, 600, 7000, and 40000 make the next 5 Lines: And 300 (the third Figure in the Multiplier) times 5, 20, 600, 7000, and 40000 make the last 5 Lines or Numbers in the Operation.

In the second Example you see how the Work stands when the unnecessary Cyphers are cut off, and thrown out of the Account. And

In the third Example you see that the Sum of the Numbers separated from the Cyphers do respectively make the Numbers, and fall in the same order as in the second Example of *Prop.* 2. So that here you see not only the Reasons why the Tens are carried from one single Product to another, when 1 Line or Number is only made instead of 5: but you also see plainly the Reason why the Units place of every Line stands under Tens place of the Product or Line of Figures preceeding.

Thus much for the Theory of Multiplication, I shall next shew,

II. *The Use and Application of Multiplication.*

1. *Of Money.*] *Pounds* are reduced immediately into Shillings, Pence, or Farthings, by multiplying the given Pounds by 20, 240, or 960, as *per Table* 2. in Addition.

Example, in 7873 *l.* how many Shillings? &c.

	Example 2.	*Example* 3.
7873 Pounds } Multi-	*l.* 7873	7873 *l.*
20 Shill. in *l.* 1. } ply.	240 *d.* in *l.* 1.	960 *qr.* in *l.* 1.
Sh. 157460 = the Answer.	31492	47238
	15746	70857
	Pence for Answer 1889520	Farth. 7558080 Answer.

Note, These Shiliings may be reduced into the Pence by multiplying them by 12; and these Pence into the Farthings by multiplying by 4. And the like of any other Pounds, Shillings and Pence.

Quest. 2. In *l.* 7873 : 18 : 11¼ how many Farthings?

Multiply 7873 by 960, as in the third Example; to the Product whereof add 911 = the Farthings in 18 *s.* 11 *d.* ¼ and the Sum is the Answer, as *per* Margin.

l. 7873 : 18 : 11¼
960
47238
70857
7558080
Add . . . 911 *qr.* in 18 *s.* 11 *d.* ¼
Sum = 7558991 = Answer.

2. *Of*

2. *Of Averdupois Weight.*] By the first Table in Addition it appears that Tons are reduced into Hundreds by multiplying them by 20; into Quarters of Hundreds by 80; into Pounds by 2240; into Ounces by 35840, &c.

Example, In 85 Ton how many Ounces?

Ounces in 1 Ton = 35840
Tons given 85

179200
28672

Answ. Oun. 3046400

3. *Of Liquid Measures.*] In 157 Ton of Wine, how many Quarts? By the third Table in Addition, you see that in 1 Ton are 1008 Quarts: therefore the Tons being multiplied by 1008, gives the Product 158256 Quarts.

157 Ton.
1008 = Quarts in 1 Ton.

1256
157

158256 Quarts, Answer.

4. *To reduce Time.*] How many Minutes may we say it is since the Creation, supposing the Years, according to Sacred Chronology, to be 5716?

By the eleventh Table in Addition, there are in 1 Year — — 525960 Minutes, 5716 Years } Multiply.

315576
52596
368172
262980

3006387360 Minutes for Answer.

5. *To reduce Square or Superficial Measure.*] How many Diamonds, of 16 in a square Inch, will pave the Globe we live on, supposing it a compleat Sphere or Globe all of Earth?

The Ambit (or Circumference) of the Earth, according to Mr *Norwood*'s measured Degree of near 70 *English* Miles in a Degree, is Miles — — — 25040

A fourth of which is Miles 6260

Which multiplied by the Earth's Diameter = 7967

4382
3756
5634
4382

The Product is the superficial Content of a Circle, whose Circumference is that of the Earth, *viz.* — 49873420 Product

Product brought over — — —	49873420
	4
Which multiplied by 4, produceth the convex Area of the Earth (or whole superficial Content) in Square Miles — — — — — — —	199493680
Which multiplied by the square Yards in one square Mile, which are (as *per Table the 7th*) — — —	3097600
	119696208
	139645576
	179544312
	59848104
The Product is the square Yards on the Earth=	617951623168000
Which multiplied by the Inches in a square Yard, which are — — — —	1296
	3707709739008
	5561564608512
	1235903246336
	617951623168
Produceth the square Inches on the Earth's Surface — — —	=800865303625728000
Which multiplied by the Diamonds in 1 square Inch, *viz.*	16
	4805191821754368
	800865303625728
Produceth the Number of Diamonds which answers the Question. — —	12813844858011648000
	100
And the Value of the Diamonds (as aforesaid) that would pave the Globe at 100 *l.* each Diamond, is Pounds Sterling. — — — —	1281384485801164800000

This Value is numbred as the 16*th* in the Numeration Table.

By all which Examples it appears, that in this kind of Reduction, which some call Reduction descending, one general Rule is to be observed, *viz.*

Multiply any Number of any Denomination by any Number of Units of a smaller or inferior Name, that make a Unit of that given, and the Product shews how many of the later are contained in all the former.

For

For Proof of Multiplication, see at the end of *Division.*

Note, *That many new and very brief Rules and Examples for Multiplication may be seen in Decimals.*

SECT. V. *Division of Intire Numbers four several Ways.*

DIVISION is that Part of Arithmetic by which a Number given is divided, separated, or distributed into any Number of Parts assigned.

Hence 'tis plain, that it is the Reverse of Multiplication; for as that produceth an Increase, so this produceth a proportionable Decrease: And consequently the Truth of any Product of Multiplication, is proved by dividing it by one of the Factors, of which more hereafter.

Division performeth the Work of many Substractions (as Multiplication does that of many Additions) by a few Figures and in a small Time; which Substraction would require an incredible deal of Time, Figures and Paper, to effect. To instance in the former Numbers: If it were proposed to find out how many Years are in 175320000 Hours, there being 8766 Hours in 1 Year, therefore to do this by Substraction, would require that you deduct 8766 so often from 175320000, till nothing remain, which would be after 20000 Deductions. But the Work of Division shews the Answer with that Brevity which you see in the Margin.

8766) 175320000 (20000
            ———
              0

It appears from hence, That in Division there are always 2 Numbers given, and a third sought for. The Numbers given, are, 1*st*, That which is required to be divided, which we call the *Dividend.* 2*dly*, The Number of Shares, Parts, or Portions, into which the said Dividend is assigned to be parted or divided, which we call the *Divisor.* 3*dly*, The Part or Share arising from the Work of Division (being the Number sought for) we call the *Quotient*, (from *quotiens*, *i. e.* how many times) which shews how often the Divisor is contained in the Dividend.

And if the Dividend contain somewhat more than the Quotient expresseth, (but not so much more as the Divisor amounts to) that Surplusage is called the Remainer or Remainder.

So that the Relation which the two Numbers given and that sought (supposing no Remainer) have to each other, is, That the Dividend contains either the Divisor or Quotient so often as the

 other

other doth a Unit: For as a Unit to the Diviſor : : ſo the Quotient to the Dividend.

Or as a Unit to the Quotient : : ſo the Diviſor to the Dividend; and conſequently,

The Product of the Diviſor and Quotient is equal to the Dividend by 1; any of which Conſiderations proves the Truth of the Work of Diviſion.

The Operation of Diviſion conſiſteth in theſe five things.

1. In conſidering how many places toward the left hand of the Dividend the Diviſor can be taken from: If from the like Number of Places that are in the Diviſor, then,

2. To conſider how often the firſt Figure in the Diviſor can be had in the firſt of the Dividend towards the left hand. But if the Diviſor cannot be taken from the like Number of Places with itſelf from the Dividend; then to aſk how often the firſt Figure in the Diviſor (as before) can be had in the two firſt places towards the left hand of the Dividend; in either of which Caſes,

3. We put the Anſwer in the Quotient, (as you ſee in the Example.)

4. By that Figure ſo put in the Quotient, we multiply the Diviſor, and put the Product under that part marked out in the Dividend.

5. We deduct that Product from the ſaid marked Part or Dividual, and put the Remainer under a Line, as in the Examples.

Propoſition 1. To divide by any ſingle Figure or Digit.

Example 1.

```
Diviſor  Dividend   Quotient
   7 ) 87654321 ( 12522045
       ........
       7
       --
       17
       14
       --
        36
        35
        --
         15
         14
         --
          14
          14
          --
          032
           28
           --
            41
            35
            --
             6 Remainer
```

Example

Example 2.

```
8) 1743219 (217902
   ......
  16
  --
   14
    8
   --
    63
    56
   ---
     72
     72
     --
      019
       16
      ---
        3 remains.
```

Example 3.

```
9) 90123456 (10013717
   ........
   9
   ----
   0012
      9
    ---
     33
     27
    ---
      64
      63
     ---
       15
        9
      ---
       66
       63
      ---
        3 remains
```

Prop. 2. To divide by 2, 3, or more Figures in the Divisor.

Example 1.

```
21) 7334565 (349265
    ......
    63
    --
    103
     84
    ---
     194
     189
     ---
       55
       42
       --
       136
       126
       ---
        105
        105
        ---
          0
```

Example 2.

```
136) 1234567 (9077
     ....
     1224
     ----
      1056
       952
      ----
       1047
        952
       ----
         95 rests.
```

Example 3.

```
9876) 1234567 (125
        ...
       9876
       -----
       24696
       19752
       -----
        49447
        49380
        -----
           67 rests.
```

Prop. 3. To divide by a Divisor having Cyphers in the first, second, &c. places thereof.

Example 1.	*Example* 2.	*Example* 3.
350)987654(2821	12000)987654(820	1000)976432 = Answ.
70	96	Quote = 976
287	27	Remains 432
280	24	
76	Rests 3654 to divide by 12000	
70		
65		
35		
Rests 304		

Rules for performing the Work of the three last Propositions.

Those by 1 Figure under *Prop.* 1. are so easy, that any one may divide, observing the five general Rules for proceeding. To instance in Example 2. I ask how often 8, the Divisor, can be had in 17, the two first Figures to the left hand of the Dividend, (because I cannot have 8 in one place of the Dividend) the Answer is 2 times; which 2 I put in the Quotient, and say 2 times 8 (the Divisor) is 16; which placing under the 17, and taking the 16 from 17 the Remainer is 1. To which bring down the next Figure 4 in the Dividend, (and as you bring down any Figure to a Remainer, mark it with a Point under, lest you bring it down twice) and then say how often 8, the Divisor, can be had in 14, the Answer is 1; which put in the Quotient, and say 1 time 8 is 8, which put under the Deduct from 14, and the Remainer is 6; to which bring down 3, (the next Figure in the Dividend) and ask how often 8 in 63, the Answer is 7, which put in the Quotient; and so go on to the end of the Work, observing this Rule, That if after you have brought any Figure down to a Remainer, you cannot have the Divisor in it; you must put a (o) in the Quotient, and bring down the next Figure, as in the last Figure but one of the Example we are upon.

In

In the Examples to Prop. 2. It is almost as easy to divide by 2, 3, or 4 Figures as 1, observing chiefly this Rule; as in Example 2. of this Prop. I ask how often 1 (the first Figure in the Divisor) may be had in 12, (the two first Figures in the Dividend) the Answer would be 12 times; but you must never take it above 9, that is, you must never put above 9 in the Quotient at one time. And also observe, that you do not put that 9 in the Quotient till you have tried on a piece of waste Paper whether 9 times 136 do not exceed 1234, (the first part of your Dividend) which finding it less, may consequently be taken from it: So 9 times 136 is 1224, from 1234, and there rests 10; to which bring down the next Figure (5) in the Dividend, and consider how often 136 (the Divisor) can be had in 105, the Answer is (0) which put in the Quotient, and then bring down the next Figure 6 makes 1056. Then say how often 1 (the first of the Divisor) can you have in 10, (the two first of that part of the Dividend, because that has one place more in it than is in the Divisor:) If you should say, 1 may be had 9 times or 8 times in 10, you will find that the Divisor multiplied by either of those Figures will exceed 1056, and so cannot be taken from that Number. Wherefore finding that 7 times 136 will be less than 1056, I put 7 in the Quotient, and multiplying 136 by 7, it produceth 952; which take from 1056, and the Remainer is 104: To which bring down the 7 (the last in the Dividend) makes 1047, and say how often 1 (the first of the Divisor) can be had in 10 (the two first of the said 1047) the Answer is 7 times, (for it will bear no more, without making the Product of the Divisor thereby to exceed the 1047) so 7 times 136, which is 952, deducted from 1047, the Remainer is 95: So that I find I can have 136 in 1234567, 9077 times, and 95 rests. By observing these Rules you'll easily see how to perfrom the Work of this Example, and by this any other; and I have been particular in giving Rules for that end.

In the Examples under Prop. 3. When Cyphers are in the first, second, *&c.* places of the Divisor, cut off the Cyphers, and as many Figures towards the right hand of the Dividend, and divide those remaining towards the left hand of the Dividend, by those that remain towards the same hand in the Divisor: and when the Work is ended, put the Figures cut off the Dividend to the right hand of the Remainer.

And for the third Example, to divide any Number by 10, 100, 1000 10000, *&c.* You have nothing to do but only cut from the right hand of the Dividend so many places as there are Cyphers at the end of the Divisor: so those remaining towards the left hand of the Dividend.

Dividend are the Quotient, and thoſe cut off towards the right hand are the Remainer.

But there is

A Second Way of Diviſion:

Where by omitting to put down the ſeveral Products, and deducting gradually as you multiply each Digit, you do it with near half the Figures.

```
9876)1234567 (125
        ...
     24696
      49447
          67 reſts.
```

Thus the third Example under *Prop.* 2. is done as in the Margin. For I ſay how often 9 in 12, the Anſwer is 1; then inſtead of ſaying once 9876 is ſo much, and putting it under 12345, I ſay 1 time 6 from 15, (where 10 is added to the 5 in Units place of 12345) and there reſts 9, which put under, and ſaying 1 time 7 and 1 borrowed is 8 from 14 reſts 6; 1 time 8 (in the Diviſor) is 8, and 1 borrowed is 9, from 13 (borrowing 10 as before) and there reſts 4, which put under, and ſay 1 time 9 (in the Diviſor) is 9, and 1 borrowed is 10 from 14, and there reſts 2. Then to the Remainer 2469 bring the next Figure in the Dividend, *viz.* 6, and ſay how often 9 in 24, the Anſwer is 2; then proceed as before, 2 times 6 is 12 from 16 reſts 4, 2 times 7 is 14, and 1 is 15, from 19, reſts 4; 2 times 8 is 16, and 1 borrowed is 17, from 26, and there reſts 9, (you borrowing 2 to add to the 6 is 26) 2 times 9 is 18, and 2 borrowed is 20, from 24, leaves 4, or 4944; to which bring down the 7 in the Dividend, and proceed as before.

I deſire this Method laſt mention'd may be well underſtood, for that the ſubſequent Examples will be performed by it, as being the ſhorteſt and eaſieſt Way, tho' ſome prefer this

Third Way of Diviſion.

Divide 1234567 by 136.

```
     109
    10045
  1234567( 9077
   1366 6
    13 3
     1
```

In this kind of dividing, the Diviſor is put under ſo much of the the Dividend as it can be taken from; for 136 can't be taken from 123, therefore I put it under 1234, the firſt part of the Dividend, and then aſk (as in the ſecond Example of *Prop.* 2) how often 1 in the Diviſor can be had in 12 in the Dividend, the Anſwer is 9 times; which 9 put in the Quotient as uſual,

usual, and say 9 times 6 in the Divisor is 54, from 54, (borrowing 5 to add to the 4 standing over the 6, and as you mention 6 and 4, dash them out with your Pen) there rests 0; 9 times 3 in the Divisor is 27, and 5 borrowed is 32, from 33 (borrowing 3 Tens to add to the 3 standing over the 3 in the Divisor) and there rests 1, which put over the 3, as you see. Then remove your Divisor a degree towards the right hand, as in the Example, and consider how often 136 you can have in 105, (which is 10 that remained, and 5 next the 4 in the Dividend) the Answer is 0, which put in the Quotient, and taking the next Figure, *viz.* 6, into the 105, makes 1056; say how often 136 in 1056, or 1 in 10, the Answer is but 7 times; then 7 times 6 is 42, from 46, and there remains 4, which put over the 6, (dashing it out, and also the 6 in the Divisor) saying 7 times 3 is 21, and 4 borrowed is 25, from 25, (borrowing two Tens) and there rests 0, which put down over the 5, saying 7 times 1 is 7, and 2 is 9, from 10 leaves 1, which put over the Cypher as you see, dashing out the Figures in the Divisor as you multiply them, and of the Dividend as you deduct therefrom. Then remove the Divisor, and proceed as before, and as you see in the Example.

But there are two things which render this way of Division inferior in Estimation to the second, *i. e.* the repeating the Divisor for every Figure put in the Quotient; and also the cancelling the Figures, makes it very difficult to examine your Work in case of a Mistake.

The Illustration and Rationale of the Work of Division.

I shall instance in admitting that 9876543210 were to be divided by 45678.

A Fourth

A Fourth Way of Diviſion; or, The Operation of Diviſion illuſtrated.

Products of the Diviſor.	*Diviſor repeated*		
by 1 = 45678	45678	(9876543210	(200000 = firſt Quotient.
		9135600000	
2 = 91356			
3 = 137034	45678	740943210	—10000 = ſecond Quote.
		456780000	
4 = 182712	45678	284163210	—6000 = third Quote.
5 = 228390		274068000	
6 = 274068	45678	10095210	—200 = fourth Quote.
		9135600	
7 = 319746			
	45678	959610	—20 = fifth Quote.
8 = 365424		913560	
9 = 411102	45678	46050	—1 = ſixth Quote.
The Tarif.		45678	
			216221 = Sum or General Quotient.
		reſteth 372	

In the beginning of this Part or Section, Diviſion is ſaid to be the Work of many Subſtractions, and ſo it is plain: But then we are taught here how to go a nearer way to work than to deduct the Diviſor ſingly; for in the firſt Operation above, we deduct 200000 times the Diviſor at one time, (which are all the 100000's of the Diviſor that are contained in the Dividend.) At the ſecond Working we deduct 10000 times the Diviſor, (which is all the 10000's of the Diviſor that is in the Dividend.) The third time we take 6000 times the Diviſor from the Dividend, then 200 times, then 20 times, and then 1 time the Diviſor from the Dividend: So that by this Art of Diviſion you deduct at 6 times what by Subſtraction would require 216221 times to perform.

Now to know how many times the Diviſor to deduct the firſt time, I conſider what part or places of the Dividend I can take the Diviſor from, and find it the five firſt towards the left hand; therefore I mark that, by putting a Point under it, and I conſider how often

often the Divisor can be had in those five places, or how often 4 (the first of one) in 9 (the first of the other) and find it 2 times. Now to know what Denomination to give this (2) I consider what place that Figure which I made the Point under possesseth, which being the Hundred thousands, therefore this 2 is 200000: so that multiplying the Divisor by 200000, and deducting the Product from the Dividend, the Remainer is 740943210, which is an absolute new Dividend, to be divided by 45678 the Divisor. So having repeated the Work by the same Rules, at last a Number less than the Divisor remains, so the Work is ended.

And thus by dividing the whole into 6 distinct Dividends, there ariseth 6 Quotients; the Sum of which is the general or true Quotient.

But because Brevity is most commendable in this Art, therefore all superfluous Figures being omitted, as the repeating of the Divisor, the Cyphers at the ends of the Quotients and Subtrahends, the Work is then the same as in the two first Propositions: And if the whole Subtrahends be omitted to be put down, deducting as you gradually multiply the Divisor by the Figure put in the Quotient, the Work is then contracted as much as may be, and will stand as in the Example under the second Way of Division.

The Tarif shews you by Inspection how often the Divisor can be had in each Dividend (without trial or guessing) and the Digits towards the left hand shew what must be put in the Quotient.

The Use of Division.

Farthings, Pence, or Shillings, are immediately reduced into Pounds, by dividing the given Number by 960, 240, or by 20, (as by the second Table in Addition.)

In 157460 Shillings how many Pounds? Cut off Units place, and take half the remaining Figures toward the left hand; and where the Number to be halfed is odd, take the less half, and put 10 to the next Figure, *&c.* Thus ½ of 15746|0 is *l.* 7873.

In 1889520 Pence how many Pounds?

```
                l.
240)1889520(7873 Answer.
     209
      175
       072
         0
```

In 7558080 Farthings how many Pounds?

```
                 l.
960)7558080(7873 Answer.
    838
     700
      288
        0
```

 Note,

Note, Farthings are reduced into Pence by dividing by 4, and Pence into Shillings by dividing by 12.

In 7558991 Farthings, how many Pounds, Shillings, Pence, and Farthings?

```
                  d.        s.        l.    s.   d.
4) 7558991 (1889747 (157478 (7873 : 18 : 11 ¾ Answer.
    ......  12) ......  20)
   35         68
   35         89
   38         57
   29         94
   19        107
   31         11 Pence rest.
   3 Farthings rest.
```

Shillings are made Pounds by the above Rule of halving.

Example 5. In 3046400 Ounces, how many Ton?

```
Ounces in a Ton    }
per Table 1. in    }  35840) 3046400 (85 = Ton Answer.
Addition,          }
                             17920
                                 0
```

Example 6. In 158256 Quarts, how many Ton of Wine?

```
Quarts in a Ton }  1008) 158256 (157 = Tons of Wine, Answer.
per Table 3.    }
                         5745
                         7056
                            0
```

These six Examples are the Reverse, and prove the Truth of the six first in the Use of Multiplication.

And as a seventh Example, I shall give the Use of Multiplication and Division, in shewing how to find all the aliquot Parts into which any Number is capable of being divided: and the Use thereof.

Example

Example 1.] To find how many even Parts into which 360 is divisible. See the whole Operation.

2) 360	The Divisors and last Quote	= 2, 2, 2, 3, 3, 5,
2) 180		— 4, 8,
2) 90		6, 12, 24, 9,
3) 45		18, 36, 72,
3) 15		—
5		10, 20, 40, 15,
		30, 60, 120, 45,
		90, 180, 360.

1. I divide the given Number by 2, 3, 5, 7, or other that will divide without a Remainer, and then place the Divisors and last Quote as in the Example.

2. I multiply the 2 next the left hand by the next 2, which produceth 4, placed under the 2, and that Product by the first 2 gives 8.

3. I multiply the 3d (2) the 4, 8, and 2d (3) by the 1st 3, which produceth 6, 12, 24, and 9 placed under the respective Multiplicands, (as I do all the rest following.)

4. In like manner I multiply 6, 12, and 24, by the 3, (because that Digit is repeated, otherwise I should have multiplied the 2, 2, 2, 3, and the 4, 8, also by it) and the Products are 18, 36, and 72.

5. I multiply all that is before by 5, (except where the Products would be the same) as 2, 4, 8, and 2d (3) in the first and second Lines, which produceth 10, 20, 40, 15, in the fifth Line. Also by the same 5 (next the right hand) I multiply 6, 12, 24, and 9, which produceth 30, 60, 120, and 45 in the sixth Line; and 18, 36, and 72 in the fourth Line (of the Numbers above) by the same 5 produceth 90, 180, and 360.

And if there were any more different Digits in the first Line towards the right hand, I should multiply all the above 7 Lines thereby (where the Products would not be the same, for I omit repeating one and the same Product). But because the last Number is 360, and cannot therefore be properly said to be a part of the Number given, I therefore omit that, and put 1 always instead of the Number given.

And tho' you change the places in the Figure in the first Line, the Answer will be true; as in the subsequent Example will appear, as by

by the above Rules the aliquot Parts in the laſt Example of 360 are, found 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, and 180.

Example 2.] To find all the aliquot Parts into which 10350 may be divided.

10350 divided by 2=5175; that by 3=1725; that by 3=575; that by 5 = 115; that by 5 = 23.

Which Diviſors and the laſt Quote are } 3, 3, 2, 5, 5, 23 (the 2 being tranſpoſed)

	9,			= 3 by the 3.
6,	18,			= 3, and 9 by the 2 in the upper Line.
15,	45,	10,		= 3, 9 and 2 by 5 in the upper Line.
30,	90,			= 6 and 18 by 5.
75,	225,	50,	25,	= alſo the 15, 45, and 10 by ſaid 5; it being repeated.
150,	450,			= 30 and 90 by 5, for the ſame reaſon.
69,	—,	46,	115,	= 3, 2 and 5 in 23, the firſt in the upper Line.
	207,			= 9 in 2d Line by ſaid 23.
138,	414,			= 6 and 18 in the 3d Line by ſaid 23.
345,	1035,	230,		= 15, 45, and 10 in the 4th Line by 23.
690,	2070,			= 30 and 90 in the 5th Line by the 23.
1725,	5175,	1150,	575,	= the 6th line by 23, the firſt Number in the firſt Line.
3450,	10350.			= 150 and 450 in the 7th Line by ſaid 23.

So the even Parts of 10350 are 1, 2, 3, 5, 6, 9, 10, 15, 18, 23, 25, 30, 45, 46, 50, 69, 75, 90, 115, 138, 150, 207, 225, 230, 345, 414, 450, 575, 690, 1035, 1150, 1725, 2070, 3450, and 5175.

The

The Use of the Rules for finding the Aliquot Parts of a Number.

This will appear thus: Suppose I would find all the aliquot Parts of a Pound Sterling, I reduce it to its least Denomination, as 960 Farthings; the aliquot Parts of which are found 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 30, 32, 40, 48, 60, 64, 80, 96, 120, 160, 192, 240, 320, 480, and 960.

2,	2,	2,	2,	2,	2,	3, 5.
—	4	8	16	32	64.	
6,	12,	24,	48,	96,	192.	
10,	20,	40,	80,	160,	320,	15,
30,	60,	120,	240,	480,	960.	

Which Numbers being made Denominators, and 1 the Numerator, there will arise the Parts of Coin following; which being supposed the Price of any Integer, the Value of any Number of them is found by dividing the Number of Integers by the respective Denominators at once.

Parts of a Pound.	s. d.	Parts of a Pound.	s. d. q.	Parts of a Pound.	s. d. q.	Parts of a Pound.	d. q.
$\frac{1}{2}$	10 : 0	$\frac{1}{12}$	1 : 8 : 0	$\frac{1}{40}$	0 : 6 : 0	$\frac{1}{120}$	2 : 0
$\frac{1}{3}$	6 : 8	$\frac{1}{15}$	1 : 4 : 0	$\frac{1}{48}$	0 : 5 : 0	$\frac{1}{160}$	1 : 2
$\frac{1}{4}$	5 : 0	$\frac{1}{16}$	1 : 3 : 0	$\frac{1}{60}$	0 : 4 : 0	$\frac{1}{192}$	1 : 1
$\frac{1}{5}$	4 : 0	$\frac{1}{20}$	1 : 0 : 0	$\frac{1}{64}$	0 : 3 : 3	$\frac{1}{240}$	1 : 0
$\frac{1}{6}$	3 : 4	$\frac{1}{24}$	0 : 10 : 0	$\frac{1}{80}$	0 : 3 : 0	$\frac{1}{320}$	0 : 3
$\frac{1}{8}$	2 : 6	$\frac{1}{30}$	0 : 8 : 0	$\frac{1}{96}$	0 : 2 : 2	$\frac{1}{480}$	0 : 2
$\frac{1}{10}$	2 : 0	$\frac{1}{32}$	0 : 7 : 2			$\frac{1}{960}$	0 : 1

Example 1. What doth 7358 amount to, at 1 *d.* 1 *q.* each?

192) 7358 (38 *l.* 6 *s.* 5 *d.* $\frac{1}{4}$ = Answer.

1598

62 rest so many 5 Farth. or 6 *s.* 5 *d.* $\frac{1}{4}$.

Example 2. What doth 31987 come to at 3 *d.* 3 *q.* each?

64) 31987 (499 *l.* 15 *s.* 11 *d.* $\frac{1}{4}$. = Answer.

638

627

rests 51 so many 3 *d.* 3 *q.* or 15 *s.* 11 *d.* $\frac{1}{4}$.

Note, 1 *d.* 1 *q.* in the first is $\frac{1}{192}$; and 3 *d.* 3 *q.* in the second Example, is $\frac{1}{64}$ of a Pound.

Also by the same Rule, what follows are the even Parts of a Ton in Averdupois greater Weight.

Parts of a Ton.	C. qr. lb.	Parts of a Ton.	C. qr. lb.	Parts of a Ton.	C. qr. lb.	Parts of a Ton.	C. qr. lb.
$\frac{1}{2}$	10 : 0 : 0	$\frac{1}{16}$	1 : 1 : 0	$\frac{1}{64}$	0 : 1 : 7	$\frac{1}{280}$	0 : 0 : 8
$\frac{1}{4}$	5 : 0 : 0	$\frac{1}{20}$	1 : 0 : 0	$\frac{1}{70}$	0 : 1 : 4	$\frac{1}{320}$	0 : 0 : 7
$\frac{1}{5}$	4 : 0 : 0	$\frac{1}{28}$	0 : 2 : 24	$\frac{1}{80}$	0 : 1 : 0	$\frac{1}{448}$	0 : 0 : 5
$\frac{1}{7}$	2 : 3 : 12	$\frac{1}{32}$	0 : 2 : 14	$\frac{1}{112}$	0 : 0 : 20	$\frac{1}{560}$	0 : 0 : 4
$\frac{1}{8}$	2 : 2 : 0	$\frac{1}{35}$	0 : 2 : 8	$\frac{1}{140}$	0 : 0 : 16	$\frac{1}{1120}$	0 : 0 : 2
$\frac{1}{10}$	2 : 0 : 0	$\frac{1}{40}$	0 : 2 : 0	$\frac{1}{160}$	0 : 0 : 14	$\frac{1}{2240}$	0 : 0 : 1
$\frac{1}{14}$	1 : 1 : 20	$\frac{1}{56}$	0 : 1 : 12	$\frac{1}{224}$	0 : 0 : 10		

The

The Proof of Multiplication.

This can only be done by Diviſion: as in any of the 6 Examples foregoing, the Products of thoſe in the Uſe of Multiplication are proved to be true by dividing thoſe Products by the Multiplier, the Quotient is the Multiplicand; or if you divide the Product by either of the Factors, the Quotient will be the other. But to pretend to prove Multiplication by caſting out the Nines, is a Miſtake, as I have elſewhere demonſtrated; for why divide by 9 more than 2, or any other Digit, which would prove the Work as well? But the eaſieſt way is to divide the Factors by 10, and the Product of the Remainers by 10, which will leave a Remainer equal to that of the Product divided by 10. But the miſchief is, that if there be a Miſtake in the Product of juſt your Diviſor, or any Power thereof, this Way of proving will not ſhew it.

The Proof of Diviſion.

This is either performed by Multiplication or Diviſion, as appears in the Margin. For Example, I have divided 17002125 by 357, and find the Quotient 47625. And if 17002125 be divided by that Quotient, the later Quotient will be 357 = the former Diviſor. This is proved by Multiplication of the Quotient 47625 by 357 the Diviſor. See *Example* 2. *Prop.* 2. of *Sect.* 4.

```
357) 17002125 (47625) 17002125 (357 = Quotient
       .....                ...        or former
     -------            -------        Diviſor.
      2722              271462
     -------            -------
       2231              333375
      -------           -------
        892                  0
       -------
        1785
       -------
           0
```

SECT. VI. *Of Extracting the Roots of Numbers, called Evolution.*

IN this Section I ſhall ſhew,

I. The Extraction of the Square Root.

II. The Extraction of the Cube Root.

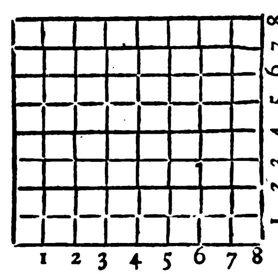
III. The Extraction of the Biquadrate Roots of Numbers.

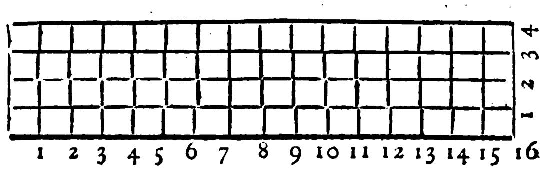
I. The

I. The Square Root of a Number is such a one, as being multiplied in itself, produceth the Number given (for to square any Number, is to multiply it by itself) Thus the square Numbers, whose Roots are the 9 Digits, are as follows in the Margin.

Squares.	Their Roots.
81	9
64	8
49	7
36	6
25	5
16	4
9	3
4	2
1	1

This may be illustrated by any square Superficies, as Glass, Board, &c. for if the superficial Content of a Square thereof (whose 4 Sides are equal) be 81, then one of the 4 Sides is 9; if the square Superficies be 64, then 1 of the 4 equal Sides is 8, &c. as appears by the Table. And the square Figure whose Surface is 64 square Inches, Feet, or other thing; each Side is therefore 8, which is called the Root of that Superficies, as you see. So that when the square Root of any Number is demanded, it is as much as to require what the Side of a Geometrical Square is, whose Area (or superficial, or outside Content) is any Number given. Hence it follows, that by extracting the Square Root of any Superficies, you reduce it to a compleat (or Geometrical) Square. Thus the long Quadratick Figure being 16 in length, and 4 in breadth, the Superficies is upon the flat 64, (or 4 times 16) and the square Root of 64 by the Table above is 8. Therefore a Geometrical Square, one of whose four equal Sides is 8, (as that above) is equal to the long Square, or any other Figure whose flat Surface is 64.





Such Numbers as the above, when the Root may be extracted without a Remainer, may be called compleat or perfect square Numbers; but there are abundantly more Numbers, whose Roots cannot be precisely extracted: and these are called imperfect Squares, or Surd Numbers, of which more in Decimals, Logarithms, and especially in Algebra, hereafter treated on.

Question

```
          .   .    .  .   .    Root
         10|27|45 89|16|(32054
       ——
    6)    127
  640)     34589
 6410)      256416
                   0 remains.
```

Question 1. What is the Square Root of 10274589?

Place the Number so that you may conveniently perform the Work, and point over the first and every second afterward; which Points shew how many places the Roots will consist of.

Then consider what square Number in the foregoing little Table is next to and can be taken from the first branch towards the left hand of the square Number given, (as here 10;) 9 I find is the next, whose Root (3) I put down (like a Quotient) as you see, and substracting the Square of 3 (or 9) from 10, there remains 1.

To which Remainer 1 I bring down the two next Figures (27) and divide that 127 (except the Figure next the right hand) by 6, which is double the Root (3) saying the 6's in 12 is 2; put that in the Root, makes it 32. Then square 2, and deduct from 7, (in the 127) and the rest is 3; then multiply 6, the double Root, by 2, and deduct the Product from 12 (in the 127) and there rests 0 the rest is only Repetition.

Then double the Root 32 makes 64, and that is your next Divisor. Then to the 3 bring down the next Branch 45, marking it as you do in Division, and you have 345 for a Dividend.

But because (if Units place, here 5, be excluded) you cannot have 64 in the rest, which is 34; therefore put (0) in the Root, and also in the Divisor, (to 64, as you see) and bring down the next Branch 89, makes a Dividend 34589 to divide by 640. And finding I can have 6 in 34, 5 times, I put 5 in the Root; and squaring 5 (as was taught for the other Figures of the Root) I say 25 from 29, (of the 34589) and there rests 4, and carry 2; 5 times 0 is 0, but 2 from 8 rests 6, and so 5 times 4, proceed as in Division, and the whole Remainer is 2564.

Lastly, To this bring down the last Branch (16) and divide the 256416 by double the Root 3205, *viz.* 6410, and the Quote being 4, I put it in the Root, and substracting the square thereof from 16, (in the 256416) and also the Product of the Divisor thereby, as before shewn, there remains (0) and 32054 is the Answer.

A second

A second Example.

```
                 Square
 . . . . . . .    Root.
 1|1473|25|96|43|(107113
20) 1473
214)   2425
2142)   28496
21422)   707543
           64874 rests.
```

A third Example.

```
                     Square
  . . . . . . .       Root.
  97|65|43|21|01|23|(988202
18)  1665
196)  16143
1976)   39921
197640)   3970123
             17319 rests.
```

II. *The Extraction of the Cube Root.*

The Cube Root of any Number is that whose Square multiplied by the Root, produceth the Cube Number given. Thus the Cube Numbers, whose Roots are the 9 Digits, are as follows.

A Cube is a solid Body bounded by six Geometrical square Superficies's, as a Dye, whose Length, Breadth, and Depth, (or Thickness) are all equal: And to extract the Cube Root of any Number, is to suppose that Number the Content of a Cube in Feet, Inches, &c. given, to find the Side of one of the six Squares that bounds it. So that as the Extraction of the Square Root is chiefly used in measuring and proportioning of Surfaces; so is the Cube Root in doing that of Solids. And as there are Surd Numbers, whose Square Roots cannot be extracted without a Remainer, so it is in Cube, and other Roots.

Cube Numbers.	Squares.	Roots of the Squares and Cubes.
1	1	1
8	4	2
27	9	3
64	16	4
125	25	5
216	36	6
343	49	7
512	64	8
729	81	9

I shall give but one Example, but 'tis so plainly demonstrated, as may be sufficient to enable any one to extract the Root of any whole Number: And where there are Remainers, I shall shew how to proceed, when I shew the Use of Decimals in extracting the Cube Root.

What is the Cube Root of 329341680093464?

The whole Operation follows, with the Name of each Line or how it ariseth.

	32|934|168|093|464(32054	= Cube Root fought.
	27	The Cube of 3 (1ſt in the Root) deduct.
279)	5934	The firſt Reſolvend or Dividend.
	9	The treble Root (3).
	27	Treble the Square of that 3.
	279	Sum, or the firſt Diviſor.
	8	Cube of 2, the ſecond put in the Root.
	36	Squ. of that 2, multip. in laſt treb. Root.
	54	The treble Square of the Root in 2.
	5768	The Sum of the 3 laſt Lines = a Subtrah.
30816)	166168	The 2d Reſolvend, being 166, the Rem. and 168 carried down to that.
	96	Treble the Root 32.
	3072	Treble the Square of that Root.
	30816	Sum, or the 2d Diviſor.
3072960)	166168093	The 3d Reſolvend.
	960	Treble the Root 320.
	307200	Treble the Square of that Root.
	3072960	Sum, (or the 3d Diviſor)
	125	Cube of 5 (laſt put in the Root.)
	24000	Square of that 5, in the laſt treble Root.
	1536000	Laſt treble Square of the Root in that 5.
	153840125	The Sum, or a Subtrahend.
308170365)	12327968464	The 4th Reſolvend.
	9615	Treble the Root 3205.
	30816075	Treble the Square of that Root.
	308170365	Sum, (or the 4th Diviſor.)
	64	The Cube of 4, laſt put in the Root.
	153840	Square of that 4 in the laſt treble Root.
	123264300	Laſt Treble Square of the Root in that 4.
	12327968464	The Sum (or Subtrahend) which deducted from the laſt Reſolvend,
	(0) reſts.	

The Steps in the Performance of the Work of Extraction of the Cube Root are very evident in this laſt Example: For,

1. You

1. You point over the Figure in Units place, and then over every third Figure, which divides the whole Cube Number into 5 Parts; which ſhews that the Root will conſiſt of 5 places.

2. I find that 3 being cubed, produceth 27, (as appears by the little Table foregoing) which is the next leſs Cube Number to 32, (the firſt part towards the left hand) I therefore put the Root 3, in the Cube Root ſought as you ſee, and deducting the 27 from 32, the Remainer is 5.

3. To the Remainer we always bring down the next part, (as here 934) and that makes the Reſolvend, which is always the Dividend, in order to find the next Figure in the Root.

4. And to find the Diviſor whereby to divide that Dividend, you may obſerve that it is always compoſed of the treble Root, *i. e.* 9, and of the treble Square of the Root, *viz.* 27.

5. I find I can have 279 the Sum, in 593 (for the 4 in Units place of the Reſolvend 5934, you are in this caſe to take no notice of) 2 times, which put in the Root makes it 32.

6. The next thing is to frame your Subtrahend, which is always compoſed, 1ſt, Of the Cube of the Figure laſt put in the Root: 2*dly*, The Square of that Figure multiplied in the laſt treble Root: And, 3*dly*, The laſt treble Square of the Root multiplied in the ſaid Figure laſt put in the Root: The Sum of which three Numbers, as the Example plainly ſhews, is the Subtrahend.

7. The Subtrahend muſt always be deducted from the laſt Reſolvend, as here 5768 from 5934, and the Remainer is 166; to which bring down (as before) the next three Figures (168) and you have a new Dividend (or Reſolvend): all the reſt is Repetition of the ſame Method of working; except here, that you cannot have 30816 in 16616, therefore you put a (0) in the Root, and from that 320 (the Root) you make a new Diviſor, which is 3072960; and for the Dividend, you bring down the 3 next Figures (093) to the laſt Reſolvend, and that makes your Dividend 166168093, &c. which is all very obvious in the Example.

III. *The Extraction of the Biquadrate Root.*

The Biquadrate Root is the Root of the 4th Power, as the Cube is that of the 3d, andthe Square Root that of the 2d Power, according to the following Table.

Whence it appears, that the ſecond Power of any Number is the Square of it, or Product of any Number by itſelf; the Cube

or 3d Power is produced by multiplying the Square of any Number by its Root; the Biquadrate or 4th Power is discovered by multiplying the Cube by the Root, as appears in these nine Examples.

But of the Powers of Numbers to the 10th inclusive, and the proper Name of each, you have a full Account in the Extraction of Roots in Algebra; where the Reason of that abstruse Method of extracting the Square, Cube, Biquadrate, Sursolid, &c. Roots, is fully explained from the Algebraical Canons for each, both by Numbers and Symbols.

A Table of Powers and their Roots.

The Biquadrates or 4th Powers.	The Cubes or 3d Powers.	The Squares or 2d Powers.	The Root or first Power, *viz.* of the 2d, 3d, and 4th Powers.
1	1	1	1
16	8	4	2
81	27	9	3
256	64	16	4
625	125	25	5
1296	216	36	6
2401	343	49	7
4096	512	64	8
6561	729	81	9

What is the Biquadrate Root of 2998219536?

The Example, with Rules how each Line is produced, follows.

1. You see that having pointed over Units place, and every 4th place afterwards, the Root will consist of 3 places, as there are 3 points.

2. You must consider what 4th Power in the little Table above is next to and less than the first Part or Branch 29: you see 16 is, whose Root you have there (2); then substracting the Biquadrate 16 from 29, there rests 13.

3. To that 13 bring down the next branch or Part 9821, and you have 139821 for a Dividend, of which the Units place must not be considered, in asking how often the Divisor can be had therein.

4. To find the Divisor, you see it is composed, 1st, Of 4 times the Figures then in the Root (as here 2). 2dly, Of 6 times the Square of that: And, 3dly, 4 times the Cube thereof: the Sum of which being a Divisor,

5. There ariseth from the Division, 3 in the Root.

6. You must find a Subtrahend, by adding together as the Directions against the 4 Numbers 81, 216, 216, and 96, do express; and substracting the Sum from the last Resolvend, there rests 19980: to which bringing down 9536, you have 199809536. The rest is only Repetition of the three last Steps, as to Method.

29|9821'9536	(234 = the Biquadrate Root fought.
16	The Biquadrate of 2, 1ſt put in the Root.
3448) 139821	13 (the Remainer) and 9821 brought down to it, being the 1ſt Reſolvend or Dividend.
8	4 times the 2 put firſt in the Root.
24	6 times the Square of that 2.
32	4 times the Cube of that 2.
3448	The Sum of theſe 3, a Diviſor.
81	The Biquadrate of 3, the laſt in the Root.
216	4 times the 2 in the Cube of the 3, in the Root.
216	6 times the Square of 2, in the Square of 3.
96	4 times the Cube of 2, in 3.
119841	The Sum, or a Subtrahend to take from the laſt Reſolvend above.
4898632) 199809536	A 2d Reſolvend, or Dividend.
92	4 times the 23 in the Root.
3174	6 times the Square of 23.
48668	4 times the Cube of that 23.
4898632	The Sum of the 3 Lines, a 2d Diviſor.
256	The Biquadrate of 4, the laſt in the Root.
5888	4 times 23, in the Cube of 4 in the Root.
50784	6 times the Sq. of that 23, in the Sq. of the 4.
194672	4 times the Cube of that 23, in the ſaid 4.
199809536	Sum = a Subtrahend, which taken from the Reſolvend laſt above,
(0)	remains.

I know that the Biquadrate Root is the Square Root of the Square Root, and conſequently may be performed by extracting the Square Root twice. But this ſeems a more natural way to perform it, as the Extraction of the Cube, Surſolid Root, &c. may be done from Algebraic Canons, and I have inſerted the Method here, becauſe to me it is new; for I never ſaw, nor heard of its being done thus before: and I was pleaſed when I conſidered it from the 4th Power of $a+b$ in Algebra, which wonderfully, tho' plainly, points out all theſe Rules above, which ſeem ſo intricate, as to be impoſſible firſt to diſcover. This Extraction of the Biquadrate Root is uſeful in ſome

ſome Computations of Compound Intereſt; finding 3 mean Proportionals between 2 Extremes, as in the 5th Head of Geometrical Progreſſion, *&c.*

The Proof of the Square Root (as appears from the little Table to it) is to multiply the Root in itſelf; the Proof of the Cube is to multiply that Square in the Root; and of the Biquadrate to multiply the Cube in the Root; for they reſpectively produce the Number given to have the Root extracted, provided that nothing remain; and if any thing do, add ſuch Remainer.

CHAP. II.

Contains the Application of the Fundamental Parts of Arithmetic, to Vulgar Fractions, Progreſſion, the Rules of Proportion, Practice, Loſs and Gain, Fellowſhip, Barter, Exchange, Equation of Payments, Alligation, and Rules of Falſe Poſition, in eleven Sections.

SECT. I. *Of Vulgar Fractions.*

I. NOTATION *and Numeration,*] Teacheth what a Fraction is, and how to read or write down any one.

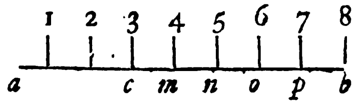
By a Fraction here is meant a broken Number, that is to ſay, one or more Parts of a Unit; for as there is Infinity or Units, ſo a Unit may be, or be ſuppoſed to be, divided into any Number of Parts.

A Fraction conſiſts of two Parts, a *Denominator*, and a *Numerator:* The former ſhews how many Parts the Unit is divided into, which is wrote below the Line; the later ſhews how many of thoſe Parts are contained in the Fraction, which is wrote above the Line; a

$\frac{3}{8}$ of

$\frac{3}{8}$ of a Pound Sterling, or 7 *s.* 6 *d.* here $\frac{3}{8}$ is the Numerator. the Denominator.
And these 2 Parts are called the Terms of a Fraction.

Or, to be more plain yet; the Line (*ab*) represents 1 or a Unit divided into 8 parts, and doth represent the said Denominator of the Fraction given;



and the Line (*ac*) is 3 of those 8 Parts; (*am*) = 4 Eights or $\frac{4}{8}$; (*ao*) = $\frac{6}{8}$; (*ap*) = $\frac{7}{8}$, &c.

Having thus shewn what a Fraction is, you are next to know how to read it; and that is, by first mentioning the Numerator, and then the Denominator. As by this Example you see where 1|12th of the Unit (or Line *m n*) is wrote thus $\frac{1}{12}$, and read One Twelfth, &c. $\frac{2}{12}$ = 2 Twelfths, &c. But the Line might more commodiously be made downright, and then Fractions would stand as in the Examples next the left hand, whereby a Line in Printing might often be gained, if $\frac{1}{12}$ were wrote 1|12; $\frac{2}{12}$, 2|12, &c. But let Custom have its own way; and then Fractions are wrote and read as in the foregoing Tabulet, and by the same Rule $\frac{17}{25}$ is seventeen 25th Parts; $\frac{365}{1728}$ is 365, seventeen hundred and 28 Parts; that is, if a Unit were divided into 1728 Parts, this Fraction does contain 365 of those Parts.

A Table of Simple Fractions.

m				Read thus.
1—	—12th,	wrote thus	$\frac{1}{12}$	One Twelfth.
2—	—12ths,	——	$\frac{2}{12}$ —	Two Twelfths.
3—	—12ths,	—— ——	$\frac{3}{12}$	Three Twelfths.
4—	—12ths,	——	$\frac{4}{12}$ —	Four Twelfths.
5—	—12ths,	—— ——	$\frac{5}{12}$	Five Twelfths.
6—	—12ths,	——	$\frac{6}{12}$ —	Six Twelfths.
7—	—12ths,	——	$\frac{7}{12}$	Seven Twelfths.
8—	—12ths,	— —	$\frac{8}{12}$ —	Eight Twelfths.
9—	—12ths,	—— ——	$\frac{9}{12}$	Nine Twelfths.
10—	—12ths,	——	$\frac{10}{12}$ —	Ten Twelfths.
11—	—12ths,	——	$\frac{11}{12}$	Eleven Twelfths,
12—	—12ths,	— —	$\frac{12}{12}$ or 1	Twelve Twelfths.
n				

But there are various kinds of Fractions, as Proper, Improper, Simple, and Compound.

A Proper Fraction is one whose Numerator is less than the Denominator, as those above.

An Improper Fraction is when the Numerator is greater or equal to the Denominator, as $\frac{7}{5}$, $\frac{7}{7}$, &c.

A Simple

A Simple or Single Fraction, as any of thoſe foregoing is immediately the Fraction of a whole Unit. But

A Compound Fraction, is a Fraction of a Fraction, or Part of another Part of a Unit, as $\frac{2}{3}$ of $\frac{3}{4}$, or $\frac{3}{5}$ of $\frac{9}{14}$, &c. and is illuſtrated in the following Examples.

Where the whole Line (*ln*) is divided into 10 equal Parts, and each of thoſe are ſubdivided into two Parts: ſo that ſuppoſing the Unit (*ln*) to be 1 *l.* each 10th is 2 *s.* and every half of a 10th is 1 *s.* ſo that $\frac{13}{14}$ of $\frac{7}{10}$ is a Fraction of a Fraction, whoſe Value is 13 *s.* But the Value here is not intended to be ſo obſervable as the Nature of the Compound Fraction; for here $\frac{7}{10}$ is $\frac{7}{10}$ of the Line *ln*, and $\frac{13}{14}$ of $\frac{7}{10}$ is 13 *s.* for $\frac{1}{10}$ is 2 *s.* and conſequently $\frac{7}{10}$ is 14 *s.* and $\frac{13}{14}$ of 14 *s.* muſt needs be 13 *s.* Sometimes you have a Fraction of a Fraction of a Fraction, &c. of a Unit, as 1 Farthing is $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a Pound, &c.

A Table of Compound Fractions.

l		*s.*
—	— $\frac{1}{2}$ of $\frac{1}{10}$ of 1 *l.* or	1
1 —	—10 or	2
—	— $\frac{3}{4}$ of $\frac{2}{10}$	3
2 —	—10	4
—	— $\frac{5}{6}$ of $\frac{3}{10}$	5
3 —	—10	6
—	— $\frac{7}{8}$ of $\frac{4}{10}$	7
4 —	—10	8
—	— $\frac{9}{10}$ of $\frac{5}{10}$	9
5 —	—10	10
—	— $\frac{11}{12}$ of $\frac{6}{10}$	11
6 —	—10	12
—	— $\frac{13}{14}$ of $\frac{7}{10}$	13
7 —	—10	14
—	— $\frac{15}{16}$ of $\frac{8}{10}$	15
8 —	—10	16
—	— $\frac{17}{18}$ of $\frac{9}{10}$	17
9 —	—10	18
—	— $\frac{19}{20}$ of $\frac{10}{10}$	19
10 —	—10	20
n		

Having thus ſhewn what a Fraction is, and how to read the ſame, I proceed to

II. *Reduction of Vulgar Fractions.*

This Rule muſt neceſſarily be taught before Addition and Subſtraction, becauſe they cannot be performed till the Fractions given to be added or ſubſtracted, are fitted by Reduction for that purpoſe.

Caſe 1. *To reduce a mix'd Number to an Improper Fraction.* As for *Example* 12 $\frac{3}{7}$.

Rule.] Multiply the intire Part by the Denominator of the Fraction, and to the Product add the Numerator (3), and the Sum placed

placed over the Denominator (7) is the Anſwer, and will ſtand thus $\frac{87}{7}$.

Caſe 2. *To reduce an improper Fraction to a whole or mix'd Number.* *Example*, reduce $\frac{87}{7}$.

Rule.] Divide the Numerator (87) by (7) the Denominator, and the Quotient is the intire Number, and the Remainer (3) is the Numerator to place over the Denominator (7); ſo the Anſwer is $12\frac{3}{7}$, and proves the firſt Caſe true. And by the ſame rule $\frac{36}{5}$ is $= 7\frac{1}{5}$; $\frac{144}{12}$ is $= 12$; $\frac{81}{9}$ is 9; $\frac{44}{11}$ is 4, &c.

Caſe 3. *To reduce Compound Fractions to Simple.*

Example. Reduce $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{7}{8}$ into one ſimple Fraction.

Rule.] Multiply the Numerators together for a new Numerator, and alſo the Denominators together for a new Denominator, and it ſtands thus $\frac{42}{96}$; or this is $\frac{1}{2}$ of $\frac{7}{8}$ or $\frac{7}{16}$, by changing the Parts of the firſt and ſecond Fractions, and omitting thoſe which are the ſame; (as 3 in each) and $\frac{17}{24}$ of $\frac{1}{2}$ of $\frac{12}{15}$ of $\frac{3}{4}$ is $\frac{612}{2880}$.

The Truth of this Rule is eaſily *proved* from any ſelf-evident Inſtance, as $\frac{1}{2}$ of $\frac{2}{3}$ of a Pound Sterling is 6 *s.* 8 *d.* ſo by the Rule it is $\frac{2}{6}$. Now $\frac{1}{6}$ being 3 *s.* 4 *d.* $\frac{2}{6}$ muſt be 6 *s.* 8 *d.* or $\frac{1}{2}$ of $\frac{2}{3}$ is $\frac{1}{3}$: And if the Parts be changed, and the two of a ſort be omitted, it proves the ſame; and the Reaſon is plain, for the Terms of a Fraction multiplied by the ſame Number, does not increaſe or alter the Value of ſuch Fraction.

Caſe 4. *To reduce a Fraction to its loweſt Terms.*

Example. Reduce $\frac{36}{45}$ to its loweſt Terms.

Rule 1.] Divide the Parts of the Fraction by any Number that will divide both without a Remainer: ſo this $\frac{36}{45}$ is reduced to $\frac{4}{5}$, for Anſwer.

for $\frac{36}{45}$ divided by 3 is $\frac{12}{15}$, and $\frac{12}{15}$ divided by 3 is $\frac{4}{5}$

```
36)45 (1
   ---
    9)36 (4
      ---
       0
```

Rule 2.] Divide the greater by the leſſer Part of the Fraction, and if any thing remain, divide the laſt Diviſor by that; and if any thing yet remain, divide the laſt Diviſor by that, till nothing remain; and then the laſt Diviſor divideth both Parts of your Fraction, ſo as to reduce it to its loweſt Term. See the Work in the Margin, where 9 the laſt Diviſor reduces $\frac{36}{45}$ to $\frac{4}{5}$; ſo $\frac{144}{2016}$ is $\frac{1}{14}$; $\frac{84}{96}$ is $\frac{7}{8}$; and $\frac{315}{420}$ is $\frac{3}{4}$, and $\frac{140}{175}$ is $\frac{4}{5}$, 35 dividing both Parts of the Fraction; found as *per* the 2d Rule.

The Truth of this Rule: That a Fraction reduced to its loweſt Terms is of the ſame Value with that given, is thus *proved*; $\frac{8}{12}$ of a Pound in its loweſt Terms is $\frac{2}{3}$ of a Pound, or 13 *s.* 4 *d.* And $\frac{1}{12}$ of a Pound being 20 *d.* 8 times that is 13 *s.* 4 *d.*

Caſe 5. *To find the Value of any Fraction of Weight*, &c.

Example. What is the Value of $\frac{31}{42}$ of a Pound Averdupois?

Rule.] Multiply the Numerator of the Fraction by ſuch a Number of Units of the next leſs Denomination, as is equal to a Unit of that name which the Fraction is of, and divide the Product continually by the Denominator, and the Quotient or Quotients anſwer your Queſtion. See the Work of the following Examples.

Example 2. What is the Value of $\frac{17}{26}$ of a Pound Sterling?

17 Numerator
20 Shill. in 1 *L.* } Mult.

26)340 (13 *s.*
80
2 remains Sh.
12 Pence in 1 *s.* } Mult.

26)24 (0 *d.*
4 Farth. in 1 *d.* Mult.

26)96 (3 $\frac{9}{13}$ Farthings.
18

So the Anſwer is
s. *d.* *qr.*
13 : 0 : 3 $\frac{9}{13}$

Example 1. to *Caſe* 5.

31 Pounds wt.
16 Ounces in 1 ℔
186
31

42)496 (11 Ounces.
76
34 remains.
16 Drams in 1 Ounce multiply.
204
34

42)544 (12 Drams.
124
40 remains to place over the 42.

So the Anſwer is
Oun. *Dram.*
11 : 12 $\frac{40}{42}$ or $\frac{20}{21}$.

And by the ſame Rule any other Fraction of Money, Weight, Meaſure, *&c.* hath its Value found.

The

The Truth of this Rule is manifest; for suppose $\frac{7}{8}$ of a Pound Sterling we know is 17 *s.* 6 *d.* or 7 Half Crowns; and it will appear to be so by the Rule, if work'd as the two Examples above are.

Case 6. *To reduce Fractions of different Denominators to those of the same Value, which have a Common (or one and the same) Denominator.*

Example. Reduce $\frac{2}{9}$, $\frac{3}{7}$, and $\frac{4}{5}$ to a common Denominator.

Rule.] Multiply the Denominators one in another for the common Denominator, as 9 times 7 is 63, and 5 times 63 is 315 = the common Denominator.

Then for the 3 new Numerators, multiply every Numerator into all the Denominators except its own, and the Product is a Numerator answering to the Fraction whose Numerator you multiplied: as the Numerator 2 in 7 and 5 produceth 70, so is $\frac{70}{315}$ equal to $\frac{2}{9}$. Then 3 multiplied in 9 and 5 gives 135, so is $\frac{135}{315}$ equal to $\frac{3}{7}$. And lastly, 4 multiplied in 7 and 9 is = 252, so is $\frac{252}{315}$ equal to $\frac{4}{5}$. So the 3 new Fractions have each the same Denominator, and are in value the same as those given. So also $\frac{9}{11}$ and $\frac{2}{3}$ are = $\frac{27}{33}$ and $\frac{21}{33}$; $\frac{2}{3}$ and $\frac{1}{7}$ are = $\frac{2}{21}$, and $\frac{14}{21}$, &c.

The Truth of this Rule will be evident by reducing any of the Fractions which have the same Denominator to its lowest Term, and that you'll find the primitive Fraction given: as $\frac{70}{315}$ in its lowest Term is $\frac{2}{9}$ = the Fraction given, and so of all the rest.

This Rule ought to be well minded, it being of principal Use in Addition and Substraction of Fractions.

Case 7. *To reduce Fractions of a smaller Denomination to Fractions of a greater.*

Example. What Fraction of a Pound Sterling is $\frac{3}{7}$ of a Farthing?

Rule.] Consider that $\frac{3}{7}$ of a qr. is $\frac{3}{7}$ of $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a Pound: then reduce this compound Fraction to a simple, by Case 3. and you'll find it $\frac{3}{6720}$ of a Pound. And by the same Rule $\frac{15}{19}$ of an Ounce is $\frac{15}{304}$ of a Pound, &c.

The Truth of this appears by the next Case.

Case 8. *To reduce Fractions of a greater to a smaller Denomination.*

Example. What Fraction of a Farthing is $\frac{3}{6720}$ of a Pound Sterling?

Rule.] Multiply the Numerator of the Fraction by such a Number of Units of the lesser, as make one of the greater Denomination: So here 960 Farthings making 1 *l.* I multiply the Numerator 3 in 960, and place the Product for a Numerator to the Denominator of the Fraction given, which makes the Answer $\frac{2880}{6720}$ of a Farthing; which Fraction in its lowest Terms is $\frac{3}{7}$, and proves the last Case 7.

 to

to be *right*. And $\frac{15}{304}$ ℔ Averdupois is $\frac{240}{304}$ of an Ounce, which in its loweſt Term is $\frac{15}{19}$, as above. So that theſe two Caſes 7 and 8, prove the Truth of each other.

III. *Addition of Vulgar Fractions.*

Example 1. What is the Sum of $\frac{7}{12}$ and $\frac{3}{8}$?

Rule.] Reduce the Fractions to the ſame Denominator by Caſe 6. of Reduction, (which you'll find $\frac{56}{96}$ and $\frac{36}{96}$.) Then add the Numerators 56 and 36 make 92, which placed over the common Denominator 96 is $\frac{92}{96}$, the Anſwer, or $\frac{23}{24}$.

The Truth of this, &c. *is thus proved:* Suppoſe the given Fractions be of a Pound Sterling; 7 12ths (or 7 times 20 *d.*) is 11 *s.* 8 *d.* and $\frac{3}{8}$ of a Pound is 7 *s.* 6 *d.* the Sum of which is 19 *s.* 2 *d.* which you'll find to be the Value of the Anſwer $\frac{92}{96}$, by the fifth Caſe of Reduction of Fractions.

Example 2. What is the Sum of $\frac{3}{5}$, and $\frac{7}{8}$ of $\frac{2}{7}$ of $\frac{5}{9}$?

Rule.] Firſt reduce the compound Fraction $\frac{7}{8}$ of $\frac{2}{7}$ or $\frac{1}{4}$ of $\frac{5}{9}$ to a ſingle Fraction, which you'll find $\frac{70}{504}$ or $\frac{5}{36}$; to which add the $\frac{3}{5}$, as *per* Example 1. and you'll find the Sum $\frac{1862}{2520}$, or $\frac{3}{36}$, (the Anſwer by Caſe 3. of Reduction.)

Example 3. What is the Sum of $17\frac{3}{4}$ and $\frac{2}{3}$ of $\frac{3}{4}$?

Rule.] The compound Fraction in a ſimple is $\frac{6}{12}$, which added to the Fraction part of the mixt Number, maketh $\frac{60}{48}$, which by the ſecond Caſe of Reduction is $1\frac{12}{48}$ or $1\frac{1}{4}$; which added to 17, gives the Sum $18\frac{1}{4}$. *The Truth of which is proved* by ſuppoſing $17\frac{3}{4}$ to be 17 *s.* 9 *d.* and $\frac{2}{3}$ of $\frac{3}{4}$ to be ſo of a Shilling, which is 6 *d.* the Sum of which is 18 *s.* 3 *d.* or $18\frac{1}{4}$, as above.

IV. *Subſtraction of Vulgar Fractions.*

Example 1. From $\frac{92}{96}$ take $\frac{3}{8}$.

Rule.] The Fractions in a common Denominator are $\frac{736}{768}$ (equal to $\frac{92}{96}$) and $\frac{288}{768}$ (equal to $\frac{3}{8}$) therefore 288 deducted from 736, the other Numerator, the Remainer is 448; ſo the Anſwer is $\frac{448}{768}$ (which in its loweſt Term is $\frac{7}{12}$, and *proves* the firſt Example in Addition of Vulgar Fractions). And this Remainer (as in Whole Numbers) add to the Subtrahend $\frac{3}{8}$, gives the Sum $\frac{92}{96}$, *which proves (that way alſo)* this and the firſt in Addition of Vulgar Fractions *to be truly performed.*

Example 2. From $\frac{1862}{2520}$ take $\frac{7}{8}$ of $\frac{2}{7}$ of $\frac{5}{9}$.

Rule.]

Rule.] 1*st*, Reduce the compound Fractions to a single, which you'll find by the third Case in Reduction to be $\frac{70}{504}$.

2*dly*, Reduce $\frac{1862}{2520}$ and $\frac{70}{502}$ to one and the same Denominator by the sixth Case of Reduction, which are $\frac{938448}{1270080}$ (equal to $\frac{1862}{2520}$) and $\frac{176400}{1270080}$ (equal to $\frac{70}{504}$.)

3*dly*, Deduct the Numerator 176400 from the other Numerator 938448, and the Remainer is 762048 : so the Answer is $\frac{762048}{1270080}$. And that in its lowest Terms (by the fourth Case of Reduction) you shall find $\frac{3}{5}$, by dividing each part of the Fraction by the common Measurer or Divisor 254016. *And this proves not only the Truth of the second Example in Addition, but also that this Example is rightly performed.*

Example 3. From $18\frac{1}{4}$ take $\frac{2}{3}$ of $\frac{3}{4}$.

Rule.] 1*st*, Reduce $\frac{2}{3}$ of $\frac{3}{4}$ to 1 Fraction as before, which is $\frac{6}{12}$.

2*dly*, Reduce $\frac{6}{12}$ and $\frac{1}{4}$ (the Fraction-part of the mixt Number given) to a common Denominator, which are $\frac{12}{48}$ (equal to $\frac{1}{4}$) and $\frac{24}{48}$ (equal to $\frac{6}{12}$.)

3*dly*, Now you should take $\frac{24}{48}$ from $\frac{12}{48}$, but you cannot, as being less; therefore borrow 1 or $\frac{48}{48}$ from the 18, will leave 17. Then add $\frac{48}{48}$ to the $\frac{12}{48}$ makes $\frac{60}{48}$, from which take the $\frac{24}{48}$, and there remains $\frac{36}{48}$, which in its lowest Term is $\frac{3}{4}$; so the Remainer or Answer is $17\frac{3}{4}$, and *proves* the third Example in Addition. But this is done shorter by omitting the Numerators and Denominators which are the same Digits as $\frac{2}{3}$ of $\frac{3}{4}$ is $\frac{2}{4}$ or $\frac{1}{2}$, *&c.* as under Case 3. of Reduction.

Which 3 Examples in Substraction, and those 3 in Addition, do mutually prove each other.

And thus I have given as many Examples as are necessary, in order to the perfect understanding of Addition and Substraction, which will prove very easy, (as they are so plainly expressed) especially to such as have a due knowledge of Reduction of Vulgar Fractions. I shall therefore proceed to

V. *Multiplication of Vulgar Fractions.*

Example 1. Multiply $\frac{7}{15}$ by $\frac{9}{10}$.

Rule.] Multiply the Numerators together for a new Numerator, as here 7 by 9 is 63; and the Denominators together for a new Denominator, as 15 by 10 is 150: so that $\frac{63}{150}$ is the Answer.

Example 2. Multiply $\frac{5}{7}$ of $\frac{2}{3}$ by $\frac{7}{8}$ of $\frac{8}{9}$.

Rule.]

Rule.] Work as before; for the Product of 5 in 2 in 7 and in 8, is 560 the Numerator; and 7, 3, 8, and 9 together, gives 1512: so is $\frac{560}{1512}$ the Answer, or $\frac{10}{27}$ in its lowest Terms.

Example 3. Multiply 25 by $\frac{7}{12}$.

Rule.] The whole Number wrote Fraction-ways is $\frac{25}{1}$, which multiply as by the Rule 1. and the Product is 175, (of the Numerators) and that of the Denominators is 12: so the Answer is $\frac{175}{12}$, or by the second Case of Reduction $14\frac{7}{12}$.

Thus you see in Multiplication of Fractions the Product is less than one of the Factors; and 'tis so much less, as the Numerator of the Multiplier is less than the Denominator, (in this Example) or so much as 7 is less than 12 its Denominator: for if the Numerator 7 were 12, it is plain that $\frac{12}{12}$ (as is said before) is 1, and 1 time 25 would be 25, which would make the Product equal to the Multiplicand; and therefore as 7 is less than 12, so must the Product $14\frac{7}{12}$ be less than 25.

Example 4. Multiply $34\frac{7}{8}$ by $13\frac{11}{12}$.

Rule.] Reduce both the mixt Numbers to improper Fractions, and then proceed as by the first Rule in Multiplication of Fractions.

Thus $34\frac{7}{8}$ is $\frac{279}{8}$, and $13\frac{11}{12}$ is $\frac{167}{12}$, and the Product of the Numerators 279 and 167 is 46593, and of the Denominators (96) so the Answer is $\frac{46593}{96}$, or $485\frac{33}{96}$.

VI. *Division of Vulgar Fractions.*

Example 1. Divide $\frac{63}{150}$ by $\frac{7}{15}$.

Rule.] Place the Fractions as in the Margin; then multiply the Numerator of the Divisor by the Denominator of the Dividend, and the Product is 1050, the Denominator of the Quotient. And the Denominator of the Divisor in the Numerator of the Dividend produceth 945, the Numerator of the Quotient; which is therefore $\frac{945}{1050}$, in its least Terms $\frac{9}{10}$: *And proves the Truth* of the first Example in *Multiplication* of Vulgar Fractions.

$\frac{7}{15}$) $\frac{63}{150}$ ($\frac{945}{1050}$

Example 2. Divide $14\frac{7}{12}$ by 25.

Rule.] Reduce the mixt Number to an improper Fraction, and put a Unit under the 25, then work as *per* the last Rule, and as in the Margin. Where you see the Quotient is $\frac{175}{300}$, or in its least Terms is $\frac{7}{12}$. *And this proves the Truth* of Example 3. in Multiplication, as *that proves this true.*

Divis.	Dividend	Quot.
$\frac{25}{1}$)	$14\frac{7}{12}$ or $\frac{175}{12}$	($\frac{175}{300}$ or $\frac{7}{12}$.

Example

Example 3. Divide $485\frac{33}{96}$ by $13\frac{11}{12}$.

Rule.] Reduce the mixt Numbers to Fractions, and you'll have $\frac{46593}{96}$ equal to $485\frac{33}{96}$ = the Dividend, $\frac{167}{12} = 13\frac{11}{12}$ = the Divisor, and $\frac{559116}{16032}$ = the Quotient; which being reduced to a mixt Number, is $34\frac{14028}{16032}$, or $34\frac{7}{8}$, the Fraction being in its lowest Terms, (dividing each part by the common Measurer 2004).

$\frac{167}{12}$) $\frac{46593}{96}$ ($\frac{559116}{16032}$

or $34\frac{7}{8}$ = the Answer.

And this manifestly proves the Truth of the 4th Example in Multiplication of Fractions.

I have in the *Merchant's Magazine* shewn the Reason of all the Rules abovementioned, for Reducing, Adding, *&c.* Vulgar Fractions; and have not room to repeat that here: but have fully shew'd how one Rule proves the Truth of another, which is a good Demonstration of the Genuineness of all.

VII. *To Extract the Roots of Vulgar Fractions.*

Rule 1.] You must always reduce your Fraction into its least Terms: for if the Root can be justly extracted in any Terms, it can be so in its lowest. Tho' when you can see, as sometimes it happens, that the Roots may be immediately extracted of the Fraction in the Terms given, then you need not reduce it to its lowest. Thus for

Example, The Square Root of the Fraction $\frac{100}{144}$ is = $\frac{10}{12}$, the Answer = $\frac{5}{6}$. And if the $\frac{100}{144}$ had been reduced to its least Terms, they would have been = $\frac{25}{36}$, whose Square Roots are $\frac{5}{6}$, as before.

Example 2. When the Square Root of a mixt Number is required, as suppose of $56\frac{1}{4}$; reduce the mixt Number to a Fraction, and extract the Root of both Terms as before: so $56\frac{1}{4}$ is = $\frac{225}{4}$, whose Square Root is $\frac{15}{2} = 7\frac{1}{2}$ the Answer, (for the $\frac{225}{4}$ is given in its least Terms) and if you multiply $7\frac{1}{2}$ by $7\frac{1}{2}$, the Product is $56\frac{1}{4}$ *for Proof.* But

Note, That when a Fraction is given, whose Root cannot be extracted without Remainer, you must reduce it to a Decimal, and proceed to get the Root thereof, as *per* the Rules and Examples at the End of Decimals; or it may in most Cases be done near enough by Logarithms, as at the end of *Sect.* 3. of *Chap.* VI.

II. *To extract the Cube Root of Vulgar Fractions.* Reduce the Fraction to its least Terms, and then extract the Cube Root of each for those of the Root, unless you can see that the Roots may be immediately

immediately extracted of the Terms given. For Example, to get the Cube Root of $\frac{729}{3375}$, this Fraction in its least Terms is $\frac{27}{125}$, whose Cube Root is $\frac{3}{5}$. Or if you had done it without reducing to its lowest Terms, the Roots of the Terms given would be 9 and 15, or $\frac{9}{15}$, which is equal to $\frac{3}{5}$, as before.

For the Root of a mixt Number, reduce it to an improper Fraction, and proceed as by the last Example. But when the Root cannot be accurately enough extracted without trouble, you may proceed by Decimals, or Logarithms, as is said above.

Note, That the Use of the Square and Cube Roots you have partly in the two next Sections.

VIII. *The Application or Use of Vulgar Fractions.*

Quest. 1. Two Bags contain $470\frac{7}{8}$ Dollars, but the greater exceeds the lesser $32\frac{9}{13}$ Dollars: how much is in each Bag?

Rule.] Substract the Difference from the Sum, and there rests $438\frac{12}{104}$ Dollars; half of which is $219\frac{9}{208}$, the Content of the lesser Bag; to which add the Difference $32\frac{9}{13}$, and the Sum is $251\frac{163}{208}$ Dollars in the greater Bag: *The Sum of which for Proof* is $470\frac{7}{8}$.

Quest. 2. A Brick-Wall contains $1792\frac{13}{71}$ square Feet on the Superficies; how many Rod is that of $272\frac{1}{4}$ Feet to the Rod?

Rule.] Divide $1792\frac{13}{71}$ by $272\frac{1}{4}$, and the Quotient is $\frac{508980}{77319}$, or $6\frac{45066}{77319}$ Rod for Answer.

Quest. 3. In $846\frac{3}{8}$ Pole, how many Yards of $5\frac{1}{2}$ Yards to the Pole?

Rule.] Multiply the one by the other gives the Answer $4655\frac{1}{16}$ Yards.

Quest 4. What is the Value of 87 *C.* — *q.* 13 ℔ at 1 *l.* 9 *s. per C?*

Rule.] The Weight is 87 $\frac{13}{112}$ *C.* or $\frac{9757}{112}$ *C.* the Money is $\frac{29}{20}$ *l.* multiply the Fractions together gives $\frac{282953}{2240}$ *l.* which by the fifth Case of Reduction of Fractions is = 126 *l.* 6 *s.* 4 *d.* 1 *q.* $\frac{4}{7}$.

Quest. 5. If I give *l.* 126 : 6 : 4 : 1 $\frac{4}{7}$ for 87 *C.* 0 *q.* 13 ℔ of Sugar, what is that *per* Hundred weight of 112 *per C?*

Rule.] Divide the Money (as $126\frac{2139}{6720}$) by the Weight $87\frac{13}{112}$ *C.* and the Quotient is exactly *l.* 1 : 9 : —, the Answer.

And this proves the Truth of the last Question.

Quest. 6. What is the Product of 5 *s.* 7 *d.* by 7 *s.* 5 *d?*

Rule.] Multiply $5\frac{7}{12}$ *s.* by 7 $\frac{5}{12}$ *s.* (the Pence being so many 12ths of a Shilling) and the Product is $41\frac{59}{144}$ Shillings, or *l.* 2 : 1 : 4 : $3\frac{2}{3}$.

Quest. 7 Divide *l.* 2 : 1 : 4 : $3\frac{2}{3}$ by $7\frac{5}{12}$ Shillings, (or 7 *s.* 5 *d.*) The first, in a mixt Number is $41\frac{59}{144}$ *s.* which divided by $\frac{89}{12}$, (41 $\frac{59}{144}$ being

being first reduced to an improper Fraction) the Quotient is $\frac{71556}{12316}$ or $5\frac{7}{13}$ *s.* *This proves the last Question* to be truely wrought.

Quest. 8. What is the Product of *l.* 5 : 7 : $10\frac{1}{4}$ multiplied in itself?

Rule.] Reduce the 7 *s.* 10 *d.* $\frac{1}{4}$ into Farthings, and those Farthings into the Fraction of a Pound by the seventh Case of Reduction of Fractions: then you have *l.* $5\frac{377}{960}$ to square, (or multiply by itself) which produceth *l.* 29 : 1 : 7 : 2 $\frac{47040}{921600}$, the Answer.

I should not have inserted this and the sixth Question above, but that there is much noise (tho' little real Use) of them; some Persons being deceived by thinking to answer them (for example) by reducing in the 6th Question the 5 *s.* 7 *d.* into Pence, and multiplying them by the Pence in 7 *s.* 5 *d.* and then reducing the Pence of the Product into Shillings or Pounds by dividing by 12, *&c.* But this way gives 12 times the Answer; because, as appears, the Pence should be divided by 144, or 12 times 12 = the Product of the Denominators of the Fraction. And so in the last Example they will divide the Farthings in the Product of those in the Number given multiplied in itself by 960; whereas it ought to be divided by the Square of that, *viz.* 921600.

On the contrary, others think to perform the Work of the sixth Question by reducing the 67 and 89 Pence to the Fraction of a Pound, multiplying $\frac{67}{240}$ by $\frac{89}{240}$. But here 'tis plain the Answer will be so much too little, as $\frac{5963}{57600}$ *l.* is less than $\frac{5963}{2880}$ *l.* *i. e.* it will be but $\frac{1}{20}$ of the true Answer.

Hence it appears, that such Questions are only naturally and accurately resolved by Fractions, as in the said Examples; for which reason I have given those two above in this part of the Use of Fractions, but hope the ingenious Reader will excuse this Digression on so trivial an Instance.

SECT. II. *PROGRESSION.*

THIS Part of Arithmetic, tho' neglected both by many Authors of Arithmetical Tracts, and Teachers of the Science, is however of excellent Use, as shewing in a great Variety of Instances the wonderful Power and Harmony of Numbers, and their Relation one to another.

This Relation of Numbers is either Arithmetical or Geometrical.

I. *Arithmetical Progression*, or Relation of Numbers, is when a Series thereof differ by the Addition of some Number to the first, second, &c. So these Numbers,

1	2	3	4	5	6	7	8	9	10	11	12,	&c.
2	4	6	8	10	12	14	16	18	20	22	24,	&c.
3	6	9	12	15	18	21	24	27	30	33	36,	&c.

Each of those in the first Series or Row differ by 1, each of those in the second Series by 2, and in the third by the common Addition of 3. And a Series of abundant Numbers are 30, 42, 54, 66, &c. which differ by 12. Now,

1. *To find the Sum of any of the Series's*, you must multiply half the Sum of the first and last Terms by the Number of Terms; or half the Number of Terms by the Sum of the first and last, and the Product gives the Sum required.

Thus 1 and 12 is 13 by 6 (half the Terms) gives 78 the Sum.
2 and 24 is 26 by 6, gives 156=the Sum of the 2d Series.
3 and 36 is 39 by 6, gives 234=the Sum of the 3d Series.

And the like is to be observed in summing up any other Rows of Numbers, tho' never so large, or however differing, if Arithmetically continued.

2. It may be observed, that the several Sums of the three Lines above, or any other of equal Numbers of Terms, beginning with successive Digits continued, do differ also in Arithmetical Proportion; the common Difference being the Sum of the first Series, as 78, 156, 234, &c.

Question 1. How many Strokes does the Hammer of a Clock strike in the 12 Hours? This is done as the first Example above, the Answer being 78, and would have been the same had you multiplied half the first and last, *viz.* 6½ by 12, the whole Number of Terms.

Quest. 2. Admit a Boy is to collect 100 Apples, which lie a Yard distant from each other, and to put each of them singly into a Basket placed one Yard from the first; how many Yards does he pass?

Here the first Term is 2 Yards, the last 200, (to the last Apple, and back to the Basket) Sum 202, which multiplied by 50, (half the Number of Terms) produceth the Sum of the Yards passed by the Boy, which is 10100, or upwards of five Miles.

Quest.

Quest. 3. How many Feet does an heavy Body fall in 11 Seconds of Time, ſuppoſing it to fall 16 Foot the firſt Second, 3 times that the next Second, 5 times that the 3d Second, &c. as in the Example, (or by continually adding 32 to the firſt Term) the 11th Term is 336 Feet, which the Body falls in the 11th Second, (ſo prodigious is the Increaſe of the Velocity) and if ½ the laſt and firſt together be multiplied by the 11, the Sum of all the Feet that it falls is 1936.

The Seconds a Body falls.	*Multipliers.*	*Feet an heavy Body falls in each Second.*
1	1	16
2	3	48
3	5	80
4	7	112
5	9	144
6	11	176
7	13	208
8	15	240
9	17	272
10	19	304
11	21	336

Thus if you would take the Depth of a Well, or the like, ſuppoſe by a Watch that vibrates Quarter Seconds, I find a Stone 44 Quarters or 11 Seconds in falling as above, the Depth of ſuch a Well, &c. is found 1936 Feet.

And in this Progreſſion 'tis plain that any of the Terms are found without the intermediate, by multiplying 16 by double the number of Seconds leſs 1. Thus I find that it falls in the 8th Second, 240 Feet, by multiplying 16 by 15.

3. *To find a mean Arithmetical Proportional* between any *two Numbers.* Take *half the Sum of the* 2 *Numbers* or Extremes for Anſwer, as in the Examples above; or *add half the Difference to the leſſer.*

A Mean between 9 and 11 in the firſt Line is 10.
20 and 24 in the 2d Series is 22.
27 and 33 in the 3d . . . . 30, &c.

4. *The common Difference and Number of Terms given to find the laſt Term.* Thus in the 2d Series the Number of Terms (12) being multiplied by the common Difference 2, gives 24, the laſt Term, &c. or in any Series multiply the Number of Terms leſs 1 by the common Difference, and add the firſt Term.

5. *Any* 2 *Numbers ſtanding together given to find a third,* &c. Take their Difference, and add to the greater, gives the 3d, &c. Or ſubſtracted from the leſſer, gives the leſſer Terms, as in the third Series 21 and 24 are given to find the 3d Term; the Difference is 3, which added to 24 gives 27, the next Term higher; or ſubſtracted from 21 gives 18, the next Term lower.

6. *If any four Numbers are in Arithmetical Proportion*, whether *continued* or *interrupted*, the Sum of the two middle Numbers are equal to the Sum of the two Extremes. Thus,

In the first Line 7, 8, 9, 10; 7 and 10 are equal to 8 and 9.

Also in 7, 8, 14, 15; 7 and 15 are equal to 14 and 8.

And in the 3d Line 12, 15, 18, 21; 12 and 21 are equal to 15 and 18.

7. *If three Numbers are in Arithmetical Proportion continued*, the Double of the Mean is equal to the Sum of the two Extremes, as 12, 15, 18; 2 times 15 is equal to 18 and 12.

8. *If three Numbers are given, a fourth may be found* by adding together the 2d and 3d, and from that Sum substracting the first, as in 14, 15, 16, the Sum of 15 and 16 is 31, from which take 14, and the Remainer is 17, the 4th in Arithmetical Proportion; and this also holds, tho' the Progression be interrupted, as 14, 15, 25, 26.

9. *The Total of the Progression, and the first and last Terms given, to find the Number of Places*; divide the Total by half the Sum of the first and last Terms, and the Quotient is the Number of Terms or Places.

10. *The last Number and common Difference given, to find the Number of Terms*; divide the last Number by the Excess or common Difference.

11. *The Sum of the Progression, and the first and last Terms given, to find the common Difference.* Divide the Total of the Progression by half the Sum of the first and last Terms, and the Quotient is the Number of Terms. Then from the last Term take the first, and the Remainer divide by the Number of Terms less 1, and the Quotient is the Excess or Difference sought, as will appear by any of the three Series's above.

The Reason of the Rule for summing up an Arithmetical Progression. According to the 3d Proposition last above, half the Sum of the two Extremes is an Arithmetical Mean; and a Mean between the first and last Terms of a Progression, according to the same Rule, is found by the first Proposition; and since that is a Mean between the extreme Terms, therefore that being multiplied by the Number of Terms, must necessarily give the Total of them all.

Or more plainly: Take any odd Number of Terms in the three Series's above, and you'll find that Term standing in the middle to be the Mean, according to the said 3d Prop. Thus in the seven first Terms in the second Series, the Mean (or middle Number) is 8, there being three on each side; so that one with another, each Term

Term is 8, as you'll find it: for 6 (the next towards the left hand) is 2 less than 8, but then 10 (the next towards the right hand) is 2 more than 8; and 4 (the 2d towards the left hand from the 8) is 4 less than 8, but then 12 (the 2d toward the right hand from 8) is 4 more than 8: so lastly, 2 (the 3d toward the left hand from the 8) is 6 less than 8; but then 14 (the 3d toward the right hand) is 6 more than 8. So that nothing is more plain, than that each Term (one with another) being 8, that multiplied by the Number of Terms must give the Sum of all the Terms (or Eights): so the Sum of 7 of the Numbers of the 2d Series aforesaid is 56 (or 7 times 8). And so of any other Series, grounded on the first *Prop.* above.

Geometrical Progression.

This is when a Series of Numbers are increased by a continual Multiplication of the first Term, and the Products arising, by some certain Number called the Ratio or common Factor: as

1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024.
3, 9, 27, 81, 243, 729, 2187, 6561, 19683, &c.
4, 16, 64, 256, 1024, 4096, 16384, 65536, &c.

Prop. 1. To sum up any Series of Numbers, whose Relation is in a Geometrical Progression.

The first Way to find the Total of a Geometrical Progression.

Rule.] Multiply the last Number by the common Multiplier.

2dly, From that Product deduct the first Number.

3dly, Divide the Remainer by the said Multiplier less one; and the Quotient is the Sum sought.

The Example shall be in the finding the Aggregate of the middle Series above, the last Number whereof is 19683; which multiplied by the common Multiplier is 59049, from which take the first Term of the Progression (3) rests 59046, which divide by the said Multiplier less (1) *viz.* by 2, and the Quotient is 29523 = the Sum required.

The second and much briefer Way to find the Total of a Geometrical Progression, without many of the intermediate Terms, or the last Term being given.

Consider that in the Series of the last Example or middle Series there are 9 Terms; therefore if you multiply the 5th (or 243) by itself,

ſelf, it will at once produce the 59049 as before, (which would be the 10th Term, if the Progreſſion had run ſo far) whence the Sum is found as above. For,

Note 1. That when the Number of Terms are odd, and the firſt is above a Unit, the Square of the middle Term gives that next above the laſt Term, as in the laſt Example.

Note 2. If the Number of Terms be odd, and the firſt Term be but a Unit, then the Square of the middle Term gives the laſt Term of the Progreſſion, as in the firſt Series the Square of 32 is the laſt Term = 1024.

Note 3. If the Terms be an even Number, and the firſt Term be One or Unity; then the Square of the Sum in the place of half the Number of Terms gives the laſt Term ſave one: as in the 10 firſt Terms of the firſt Series, the Square of the fifth Term (16) gives 256 = the laſt of the 10 Terms but one.

Note 4. If the Number of Terms be even, and the firſt Term be more than a Unit, then the Square of the Sum in the place of half the Number of Terms, gives the laſt Term of the Progreſſion, as in the third Series the Square of 256 is the laſt Term 65536.

Theſe Rules are more particular and uſeful in the ſumming up all kinds of Geometrical Progreſſions than I have any where obſerved to be exhibited, and therefore worth nothing, for they always hold, as in the two Series's above; and where the Nature of the Progreſſion is as in the firſt Rank or Line; as is thus farther demonſtrated:

0,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,
1,	2,	4,	8,	16,	32,	64,	128,	256,	512,	1024,

The upper Series being Arithmetical, the Addition of any two, or Double of any one, ſhews reſpectively what is produced by multiplying or ſquaring thoſe under them in the lower Line. Thus 4 doubled gives 8 in the upper Line, under which ſtands 256, or the Square of the Number under the 4, *&c.* So any 2 added in the upper Line, as 3 and 6, give 9; which ſtands over the Product of the 2 Numbers 64 and 8, which is 512.

The Reaſon of the Rule for ſumming up a Geometrical Progreſſion.

It will be the more eaſily underſtood by a Series of a few Terms. Thus if I would know the Sum of 4 and 8, it is 4 (or the firſt Term) leſs than the laſt Term 8 multiplied by the common Factor

(2)

(2) which plainly shews why you deduct the first Term (here 4). So also in 2, 4, 8, the Double of 8 is 2 (or the first Term) more than the Sum 14, and therefore from the 16 must be deducted the first Term (2). Lastly, in 1, 2, 4, 8, twice 8 (the last Term) is 16, which is the first Term (1) more than the Sum (15). And this is sufficient to shew the Reason why we multiply the last Term by the common Factor, and why from the Product the first Term is deducted.

Now, as when the Ratio or common Factor is (2) the Sum of the Progression is $\frac{2}{1}$ the last Number after the first is deducted from the Product; so when the common Factor is 3, the Answer is $\frac{3}{2}$; if the common Factor be 4, the Answer is $\frac{4}{3}$ of the last Number (or Term) having before Division the first Term deducted from the Product. Thus in 3 and 9, the Sum is 4 three's or 12, which is thrice the last Term (9) less the first Term (3) divided by 2. So also in 4 and 16, the Sum is 4 times 16, less the first Term (4) dividing the Remainer by 3; as might easily (from so few Terms) at first be discovered. But see more at the end of this Section.

Quest. 1. A King of much Virtue and Valour, admirably esteem'd by his Subjects, returning from the Wars with Victory and Peace, very much Emulation appear'd who should exceed in Demonstrations of Joy. Among the rest, the Governour of the Fortress intending to signalize himself, orders his Sub-Officers to make so many Discharges of Cannon; *viz.* 3 for the first Year of the King's Age, (which was 25 Years) 9 for the second Year, 27 for the third Year, &c. (not considering the Impossibility of the Performance) the Question is, how many Discharges were to be made, and how much Powder consumed, supposing the Guns to be each a Culverin, whose Requisite of Powder is lb 10?

The

The 13th Term	1594323	Mult.
Ditto	1594323	
	4782969	
	3188646	
	4782969	
	6377292	
	14348907	
	7971615	
	1594323	
This is the 6th Term (or the next above the last) *per.* Note 1. foregoing.	2541865828329	
The first Deduct	3	
Rests =	2541865828326	

Years.	Dischar.
1st	3
2d	9
3d	27
4th	81
5th	243
6th	729
	729
	6561
	1458
	5103
12th Term by Note 4.	531441
	3
13th Term . . .	1594323

Half of which is = 1270932914163, the Sum of the Progression, or the Number of Discharges order'd.

Which, at 10 Pound of Powder at each Discharge, gives 12709329141630 ℔. at 100 ℔. each Barrel, is 1270932914163 Barrels; which would lade 14184965 Ships, at 400 Ton each Ship.

Quest 2. On the same Occasion all the Bells in the King's Metropolis were directed to ring as many Changes as they were capable of. Now there being 9 Churches, 1 of which had 2 Bells, another, 3, *&c.* the last 10 Bells; how many Changes could be rung on all the Bells at each Church, and what the Sum Total of all that Progression?

Churches.	*Bells.*	*Changes to be rang at each.*
St *Peter*'s	2	2
St *Andrew*'s	3	6
St *James*'s	4	24
St *John*'s	5	120
St *Bartholomew*	6	720
St *Thomas*'s	7	5040
St *Matthew*'s	8	40320
St *Simon*'s	9	362880
St *Jude*'s	10	3628800

The Number of Bells multipli-plied (any way) one in another, gives the last Term, or Changes that can be rung thereon. Then for the Sum, multiply the last Term by the correspondent Number of Bells, and divide the Product

duct by the Number of Terms, and the Quote is *the Answer for four Bells:* For 5 Bells add 2 to the Rule, for 6 add 8, for 7 add 32, for 8 Bells add 152, for 9 add 872, and for 10 add 5912, gives the Sum of the Progressions, here 4037912.

Note, The 2 is the first Term, the 8 the Sum of the 2 first, the 32 = the 3 first, the 152 = the 4 first, the 872 = the 5 first, and the 5912 = the Sum of the 6 first Terms.

Quest. 3. Any Line or Number given, to divide the same into extreme and mean Proportionals. To the Square of the whole Line or Number given, add ¼ of that Square; extract the Square Root of the Sum, from which Root deduct half the Line or Number given, and the Remainer is the greater part. And for a Proof, the Rectangle or Product of the whole Line by the lesser part, is equal to the Square of the greater part, by *Euclid* 11. 2.

4thly, If three Numbers be given in Geometrical Proportion continued, the Product of the two Extremes is equal to the Square of the Mean, as 5, 20, 80, equal to 400.

5thly, If two Numbers be given, it follows from the last, that a mean Proportional may be found by multiplying them together, and extracting the Square Root of the Product. Thus in the last Example 5 times 80 is 400; the Square Root of which is 20 = the Mean sought.

6thly, If you would find two Geometrical mean Proportionals between two Numbers given, as suppose between 5 and 320, you must divide the greater by the less; then extract the Cube Root of the Quotient; lastly, by that Root multiply the first Term, gives the first Mean, which multiply by the said Root gives the second, so is 5, 20, 80, 320, the Numbers in order, 20 and 80 being the Means required.

7thly, Or three mean Proportionals may be found thus: Suppose between 5 and 1280. Divide the greater Extreme by the lesser, extract the Biquadrate Root of the Quotient, and by that Root multiply the lesser Extreme continually 3 times; so will the Answer be found 5. 20. 80. 320. 1280.

8thly, If 4 Numbers be given in Geometrical Proportion, the Product of the two Extremes is equal to that of the two Means: so in the last Example 5 times 320 is equal to 20 times 80, *viz.* 1600.

9thly, If two Numbers be given, a third in Geometrical Proportion may be found by dividing the greater by the lesser, and multiplying such greater by the Quotient. As *per* the 5th, 80 is found

in proportion as 5 to 20; for as 5 to 20, so is 20 to 80, as may be proved by the 3d or 6th Cases above.

10thly, Three Numbers given in Geometrical Proportion, a fourth may be found by multiplying the second and third together, and dividing the Product by the first: as in the 5th 20 times 80 is 1600, which divided by 5, gives the 4th Proportional 320. This last is the Foundation of the Rule of Three Direct, which is the next thing to be taught after I have shewed

The Process for finding out the Canon whereby to discover the Sum of a Geometrical Progression.

But first note that

: : shews a *Geometrical Proportion.*

— signifies *less.*

= signifies *equal to.*

× signifies *multiplied in.*

And in the Literal Work,

f *the first Numb. of the Progression.*

r *the Ratio or common Factor.*

u *the Sum of the Progression,* which is unknown.

l *the last Term of the Progression.*

The Sum of the Progression to be found is this Example,

1, 2, 4, 8, 16, 32.

Literally performed.	Numerally performed.
f. fr : : u—l. u—f	that is 1. 2×1 : : u—32. u—1
fu—ff=r fu—rfl.	that is u—1×1—2×1 ×u—32×2×1
u—f=ru—rl	that is u—1=2×u—2×32
ru—u=rl—f	that is 2u—u=2×32—1
$u=\frac{rl—f}{r—1}$	that is $u=\frac{2\times32—1}{2—1}$ = the Answer.

So that 'tis plain the Answer is (2) the Ratio multiplied in (32) the last Term, made less by (1) the first Term, and that Remainer divided by (2) the Ratio, less 1, the Quotient is (63) the Sum of the Progression (or u): And this Canon (or Rule) holds in all Geometrical Progressions continued. Which the Reader will more easily apprehend, when he comes to the Algebraic Part, and till then he may pass this, if he finds it difficult to understand; but I could not well omit it in this place, to which it most properly belongs.

SECT.

SECT. III. *Rules of Proportion.*

IN this Section I ſhall ſhew the Operation, 1. Of the ſingle Rule of Direct Proportion. 2. The ſingle Rule of Reverſe Proportion. 3. The double Rule of Direct Proportion. 4. The double Rule of Reverſe Proportion. 5. The Rule of Proportion by 5 Numbers Direct. 6. The double Rule by 5 Numbers Reverſe. 7. A third Variety thereof. 8. Duplicate Proportion Direct. 9. Duplicate Proportion Reverſe. 10. Triplicate Proportion. 11. Harmonical Proportion. And, 12. Seſquiplicate Proportion.

I. *The Single Rule of Direct Proportion.*

This Rule has its Foundation from the 8th and 10th Propoſitions laſt foregoing, and is demonſtrated by *Euclid* in the 16. 6.

The main difficulty is to ſtate the 3 Numbers given right; for there are always 3 given to find a 4th, and hence ſome call this *the Rule of* 3; others, becauſe of its great Uſe, have termed it *the Golden Rule:* and 'tis called the Rule of Proportion, becauſe the Number ſought bears ſuch Proportion to the 3d, as the 2d does to the firſt of the Numbers given.

For the right diſpoſing or placing the 3 *Numbers given.*

Rule.] There being always 2 of one Denomination, and 1 of another, put the laſt mentioned down firſt: 2dly, Put that towards the left hand thereof which has dependance on it: and, 3dly, Put the other next the right hand. Thus if the Intereſt of *l.* 750 were ſought for a Year at the Rate of 5 *per Cent.* it appears by the State in the Margin, that only 1 of the 3 given Numbers is Intereſt, therefore I place that in the middle. And becauſe 100 has dependance on it (as being its Principal) therefore I put 100 towards the left hand, and the 3d toward the right hand (in courſe) Then

l. Princ.	*l.* Inter.		*l.* Princ.
100.	5	: :	750
			5
			3750

37.$\frac{50}{100}$ Anſwer.

To find the 4th Proportional, or Anſwer to the Queſtion.

Rule.] *Multiply the 2d and 3d (or thoſe towards the right hand) together, and divide the Product by the firſt (or that next the left hand) and the Quotient is the Anſwer, or the 4th Proportional ſought for.* So in the Example

Example above, the 4th Proportional is $37\frac{5}{10}$ for 100. 5 :: 750. $37\frac{5}{10}$, that is, as 100 is in proportion to 5, so is 750 to $37\frac{1}{2}$. So that you may prove this Rule by comparing the Product of the 1st and 4th with the Product of the 2d and 3d, as by the sixth last mention'd Prop. in Geometrical Progression.

Case 2.] When the first Number is a Unit only, the Work is done by Multiplication alone.

Thus if 1 Piece of Broad-Cloth cost 17 *l.* what will 71 cost? Here the 4th Number is found 1207 *l.* which is the 4th Proportional sought.

Pc.	*l.*	Pc.	*l.*
1	17 ::	71.	1207
		17	
		497	
		71	
		1207	

2dly, If I give 3 *s.* 9 *d.* for an Ell of Holland, what will $1371\frac{3}{8}$ cost at that rate?

By the Rule above the Question is stated as in the Margin, and you may multiply $1371\frac{3}{8}$ by 3 *s.* $\frac{9}{12}$, as in the 4th Example of Multiplication of Fractions, and the Answer is $4936\frac{95}{96}$ Shillings; which, *by the 5th Case of Reduction of Vulg. Fractions*, is *l.* 257 : 2 : 7 : $3\frac{1}{2}$, as *per* Margin. But these Questions are sooner solved by Practice, as *per* the next Section.

Case 3.] When the 3d Number is only a Unit, the Question is answered by Division. Thus if I give *l.* 257 : 2 : 7 : $3\frac{1}{2}$ for $1371\frac{3}{8}$ Ells of Holland, what is that *per* Ell?

The Question is thus stated: and being divided (as *per* the first Example of Division of Vulgar Fractions) the 2d by the 1st, the Quotient is $\frac{3949560}{1053216}$ *s.* which by the said 5th Case of Reduction of Vulgar Fractions is 3 *s.* 9 *d.* and proves the Truth of the last Question.

Ells	Shill.	Ell
$1097\frac{1}{8}$	$4936\frac{95}{96}$ ::	1

Ell	*s.* *d.*	Ells
1	3 : 9 ::	$1371\frac{3}{8}$

96) 493695 (5142 *s.*
....
136
409
255
63 *s.* Rem.
12 Mult.
96) 756 (7 *d.*
84 *d.* Rem.
4 Mult.
96) 336 (3 *q.* $\frac{48}{96}$
48

Note, That where the first, or third Number, or both, are of diverse Denominations, you must reduce both such 1st and 3d into the least Name mentioned in either, and then

then multiply by the second Number in the lowest Denomination that is in it when given. And then having divided by the first,

Note, 2*dly*, That your first Quotient will always be of the same Denomination with the lowest mention'd and given in your second Number. To illustrate these three Cases,

Case 4.] If I give *l.* 7 for the Interest of *l.* 100 : 10 : — : what must I give for the Interest of *l.* 30 for the same time; See the Work in the Margin, where tho' the 1st and 3d be reduced to Shillings (that being the least Term in the 1st Number) yet the Answer retains the same Denomination with the 2d Number, and is *l.* $2\frac{18}{201}$; or by the 5th Case of Reduction of Vulgar Fractions *l.* 2 : 1 : 9½ *prope.*

```
   Prin.
 l.    s.  l. Int.  l. Prin.
100 : 10    7 ::     30
 20                  20
----                ----
2010 Shil.          600 Shil.
                      7
                    ----
             2010) 4200 (2 l. Inter.
                   ----
                     18
```

Case 5.] If I give 7 *l.* for 30 Ounces of Silver, what will 7 Ounces and 10 Penny-Weight cost at that rate?

Here it is plain that because the 7 ℥ 10 dw. are reduced into Penny-Weight, the 30 ℥ must be so too: Also that the Quote is of the same Denomination with the 2d Number, *viz.* *l.* 1¼; or, being reduced as aforesaid, *l.* 1 : 15.

```
 ℥.   l.    ℥.     dw.
 30.  7 ::  7   :  10.
 20         20
 ---        ---
600 dw.    150 Penny-Weight.
            7
          ----
    600) 1050 (1 l.
          ----
           450
            20
          ----
    600) 9000 (15 s.
          ----
           30
          ----
            0
```

Case 6.] If 25 Ounces of Ambergreace cost *l.* 52 : 6 : 8, what will 2 Ounces cost?

See

See the Operation, where the 2d Number in its least Denomination being 12560 Pence; therefore the Quote 1004 is Pence.

```
 ℥     l.    s.   d.   ℥
25.   52 : 6 : 8.    2
      20
     ————
     1046 s.
       12
     ————
    12560 Pence  } Mult.
        2 Ounces }
     ————
25) 25120(1004 20/25 Pence.
     ....
     ————
      120   or l. 4 : 3 : 8⅘ the Answer.
      ———
       20
```

II. *The Single Rule of Proportion Reverse.*

This teaches, upon a due stating of the 3 Numbers, to find a 4th; which shall bear such a Ratio to the 2d, as the 1st does to the 3d Number.

Quest. 1. How much Matting of 2 Foot broad will line a Gallery which is 7 Foot broad and 40 Foot long?

To state a Question in this Rule,

1. Put down the Number which has the same Denomination as that required.

2. Towards the left hand put down that Number which is joined in the Sense of the Question to the last.

```
broad.  long.  broad  long.
  7.     40.     2.   140.
          7
        ———
      2)280(140 = the Answer.

And as 140. 40 : : 7. 2.
```

3. Put down the other which has the same Denomination as that towards the left hand.

Then to find the Answer by this,

Rule.] Multiply the 2 next the left hand one in the other, and divide the Product by the 3d, or that next the right hand, and the Quotient is the Answer.

Hence the Reason is evident why this Rule is said to be Reverse: because the lesser the 3d Number is, the greater is the 4th; but in Direct

Direct Proportion the greater the 3d is, the greater is the 4th Number.

So that by this Rule it is easy to know when a Question is to be solved as Direct, and when it is Reverse: For when the 3d Number is less than the first, and yet requires more than it does, or being more, requires less than the first requires; in these Cases your Proportion is Reverse, and so you are to work by the last Rule.

Example 2.] How much Ground in length, being 12 Perches broad, will make an Acre?

To do this, consider that 4 Perches in breadth requires 40 in length to make an Acre; therefore state and work as *per* Margin.

broad.	long.	broad.
4.	40.	12.
	4	
	———	
12)	160(13⅓	Perch. long, for Answer.
	··	
	———	
	40	
	———	
	4	

But altho' Writers have prescribed the foregoing Method for stating a Question, as being very natural, and thereby have made it necessary to make this a different kind of Operation and Proportion; yet I shall shew

How to reduce Questions commonly said to be in Reverse Proportion into Direct.

And this is only to consider (to instance in the last Example) That as (12) the Breadth on which the Answer depends,
Is to (4) the Breadth belonging to the Length given ::
So is (40) the Length given,
To (13⅓) the Length required.

Breadth.	Bread.	Length.	Len.
12.	4 ::	40.	13⅓

Here the 4 Numbers are in Direct Proportion, because the Product of the 2 Means is equal to that of the 2 Extremes; *which, by the by, serves for Proof of this Direct Proportion.* But in the Reverse the Product of the 1st and 2d is equal to that of the 3d and 4th Numbers; *which proves the Reverse Proportion.*

III. *The Double Rule of Direct Proportion.*

This is when a Question requires to be twice stated, and has two such Operations, as under the single Rule. For Example,

If I give 5 *l.* for the Interest of *l.* 100 for one Year, what Interest will *l.* 700 gain at that rate in seven Months?

1st say, If *l.* 100 require *l.* 5, what will *l.* 700 require in the same time? You find as *per* the Margin that it will gain *l.* 35.

$$\begin{array}{ccc} l. & l. & l. \\ 100. & 5 :: & 700 \\ & & 5 \\ \hline & & l.\ 3500 \end{array}$$

2dly say, If 12 Months require *l.* 35, what will 7 Months require? Work and you find the Answer *l.* 20$\frac{5}{12}$, or *l.* 20 : 8 : 4.

And by the same Rule you may find the Interest of any Sum for any Number of Days; as suppose *l.* 700 for 119 Days.

$$\begin{array}{ccc} \text{Mon.} & l. & \text{Mon.} \\ 12. & 35 :: & 7 \\ & 7 & \\ \hline & & l. \\ 12) & 245\ (20\frac{5}{12} & \\ \hline & 5 & \end{array}$$

For a Year, by the first Operation, the Interest is *l.* 35 : then in the second Working say, if 365 Days requires *l.* 35, what will 119 Days require? You'll find the answer to be *l.* 11 : 8 : 2 : 2$\frac{190}{365}$.

$$\begin{array}{ccc} \text{Days.} & l. & \text{Days.} \\ 365. & 35 :: & 119 \end{array}$$

IV. *The Double Rule of Reverse Proportion.*

This is also performed at two Operations, the first Direct, the later Reverse. For instance,

What Principal will gain *l.* 20$\frac{5}{12}$ in 7 Months, as the Rate of 5 *per Cent?*

	l. Int.	*l.* Princ.	*l.* Int.	Princ.
1st say	5.	100 ::	20$\frac{5}{12}$.	$\frac{24500}{60}$ *l.*
	Mon.	*l.*	Mon.	*l.*
2dly say	12.	$\frac{24500}{60}$:	7.	700

The 1st Answer is, that $\frac{24500}{60}$ *l.* Principal will raise the 20$\frac{5}{12}$ in a Year.

The 2d, or Answer to the Question, is, That *l.* 700 is the Principal that will raise the Interest *l.* 20$\frac{5}{12}$ in 7 Months: by multiplying the 2d Number by 12, and dividing by 7. *Note*, This proves the first Question in the third Head above.

V. *The Rule of Proportion by 5 Numbers Direct.*

This Rule hath 5 Numbers given to find a 6th. It is the same with the double Rule of Proportion Direct; only whereas that was done

done by twice stating the Question, this is done by one stating: *the Rule for which is to place the 5 Numbers as followeth.* Where 'tis obvious, That the 3d must be of the same Denomination with that sought; the 1st and 2d are the 2 on which the 3d depends: so that the 4th must be of the same Denomination with the 1st, and the 5th the same with the 2d.

Example.] What is the Interest of *l.* 700 for 7 Months at the Rate of 5 *per Cent. per Ann.*? By the Work in the Margin, you see

l. Prin.	Time.	*l.* Int.	*l.* Prin.	Time.
100.	12.	5.	700.	7
12				7 multiply.

1200 = the Divisor.

4900 }
5 } multiply.

1200) 24500 ($20\frac{5}{12}$ = the Answer

The Rule for the Operation is, Multiply the 3 Numbers next the right in each other for a Dividend, and the 2 next the left hand for a Divisor, and the Quotient is the 6th Proportional sought; as appears in the Method, by double stating under Head III.

And to prove this; The Product of the 1st, 2d, and 6th, is equal to that of the 3d, 4th, and 5th.

Quest. 2.] If 1000 Men in 24 Hours can dig a Trench 18 foot broad, 9 deep, and 500 long; what length of the like Trench can 9800 Men dig in 10 Hours? The Answer as *per* Margin, is $2041\frac{2}{3}$ Feet.

Men.	Hours.	Feet.	Men.	Hours.
1000.	24.	500.	9800.	10.
24				10 multiply.

98000 }
500 } multiply.

24000) 49000000 ($2041\frac{2}{3}$ foot Answer.

100

40

16

Whence it appears, That as the Product of the 1st and 2d Numbers is to the 3d: so is the Product of the 4th and 5th to the 6th Number. *Which also serves to prove the Rule*;

For as 24000. 500 : : 98000. $2041\frac{2}{3}$.

And 24000 multiplied in $2041\frac{2}{3}$ is equal to 98000 multiplied by 500, *viz.* 49000000, as *per* Prop. 6. of Geom. Progression.

VI. *The second Variety of the Rule of Proportion by 5 Numbers.*

This is commonly called *the Rule by 5 Numbers Reverse.*

Example.] How many Men can dig $2401\frac{2}{3}$ feet in 10 Hours, at the rate of 1000 digging 500 feet in 24 Hours. *State your Question thus:*

Men.	Hours.	Feet.	Feet.	Hours.
1000 :	24 :	500.	$2041\frac{2}{3}$	10.

i. e. If 1000 Men in 24 Hours dig a Trench of 500 Feet in length, how many Men can dig a Trench of $2041\frac{2}{3}$ feet long in 10 Hours?

To answer this, multiply the 1st, 2d, and 4th Numbers together for a Dividend, which divide by 5000, the Product of the 3d and 5th Numbers, and the Quotient is 9800 Men for Answer.

VII. *A third Variety of the Rule of Proportion by 5 Numbers.*

This (altho' omitted by Authors) is as likely to be used as the last: for (to put the same Question again, that the varying may the better appear)

Men.	Hours.	Feet.	Men.	Feet.
1000 :	24 :	500.	9800.	$2041\frac{2}{3}$.

If 1000 Men in 24 Hours can dig 500 foot of a Trench, (as mention'd under the 5th Head above;) in how many Hours can 9800 Men dig $2041\frac{2}{3}$ feet (of the like Trench)?

Rule.] Multiply the 1st, 2d, and 5th together, produceth 49000000 for a Dividend: Then multiply the 3d and 4th together for a Divisor, which is 4900000. So the Quote is 10 Hours, as in the two Cases above.

In the	1st Case the Feet to be digged was sought, and found ———	$2041\frac{2}{3}$ Feet.
	2d —— the Number of Men to dig the larger Trench in the lesser Time, and found	9800 Men.
	3d —— the Time in which the larger Number could dig the larger Trench, and found	10 Hours.

For Proof of the 2d Variety or Kind of Proportion by 5 Numbers: The Product made of the 1st, 2d, and 4th, is equal to that of the 3d, 5th, and 6th.

And for Proof of the 3d Variety: The Product under the 3d, 4th, and 6th, is equal to that of the 1st, 2d, and 5th.

VIII. *Of*

VIII. *Of Duplicate Proportion Direct.*

This differs from the single Rule of Proportion Direct, in this: 2 of the 3 (the 1st and 3d) requiring to be squared, when that sought (or the 4th Number) is not a Square, and 1, *viz.* the middle or 2d Number, when that sought is a square Number (or one whose Root must be extracted to give the specifick Answer):

Or else it is when the Ratio is as the Square of one thing is to that of another, *&c.*

Examples.

The Diameter of a Circle being 2, the Area is $3\frac{1416}{10000}$: therefore What is the Area or superficial Content of a Circle, whose Diameter is 8?

The Sqr. of the Diam.	The Area.		The Sqr. of the Diam.	The Area.
4.	$3\frac{1416}{10000}$	::	64.	$50\frac{2656}{10000}$

The Rule for stating the Question is the same as for the single Rule Direct: And the Diameters being squared as you see (4 being the Square of 2, and 64 of 8) *the Rule for performing the Operation* is the same as for single Direct Proportion; for $3\frac{1416}{10000}$ multiplied by 64, and that Product divided by 4, the Quotient is $50\frac{2656}{10000}$ for Answer. For *Euclid* has demonstrated, that Circles are in proportion as the Squares of their Diameters; and accordingly, by this Proportion above we find the Area of a Circle $50\frac{2656}{10000}$, whose Diameter is 8.

Example 2.] What is the Diameter of a Circle, whose Area is $50\frac{2656}{10000}$? supposing the Diameter of Circle, whose Area is $3\frac{1416}{10000}$, be 2, (as it really is.)

This is stated as *per* Margin, and the Operation done as before. For

The Area.	The Sqr. of the Diam.		The Area.	The Sqr. of the Diam.
$3\frac{1416}{10000}$.	4	::	$50\frac{2656}{10000}$.	64

As $3\frac{1416}{10000}$ (the Area of a Circle)
Is to 4 (the Square of that Circle's Diameter:)
So is $50\frac{2656}{10000}$ (or the Area of any other Circle)
To (64) the Square of the Diameter of that Circle.

Now extract the Square Root of 64, and you have the Answer, *viz.* 8.

This and the last Example prove each other.

IX. *Duplicate Proportion Reverſe.*

Example.] Admit the Pendulum of our common Clocks is 39 Inches, (as it is very little more) we know that ſuch a Clock vibrateth Seconds (or 60 times in a Minute). Now what is the Length of a Pendulum that vibrateth Half-Seconds (or 120 times in a Minute)?

Queſtions under this Rule are moſt naturally ſtated as in ſingle Reverſe Proportion above; and being ſo ſtated, muſt be ſo wrought after the Vibrations are ſquared. By this State it appears that the Proportion is Reverſe: for if 60 Vibrations

Vibrations.	Inches.	Vibrat.	Inches.
60.	39.	120.	$9\frac{3}{4}$

require a Pendulum 39 Inches long, 'tis plain that a Clock, whoſe Vibrations are 120 in a Minute, muſt have its Pendulum ſhorter (for the longer the Pendulum, the fewer the Vibrations in a Minute; and the contrary): therefore more requiring leſs, ſhews the greater Extreme (or 120) muſt be your Diviſor, as appears by what is ſaid under the firſt Example of ſingle Reverſe Proportion. So that multiplying the Square of 60 = 3600 by 39, and dividing the Product by the Square of 120 = 14400, the Quotient or Anſwer is $9\frac{3}{4}$ Inches.

Example 2.] A Pendulum of 39 Inches vibrating 60 times in a Minute; how many times does that Pendulum vibrate in a Minute, whoſe Length is $9\frac{3}{4}$?

Here, by multiplying the two firſt towards the left hand together, *viz.* 39 and 3600 = (the

Inches.	The Sqr. of the Vibrat.	Inches.	The Sqr. of the Vibrat.
39.	3600.	$9\frac{3}{4}$	14400

Square of 60) and dividing the Product by $9\frac{3}{4}$ (as taught in Diviſion of Vulgar Fractions) the Quotient is 14400, the Square of the Anſwer: therefore extract the Square Root of 14400, and you'll find it 120 = the Number of Vibrations in a Minute of a Pendulum whoſe Length is $9\frac{3}{4}$ Inches.

Example 3.] Admit a Body on the Surface of the Earth (or 4000 Miles diſtant from the Center thereof) weigh 20 Hundred weight, what will that Body weigh if it were 12000 miles above the Surface, or 16000 from the Center of it? State your Queſtion thus;

Miles.	C. wt.	Miles.
4000	20	16000

Multiply

Multiply the Square of the 1st by the 2d, and divide the Product by the Square of the 3d, and the Quote is $1\frac{1}{4}$ *C.* Which shews, that a Ton weight, if 12000 miles high, will weigh but $1\frac{1}{4}$ *C.* and if 189314 miles high, 'twill weigh but a Pound, by the same Rule. So likewise the Velocity of Swiftness of Descent is also in proportion to a heavy Body's distance from the Center of the Earth in a duplicate Ratio: for the Celerity of the Fall on the Earth (or 4000 miles from the Center) being 16 foot in a Second; if the same Body be 16000 miles from the Center, it will fall but 1 foot in a Second, as you may easily prove by this present Rule, as abovesaid.

I have given an Example under the single Rule of Proportion Reverse, how these last, *&c.* may be performed as in Direct Proportion, by saying (in the 2d Example)

As $9\frac{3}{4}$ to 39 : : so 3600. to 14400, whose Root is 120.

But then this neither agrees, as to stating the Question, with the Rules given for that purpose under the single Rules Direct nor Reverse, and consequently these Questions are most properly and naturally to be performed as directed above, as agreeing with the Rules given for stating and working such Questions.

X. *Triplicate Proportion.*

As in Duplicate Proportion the Ratio is as the Square of one Number is to another Number, or the contrary; so this kind, the Ratio is as one Number is to the Cube of another, or as the Cube of one Number is to some other Number.

Example 1.] If a Sphere (or Ball) which is 8 Inches Diameter, weighs 48 Pounds; what will another Sphere of the same Specie of Matter weigh, whose Diameter is 4 Inches?

This is a single direct Proportion, (working with the Cubes of the Diameters) where the 4th Proportional is found ℔ 6. For

The Cube of the Diam. 8.	℔ *its Weight.*	*The Cube of the Diam.* 4.
512.	48 : :	64.

As 512 = (the Cube of the Diameter 8) is to 48 Pounds weight,
So is 64 (the Cube of the Diameter 4) to 6 Pounds, the Weight required.

Example 2.] If a Sphere weighing 48 ℔. be 8 Inches Diameter, what is the Diameter of another Sphere of the same Matter, whose Weight is 6 ℔?

Here

Here by multiplying the 2d and 3d Numbers together, and dividing the Product by the 1st; the 4th Proportional is 64, the Cube Root whereof is 4 = the Answer. For

℔ *the Weight.*	*The Cube of the Diam.* 8.	℔ *the Weight.*
48.	512 ::	6.

As 48 Pounds weight is to 512 (the Cube of the Diameter 8)
So is 6 Pounds weight to 64 (the Cube of the Diameter sought) whose Root is 4.

And as the Weight, so the Solidity of Spheres is found by the same Ratio.

Example 3.] Admit the solid Content of a Sphere be $4\frac{1888}{10000}$, whose Diameter is 2; what is the Solidity of another Sphere, whose Diameter is 4?

The Cube of the Diam. 2.	*The Solidity.*	*The Cube of the Diam.* 4.	*The Solidity.*
8	$4\frac{1888}{10000}$ ::	64	$33\frac{5104}{10000}$

Here you see that As the Cube of the Diameter 2, *viz.* 8.
Is to the Solidity of that Sphere $4\frac{1888}{10000}$:
So is the Cube of the Diameter 4, *viz.* 64.
To the solid Content of that Sphere, *viz.* $33\frac{5104}{10000}$.

Example 4.] The Diameter of a Sphere being 2, whose Solidity is $4\frac{1888}{10000}$; what is the Diameter of another Sphere, whose Solidity is $33\frac{5104}{10000}$? Answer 4; the Cube Root of 64: for as $4\frac{1888}{10000}$. 8 :: $33\frac{5104}{10000}$. 64.

XI. *Harmonical Proportion.*

In this kind of Proportion there are 3 Numbers given to find a 4th, which shall bear such direct Proportion to the first, as the Difference between the 3d and 4th bears to the Difference between the 1st and 2d.

It is called Harmonical (or Musical Proportion) probably by reason of the double Ratio of the Numbers among themselves.

The Rule to find the 4th Proportional, is, Multiply the 1st and 3d together, and divide the Product by double the 1st, less the 2d; and the Quotient is the 4th Number required.

Example.] What is the 4th Proportional Harmonical to these 3 Numbers 7 : 10 : 12? Answer 21.

See

See the Work in the Margin.

For Proof of this, as abovesaid:

21. 7 : : 9. 3

That is, as 21, the 4th Number (or that required in the Question) is to 7 (the 1st Number) so is the Difference between 12 and 21, *viz.* 9. to the Difference between the 1st and 2d, or 7 and 10, *viz.* 3.

	7 : 10 : 12 : 21		
	2		7
Product	14	= 4)	84 (21
Less	10		
Rests	4		4
			0

Hence as a Corollary it will appear, that double the 1st Number must exceed the 2d Number, otherways you'll want a Divisor, and cannot find the requisite Number without the Arithmetic of Negatives.

XII. *Sesquiplicate Proportion.*

In this kind of Ratio the Square of one Number is in proportion to the Cube of another, and the contrary.

It is used in finding the Distance of the Planets in the Solar System from the Sun their Center, about which they revolve: For

As the Square of the Time in which any Planet finisheth its Periodical Revolution,
Is to the Cube of its Distance from the Sun : :
So is the Square of any other Planet's Time
To the Cube of its Distance. Thus

	Days.		*Miles.*
The Earth finishes its Period in — —	$365\frac{1}{4}$	And its Distance from the Sun is	86000000
Mars — — —	625	— — — about	130000000
Jupiter — — —	4333	— — — — —	440000000
&c.			

SECT. IV. *Rules of Practice.*

THIS is a briefer Method of casting up any Quantity of Merchandize than the Rule (commonly used) of Proportion or Multiplication. And 'tis done by considering what even Parts of a Pound the given Price is, or is reducible to, and then framing a Rule accordingly.

It

It is called the Rule of Practice because of its excellent Use in the Practice of Merchandize, for dispatching many Computations with much ease and in a short time.

I shall not trouble the Reader with shewing him a Table of the even Parts of a Shilling or a Pound, by reason I have given an intire Table, with the Fabrick thereof, at the end of Division, and have reduced the whole Business of Practice into Rules arising gradually in the Value of the Integer in so regular and copious a Method, that any one may easily find his Rate or Value of the Yard, Ell, Pound, Ounce, *&c.* and right against it a Rule how to perform the Operation after the best and most concise manner. And he will generally know which are the even Parts of a Pound by the Shortness of the Rule, being only one Division required to bring the given Number into Pounds respectively. I have marked the Aliquot Parts of a Pound with *A. P. L.* in the Table: And I think I may affirm, that this most copious, regular, and short Way of Practice, was first (and only) shewn by myself, where I take no notice of the Parts of a Shilling, but of a Pound only.

The main thing that makes the Rules of Practice preferable, is the performing the Operation mentally, without putting down any Figures but the Quotients or Answers for the most part: And this is done by chusing proper Divisors, few or none of which exceed 12; to which Number inclusive my Multiplication-Table foregoing extends, and is supposed to be perfectly in the Reader's memory.

But before I proceed to the Tables, it will be necessary to give *these previous Cases.*

Case 1.] *When the Value of the Integer is* 2 *s.* you have the Answer by only cutting off the Units place (or dividing by 10) 2 *s.* being the 10th part of a Pound; and the Figure in Units place is so many two Shillings. Thus 3476|5 Ells at 2 *s.* each is *l.* 3476 : 10; those to the left hand Units place being so many Pounds. So also 973|4 at 2 *s.* is *l.* 973 : 8 : —; and 87|5 is *l.* 87 : 10 : —

Case 2.] From the last Case it will follow, *that if the Price is any even Number of Shillings for the Unit*; the whole Parcel will be half so many Shillings as the Value of the Unit is, multiplied in what such Parcel would amount to at 2 *s.* Thus 7876 at 18 *s.* is *l.* 787|6 at 2 *s*, therefore it is 9 times that at 18 *s.* So multiply 787|6 by 9 thus,

9 times

9 times 6 (cut off) is 54 two Shillings, put down 8 *s.* and carry *l.* 5; then 9 times 7 *l.* is 63, and 5 carried is 68; and so proceed to multiply the 78, and you will find the Answer to be *l.* 7088 : 8 : 0. And by the same Rule 19468 at 16 *s.* is *l.* 15574 : 8 : 0. I say 8 times 8 is 64, twice the 4 is 8 *s.* and carry *l.* 6, &c. Likewise 97357 at 14 *s.* is *l.* 68149 : 18 : 00, &c.

Case 3.] *When the Value of the Integer or Unit is* 1 *s.* what is the Value of 135694?

135694
|
Answer *l.* 6784 : 14 : 00

You must take a 20th part, cutting off Units place, and taking half what rests to the left hand, as half 13 is 6, ½ of 15 is 7, ½ of 16 is 8, and ½ of 9 is 4, and the 10 *s.* remaining, and the 4 cut off is 14 *s.* because the Value at 1 *s. per* Unit is half as much as at 2 *s.*

Case 4.] *When the Price of the Unit is* 6 *d.* take a 4th of the given Number, which suppose 17372, cut off the 2 from 1737|2, and take a 4th of the rest: So the Answer at one Work is *l.* 434 : 6 *s.* for 12 Sixpences remains. For 4 in 17 is 4, 4 in 13 is 3, and 4 in 17 is 4, the 434 *l.*

Case 4.] *When the Price of the Unit is* 4 *d.* and you are directed to take a 60th part of the given Number, which suppose — — — — — — — 556|7

Here the Units place being cut off, I take a 6th of what is to the left hand the dash, thus 6 in 55 is 9 and rests 1; 6 in 16 is 2, and there rests 4, which with the 7 is 47 Groats, or 15 *s.* 8 *d.* the Answer.

$\frac{1}{60}$ is *l.* 92 : 15 : 8

Case 6.] *When the Value of the Unit is* 3 *d.* and you are directed in the subsequent Rules to take $\frac{1}{80}$ of the given Number, which suppose 7539; cut off the Units place, and take $\frac{1}{8}$ of the remaining Figures, as 8 in 75 is 9, 8 in 33 is 4, and 19 Three-pences (or 4 *s.* 9 *d.*) over.

753|9

$\frac{1}{80}$ is *l.* 94 : 4 : 9

Case 7.] *When the Price of the Unit is* 2 *d.* and you are directed to take $\frac{1}{120}$ of the given Number, which suppose 19739; cut off the 9 in Units place, and take $\frac{1}{12}$ of the rest: so 12 in 19 is 1, 12 in 77 is 6, 12 in 53 is 4, and 59 Two-pences rests, or 9 *s.* 10 *d.* And that the Remainers in these

1973|9

$\frac{1}{120}$ = *l.* 164 : 9 : 10

four last Cases may not, when large, seem difficult to cast up, I shall subjoin *this little Table of Remainers* in Shillings and Pence.

Remainers.	*Two-pences.*	*Three-pences.*	*Four-pences.*	*Six-pences.*
10	1 *s.* 8 *d.*	2 *s.* 6 *d.*	3 *s.* 4 *d.*	5 *s.* 0 *d.*
20	3 4	5 0	6 8	10 0
30	5 0	7 6	10 0	15 0
40	6 8	10 0	13 4	
50	8 4	12 6	16 8	
60	10 0	15 0		
70	11 8	17 6		
80	13 4			
90	15 0			
100	16 8			
110	18 4			

Thus by this Table the 47 Groats which remained in the 5th Case above, is 15 *s.* 8 *d.* for 40 under Four-pences is 13 *s.* 4 *d.* and 7 Groats (or 2 *s.* 4 *d.*) is 15 *s.* 8 *d.*

Note 1. That the Remainers are of the same Denomination with the Dividends: As in the Example to the 5th Rule I suppose in the first Operation the Dividend Two-pences, and therefore dividing by 120, I find 47 Two-pences remaining, or 7 *s.* 10 *d.* And what remains after in taking any part of the Quotients, are Pounds, Shillings, or Pence, as those Quotients are. The Cases above, and the Rules and Examples following, make this very evident.

The

The Value of the Pound, Ell, Yard, &c.	s. d. qr.	*Here followeth 149 Rules (with Examples for such as seem the most difficult) for the brief working by Practice to give the Answer in Pounds, &c. the like not extant by any other Hand.*	*Number of Rules.*
A.P.L.	1	Take $\frac{1}{80}$ of $\frac{1}{12}$ of the given Number (or $\frac{1}{960}$ thereof.)	1
A.P.L.	2	Take an 80th of a 6th of the given Number — — —	2
A.P.L.	3	Take $\frac{1}{80}$ of $\frac{1}{4}$ of the given Number (or $\frac{1}{320}$ thereof) ——	3
A.P.L.	00 : 1 : 0	Take a 20th of a 12th of the given Number, or take an 120th and deduct $\frac{1}{2}$ thereof — — — — —	4
A.P.L.	1 : 1	To $\frac{1}{2}$ of 120 of the given Number add a 4th of that $\frac{1}{2}$ —	5
A.P.L.	1 : 2	Take a 20th of an 8th of the given Number, or $\frac{1}{160}$ —	6
	1 : 3	From 120th of the given Number take an 8th of that 120th — — — — — — —	7
A.P.L.	2 : 0	Take an 120th of the given Number —— —— ——	8
	2 : 1	To an 120th of the given Number add an 8th of that 120th —— —— — — — —	9
A.P.L.	2 : 2	To an 120th of the given Number add $\frac{1}{4}$ of that 120th, or take $\frac{1}{96}$ — — — — — —	10
	2 : 3	From an 80th of the given Number take a 12th of that 80th — — — — — —	11
A.P.L.	3 : 0	Take an 80th of the given Number, as *per* Case 6. — —	12
	3 : 1	To an 80th of the given Number add a 12th of that 80—	13
	3 : 2	To an 80th of the given Number add a 6th of that 80 —	14
A.P.L.	3 : 3	To an 80th of the given Number add a 4th of that 80th, (or take a 64th) — — — — —	15
A.P.L.	4 : 0	Take a 60th of the given Number, as *per* previous Case 5.	16
	4 : 1	To an 120th of the given Number twice put down, add an 8th of 120th — —— ——	17
	4 : 2	To a 60th of the given Number add an 8th of that 60 —	18
	4 : 3	To a 60th of the given Number add an 8th of that 60th and $\frac{1}{2}$ that 8th —— —— ——	19
A.P.L.	5 : 0	Take an 8th of a 6th of the given Number or take $\frac{1}{48}$ —	20
	5 : 1	From a 40th of the given Numb. take an 8th of that 40	21
	5 : 2	From a 40th of the given Number take a 12th of that 40th —— —— —— ——	22
	5 : 3	From an 80th of the given Number twice put down, take a 12th of an 80th —— ——	23
A.P.L.	6 : 0	Take a 40th of the given Number, as *per* previous Case 4.	24

Example to Rule 5.	*Example to Rule* 17.	*Example to Rule* 23.
19727 at 1*d.* 1*q.*	9376 at 4*d.* 1*qr.*	8935 at 5*d.* 3*qr.*
$\frac{1}{120}$ = *l.* 164 : 7 : 10	$\frac{1}{120}$ = *l.* 78 : 2 : 8	$\frac{1}{80}$ *l.* 111 : 13 : 9
$\frac{1}{2}$ of that *l.* 82 : 3 : 11 } ad	ditto *l.* 78 : 2 : 8 } ad	ditto 111 : 13 : 9
$\frac{1}{4}$ of that $\frac{1}{2}$ *l.* 20 : 10 : 11$\frac{1}{2}$	$\frac{1}{8}$ of $\frac{1}{120}$ *l.* 9 : 15 : 4	Sum, *l.* 223 : 7 : 6
Sum, *l.* 102 : 14 : 10$\frac{1}{4}$ the Answer.	Sum, 166 : 00 : 8 Ans.	$\frac{1}{12}$ of $\frac{1}{80}$ = 9 : 6 : 1$\frac{1}{4}$ ded.
		Rests = 214 : 1 : 4$\frac{1}{4}$ Ans.

The Continuation of the Table of Rules for the short Work of Practice, &c. as before.

	The Value of the Pound, Ell, Yard, &c. d. qr.		Number of Rules.
	6 : 1	To an 80th of the given Number twice put down, add a 12th of an 80th	25
	6 : 2	To a 40th of the given Number add a 12th of that 40th	26
	6 : 3	To a 40th of the given Number add an 8th of that 40th	27
	7 : 0	To a 40th of the given Number add a 6th of that 40th	28
	7 : 1	To a 40th of the given Number add a 6th of that 40th, and a 4th of that 6th	29
A. P. L.	7 : 2	To a 40th of the given Number, add a 4th of that 40th (or take $\frac{1}{32}$)	30
	7 : 3	To a 40th of the given Number add a 4th of that 40th, and a 6th of that 4th	31
A. P. L.	8 : 0	Take a 30th of the given Number	32
	8 : 1	From an 80th of the given Number put down three times, take the 4th of an 80th	33
	8 : 2	To a 60th of the given Number, put down twice, add an 8th of a 60th	34
	8 : 3	From an 80th of the given Number thrice put down, take a 12th of an 80th	35
	9 : 0	To a 40th of the given Number add half of that 40th	36
	9 : 1	To an 8th of the given Number thrice put down, add a 12th of an 80th	37
	9 : 2	To an 80th of the given Number thrice put down, add a 6th of an 80th	38
	9 : 3	To an 80th of the given Number thrice put down, add a 4th of an 80th	39
A. P. L.	10 : 0	Take an 8th of a 3d of the given Number, (or a 24th part.)	40
	10 : 1	To a 30th of the given Number add a 4th of that 30, and an 8th of that 4th	41
	10 : 2	From a 20th of the given Number take an 8th of that 20	42
	10 : 3	To a 40th and a 60th of the given Number add an 8th of that 40th	43
	11 : 0	From a 20th of the given Number take a 12th of the 20	44
	11 : 1	From a 40th of the given Number twice put down, take an 8th of a 40th	45
	11 : 2	From a 40th of the given Number twice put down, take a 12th of a 40th	46
	13 : 3	To a 30th of the given Number add an 80th, and a 4th of that 80th.	47

Example to Rule 31.
7238 at 7 *d.* 3 *q.*

$\frac{1}{40}$ =	*l.* 180 : 19 : 0
$\frac{1}{4}$ of the 40th	*l.* 45 : 4 : 9
$\frac{1}{6}$ of the 4th	*l.* 7 : 10 : 9½
Answer,	*l.* 233 : 14 : 6½

Example to Rule 35.
3929 at 9 *d.* 1 *qr.*

an 80th =	*l.* 49 : 2 : 3
ditto ——	49 : 2 : 3
ditto ——	49 : 2 : 3
$\frac{1}{12}$ of $\frac{1}{80}$ =	4 : 1 : 10¼
Sum, Ans.	151 : 8 : 7¼

Example to Rule 45.
1895 at 11 *d.* 1 *qr.*

a 40th =	*l.* 47 : 7 : 6	ad
ditto =	47 : 7 : 6	
	l. 94 : 15 : 0	*Sum.*
$\frac{1}{8}$ of $\frac{1}{40}$ =	5 : 18 : 5¼	ded.
rests	*l.* 88 : 16 : 6¼	Ans.

The Value of the Pound, Ell, Yard, &c.		A Continuation of the Table of Rules for the short Work of Practice.	Number of Rules.
	s. d. qr.		
A. P. L.	1 : 0 : 0	Take a 20th of the given Number, (as *per* previous Case 3.)	48
	1 : 0 : 1	To a 40th of the given Number, and a half of that 40th twice put down, add a 12th of one of the ½.	49
	1 : 0 : 2	To a 40th of the given Number twice put down, add a 12th of a 40th	50
	1 : 0 : 3	To a 40th of the given Number twice put down, add an 8th of a 40th	51
	1 : 1 : 0	To a 20th of the given Number add a 12th of that 20th	52
	1 : 1 : 1	To a 20th of the given Number add a 12th of that 20th, and a 4th of the 12th	53
	1 : 1 : 2	To a 20th of the given Number add an 8th of that 20th	54
	1 : 1 : 3	To a 20th of the given Number add an 8th of that 20th, and a 6th of the 8th	55
	1 : 2 : 0	To a 20th of the given Number add a 6th of that 20th	56
	1 : 2 : 1	To a 20th of the given Number add a 6th of that 20th and an 8th of that 6th	57
	1 : 2 : 2	To a 20th of the given Number add a 6th of that 20th and a 4th of that 6th	58
	1 : 2 : 3	To a 30th of the given Number add a 40th of it, and an 8th of the 40th	59
A. P. L.	1 : 3 : 0	Take half of an 8th of the given Number, (or a 16th)	60
	1 : 3 : 1	To a 20th of the given Number add a 4th of the 20th and a 12th of the 4th	61
	1 : 3 : 2	To a 20th of the given Number add a 4th of that 20th, and a 6th of the 4th	62
	1 : 3 : 3	To a 20th of the given Number add a 4th of that 20th, and a 4th of that 4th	63
A. P. L.	1 : 4 : 0	Take a 30th of the given Number, and double it (or put down twice) or take a 15th of the given Number	64
	1 : 4 : 1	To ½ of an 8th of the given Number add a 12th of that ½	65
	1 : 4 : 2	To a 20th of the given Number add a 3d of that 20th, and an 8th of the 3d	66
	1 : 4 : 3	To a 40th of the given Number twice put down, add a 60th of the given Number and an 8th of a 40th	67
	1 : 5 : 0	From a 12th take an 8th of the given Number	68
	1 : 5 : 1	From a 20th of the given Number added to ½ the 20th, take an 8th of that half	69
	1 : 5 : 2	From a 20th of the given Number added to half that 20, take a 12th of that half	70
	1 : 5 : 3	From a 20th of the given Number added to a 4th of that 20 twice put down, take a 12th of the said 4th	71

Example to Rule 65.

	893 at 16*d.* ¼
an 8th =	*l.* 111 : 12 : 6
½ an 8th,	*l.* 55 : 16 : 3
$\frac{1}{12}$ of the ½ =	4 : 13 : 0 ¼
Answer,	*l.* 60 : 9 : 3 ¼

Example to Rule 67.

	1239 at 16*d.* ¾
a 40th	*l.* 30 : 19 : 6
ditto	30 : 19 : 6
a 60th =	20 : 13 : 0
⅛ of a 40th	3 : 17 : 5 ¼
Sum,	*l.* 86 : 9 : 5 ¼

Example to Rule 71.

	952 at 17*d.* ¾	
a 20th	*l.* 47 : 12 : 00	ad.
¼ of a 20,	*l.* 11 : 18 : 00	
ditto	*l.* 11 : 18 : 00	
Sum,	71 : 8 : 00	
a 12 of a 4	— : 19 : 10	ded.
rests	*l.* 70 : 8 : 2	Ans.

A Continuation of the Table of Rules for the short Work of Practice.

	The Value of the Pound, Ell, Yard, &c. s. : d.		Number of Rules.
	1 : 6	To a 20th add half that 20th	72
	1 : 7	From a 12th of the given Number take a 20th of that 12th	73
A. P. L.	1 : 8	Take a 12th of the given Number	74
	1 : 9	To a 12th of the given Number add a 20th of that 12th	75
	1 : 10	To a 12th of the given Number add a 10th of that 12th	76
	1 : 11	Add an 80th to a 12th of the given Number	77
A. P. L.	2 : 00	Take a 10th of the given Number, as *per* previous Case 1.	78
	2 : 1	To a 12th of the given Number add a 4th of that 12th	79
	2 : 2	To a 10th of the given Number add a 12th of that 10th	80
	2 : 3	To a 10th of the given Number add an 8th of that 10th	81
	2 : 4	To a 10th of the given Number add a 6th of that 10th	82
	2 : 5	To a 12th of the given Number add a 4th of that 12th	83
A. P. L.	2 : 6	Take an 8th of the given Number	84
	2 : 8	To a 10th of the given Number add a 3d of that 10th	85
	2 : 9	To an 8th of the given Number add a 10th of that 8th	86
	3 : 0	To 10th of the given Number, add half that 10th	87
	3 : 3	From a 6th of the given Number take a 40th of that 6th	88
A. P. L.	3 : 4	Take a 6th of the given Number	89
	3 : 6	To a 6th of the given Number add a 20th of that 6th	90
	3 : 8	To a 6th of the given Number add a 10th of that 6th	91
	3 : 9	To a 6th of the given Number add an 8th of that 6th	92
A. P. L.	4 : 0	Take a 5th of the given Number, (or shorter) take 2 Tenths of the given Number	93
	4 : 3	To a 10th of the given Number twice put down, add an 8th of a 10th	94
	4 : 4	To a 10th twice put down, add a 6th of a 10th	95

Example to Rule 73.	*Example to Rule* 88.	*Example to Rule* 94.
1047 at 1 *s.* 7 *d*	895 at 3 *s.* 3 *d.*	739 at 4 *s.* 3 *d.*
a 12th = *l.* 87 : 5 : 0	a 6th = *l.* 149 : 3 : 4	a 10th = *l.* 73 : 18 : 00 Ad.
a 20 of a 12, 4 : 7 : 3	a 40th of $\frac{1}{6}$ *l.* 3 : 14 : 7 ded.	ditto *l.* 73 : 18 : 00
		$\frac{1}{8}$ of a 10th, 9 : 4 : 9
Resteth *l.* 82 : 17 : 9 The Answer.	Resteth *l.* 145 : 8 : 9 the Answer.	Sum = 157 : 00 : 9 Ans.

A Continuation of Rules for the short Work of Practice.

	The Value of the Pound, Ell, Yard, &c. s. d.		*Number of Rules.*
	4 : 6	To a 10th add an 8th of the given Number	96
	4 : 8	To a 10th of the given Number twice put down, add a 3d of a 10th	97
	4 : 9	From a 4th of the given Number take a 20th of that 4th	98
A. P. L.	5 : 0	Take a 4th of the given Number	99
	5 : 6	To a 4th of the given Number add a 10th of that 4th	100
	6 : 0	Take 3 Tenths of the given Number, as *per* previous Case 2.	101
A. P. L.	6 : 8	Take a 3d of the given Number	102
	7 : 0	Work for 6*s.* and 1*s.* as *per* previous Case 2, and 3	103
	7 : 6	Take an 8th of the given Number, and put it down thrice, or multiply by 3.	104
	8 : 0	Take 4 Tenths of the given Number, as *per* previous Case 2.	105
	8 : 6	To 4 Tenths add a 40th of the given Number	106
	9 : 0	To 4 Tenths of the given Number add a 20th thereof	107
	9 : 6	From half the given Number take a 20th of that half	108
A. P. L.	10 : 0	Take half the given Number	109
	10 : 6	To half the given Number add a 20th of that half	110
	11 : 0	To half the given Number add a 10th of that half	111
	11 : 6	From 6 Tenths take a 40th of the given Number	112
	12 : 0	Take 6 10ths of the given Number, as *per* previous Case 2.	113
	12 : 6	To half the given Number add a 4th of that half	114
	13 : 0	Take 6 Tenths of the given Number, and add to a 20th thereof	115
	13 : 4	To half add a 6th of the given Number	116
	14 : 0	Take 7 Tenths of the given Number, as *per* previous Case 2.	117
	14 : 6	To 7 Tenths add a 40th of the given Number	118
	15 : 0	To ½ the given Number add half of that half, (or from the given Number take ¼)	119

Example to Rule 104.	*Example to Rule* 112.	*Example to Rule* 115.
559 at 7*s.* 6*d.*	365 at 11*s.* 6*d.*	169 at 13*s.*
an 8th = *l.* 69 : 17 : 6 Multiply by - - - - 3	6,10ths = 219 : as *p.* Cas. 2. a 40th = *l.* 9 : 2 : 6 ded.	6,10ths 101 : 8 : 00 a 20th = *l.* 8 : 9 : 00
Prod. = *l.* 209 : 12 : 6 or Answer.	rests = *l.* 209 : 17 : 6 Ans.	Sum, *l.* 109 : 17 : 00 Ans.

Note, *That A. P. L. signifies the Aliquot Parts of a Pound.*

A Continuation of Rules for the short Work of Practice.

The Value of the Pound, Ell, Yard, &c. *s. d.*		*Number of Rules.*
15 : 6	From 8 Tenths (as *per* previous Case 2.) take a 40th of the given Number	120
16 : 0	Take 8 Tenths of the given Number	121
16 : 6	Take 8 Tenths add a 40th of the given Number	122
17 : 0	To 8 Tenths add a 20th of the given Number	123
17 : 6	From the given Number take an 8th thereof	124
18 : 0	Take 9 Tenths of the given Number	125
18 : 6	To 9 Tenths add a 40th of the given Number	126
19 : 0	From the given Number take a 20th thereof	127
19 : 6	From the given Number take a 40th thereof	128

Note, That where Pounds, Shillings, *&c.* are the Price of a Unit; you must multiply the Units given by the Pounds, and work for the Shillings, *&c.* as before directed. *See the 2d Case of the next Section.*

The following shew how Interest, Commission to Factors, Customs, &c. is cast up by the short Rules of Practice.

per Cent. or 100.		
1 *per Cent.*	Take the 100th part of the given Number, (as *per* Rule 3. Prop. 3. in Division.)	129
2	Take a 50th of the given Number	130
2$\frac{1}{2}$	Take a 40th of the given Number	131
3	To a 50th of the given Number add half that 50th	132
4	Put down a 50th of the given Number twice	133
4$\frac{1}{2}$	To a 50th add a 40th of the given Number	134
5	Take a 20th of the given Number	135
6	Take a 50th of the given Number, and put it down thrice (or multiply it by 3.)	136
7	Add a 20th to a 50th of the given Number	137
8	Multiply a 50th of the given Number by 4.	138
9	From a 10th of the given Number, take a 10th of the 10th	139
10 *per Cent.*	Take a 10th of the given Number	140

Eight Examples to the Rules for finding Interest, &c, *by Practice,* viz.

What is *l.* 1341 : 16 : 3 at 1 *per C.* 100 part is *l.* 13 : 8 : 4$\frac{1}{4}$ Answ.	What *l.* 2132 : 12 : 6$\frac{1}{2}$ at 2$\frac{1}{2}$ *p. C.* a 40th is *l.* 53 : 6 : 3$\frac{3}{4}$ Answer.	What 3235 : 17 : 11$\frac{3}{4}$ at 5 *per C.* a 20th is *l.* 161 : 15 : 10$\frac{3}{4}$ Answ.
What is *l.* 879 : 10 : 7 at 6 *per C.* a 50th = *l.* 17 : 11 : 9$\frac{3}{4}$ Multiply by - - - - 3 Produceth 52 : 15 : 5$\frac{1}{4}$ Answer.	What *l.* 933 : 8 : 10 at 7 *per C.* a 20th *l.* 46 : 13 : 5$\frac{1}{4}$ } Add. a 50th *l.* 18 : 13 : 4$\frac{1}{2}$ } Sum, *l.* 65 : 6 : 9$\frac{3}{4}$ Answer.	What 671 : 00 : 2$\frac{1}{4}$ at 8 *per Cent.* a 50th *l.* 13 : 8 : 4 3 $\frac{4}{16}$ which multiply by — 4 Prod. *l.* 53 : 13 : 7$\frac{1}{4}$ Answer.

What is *l.* 472 : 14 : 5$\frac{1}{2}$ at 9 *per Cent.* a 10th is = *l.* 47 : 5 : 5$\frac{1}{4}$ a 10th of that 10th *l.* 4 : 14 : 6$\frac{1}{2}$ deduct. resteth = *l.* 42 : 10 : 10$\frac{3}{4}$ the Answer.	What *l.* 1976 : 19 : 11$\frac{1}{4}$ at 10 *per Cent.* a 10th is *l.* 197 : 13 : 10$\frac{3}{4}$ Answer.

See more of computing Interest, in the Use of Decimals, *Chap.* III.

I ſhall next very briefly ſhew the Uſe of Rules of Practice in making Allowance for Tare; which is an Allowance for the Bag, or whatſoever a Commodity is pack'd up in, for ſome things more, ſome leſs. The Weight of the Goods, and what contains them, is together called the Groſs Weight, and the Weight of the Goods alone is the nett Weight.

Goods.	*Allowance per the* 112 ℔.	*N°. of Rules.*	*Rules for the ſhort Working.*
Currans — — — —	℔ 16	141	Take a 7th of the groſs Weight.
Almonds, Steel, Hemp —	14	142	Take an 8th of the groſs Wt.
Allum, Salt-petre, Tallow,	12	143	From an 8th take a 7th of that 8th.
Brimſtone, Copperas, Copper, — — . —	8	144	Take half of a 7th of the groſs Weight.
Iron Wire, Latten Wire —	6	145	Take half of what it comes to at 12, as above.
Cotton-Wool, Lambs-Wool, and *Poliſh*-Wool, Feathers, Hops — —	4	146	Take a 4th of a 7th of the groſs Weight.
Cotton-Yarn 5 ℔ at 100 weight.		147	Take a 20th, or half a 10th of the groſs Weight.
		148	A General Rule (eſpecially for ſuch Rates of Tare as are moſt remote from being Aliquot Parts of 112 ℔; ſuch as 3, 5, 9, 11, *&c.*) is this. Multiply the groſs Hundreds by the Pounds to be allowed for 1 *C.* and for the Quarters of Hundreds and Pounds take a Part proportionable of the Allowance *per* Hundred, and the Sum is an eaſy way of giving the Anſwer.

Example to Rule 143.

	C.	*qr.*	℔.	
Groſs	29 :	3 :	24	at 12 *per C.*
$\frac{1}{8}$ is . . .	3 :	2 :	$27\frac{1}{2}$	
a 7th of that 8,	0 :	2 :	$3\frac{3}{4}$	ded. from the 8th.
Reſteth Tare	3 :	0 :	$23\frac{3}{4}$	= Anſw.
or the Nett Wt.	26 :	3 :	$0\frac{1}{4}$	; the Groſs, leſs the Tare.

P *Example*

Example to Rule 145.	*The Example to Rule* 143. *done according to this General Rule* 148.
C. qr. ℔. Grofs 37 : 2 : 12 at 12 ℔ *per C.* ——— $\frac{1}{8}$ is . . . . 4 : 2 : 22 a 7th of that 8th, 0 : 2 : 19 deduct from the 8th. ——— Refts = 4 : 0 : 3 = the Tare at 12 *per C.* ——— $\frac{1}{2}$ of that is 2 : 0 : 1$\frac{1}{2}$ the Tare at 6 *per C.* = Anf. And if ded. there refts } 35 : 2 : 10$\frac{1}{2}$ Nett Weight	Multip. { 29 : 3 : 24 at 12 *per C.* or 112 ℔. 12 ——— Prod. = 348 ℔. for the 3 *qs.* 9 the ℔ 24, 2$\frac{1}{4}$ ——— ℔ 359$\frac{1}{4}$ Sum, or 3 *C.* — *qr.* 23$\frac{1}{4}$ ℔ as before.

For Oil imported there is allowed 18 *per Cent.* (or 112 ℔.) Tare: Now fuppofe I import 180 Hund. wt. how many Nett Tons and Gallons is there?

	The Proof, viz.
C. q. ℔. 180 : 0 : 0 Grofs. ——— Ton 9 : 00 Gall. Deduct 12 ——— Ton Nett } 8 : 240 Gal. Anf.	Multiply { 180 Hundr. Grofs. 94 ℔ *per* Nett Hun. ——— 72 162 ——— Gall. ℔ *per* Gal.=7$\frac{1}{2}$) 16920 ℔.(2256 (8 Ton. 252) ——— 240 Gal. reft.

Rule 149.] Take a 20th of the Grofs Hundreds, (is here 9 Ton.) For every Ton deduct 1$\frac{1}{3}$ Gallon; fo 9 and $\frac{1}{3}$ of 9 is 12, and the Remainer is the Anfwer, as is proved above.

Now 18 *per Cent.* is 12$\frac{1}{2}$ Gallon *per* Nett *C.* therefore what Hundreds are in Units place muft be multiplied by 12$\frac{1}{2}$; and for the Quarters of *C.* and Pounds, take a proportionable part of 12$\frac{1}{2}$: from the Sum of all which, take the 1$\frac{1}{3}$ Gallon *per* Ton.

Another Example makes this plain; which I am minded to do, becaufe it has not been before.

What Nett Tons and Gallons are in 189 *C.* 3 *q.* 21 ℔. Grofs? See the Operation.

And

C. 189 : 3 : 21

Ton. Gall.
9 : 112½ for the 189 C. }
9⅛ for 3 qrs. } Grofs.
2⅓ for the 21 ℔. }

9 : 124 Sum, Grofs.
12 = 9 and ⅓ of 9 deduct.

9 : 112 refts Nett Anfwer.

And for Proof of this Work by the longer way, do thus:

C. 189 : 3 : 21
℔ in a Nett C. 94 multiply.

756
1701

17766 ℔ in 189 C.
88 ℔ ad.

℔ 7½ *per*) 17854 Sum
Gall.
252)2380(9 Ton. Anf.

and 112 ℔ reft, as in the fhorter way.

And in 3 *qs.* 21 *lb.* or 105. *lb.* grofs, there are Nett 88 *lb.* thus *Gall.* 112. 94 :: 105. 88.

The Reafon of the fhort Rule.

There are two things that feem to fome as Difficulties in the fhorter Way, *viz.*

1. Why the Hundreds in Units place, the Quarters and Pounds, are caft up at the Rate of 12½ *per Cent.*

2. Why 1⅓ is deducted for every Ton.

As to the 1ft, you fee in the Margin that in a Nett Hundred there are 12½ Gallons, and fomewhat more. And

As to the 2d, you may obferve from the Work in the Example above, that the Units place of C. the Quarters and Pounds given, are work'd according to the Nett Meafure of 12½ Gall. *per* Nett C. But in taking a 20th of the Hundreds Grofs, (or half the Figures to the left hand the Units place of Hundreds) that half makes more than Nett Tons by 1⅓ Gallon in every Ton, as *per* Margin above; and that is the reafon you deduct 1⅓ in each Ton.

112 = the Grofs C.
deduct 18 *per* C. the Tare.

℔ 94 = the Nett C.
℔ 7½) (12$\frac{8}{15}$ Gal. *per* C.
Multiply by = 20 = the Hund. in a Ton, and it gives 250⅔, which taken from the Ton grofs 252, there

Remains — — — — — 1⅓ Gal.

Note that Tret is an Allowance in Merchandize of *l.* 4 at 104 for the Duſt or Refuſe of the Commodity; and this is deducted out of the Suttle (which is the Remainer when the Tare is deducted).

SECT. V. *Concerning GAIN and LOSS.*

THIS is a Rule whereby Merchants or other Traders know (when they have bought any Commodity by wholeſale) how they may retale the ſame out, to make any certain Gain by the whole Parcel, or at any Rate *per Cent.* &c.

Or when any Goods are damaged, they know hereby what they ſhall loſe *per Cent.* or by the whole, in ſelling the Pound, Ell, Yard, Ounce, *&c.* at any Rate leſs than it coſt.

I ſhall place this and the following Sections relating to the Buſineſs of Trade, *&c.* in that Order which I judge moſt proper for them to be learnt; not regarding what Method others have obſerved in treating thereof.

Caſe 1.] When a Merchant buyeth Goods for a certain Sum, to find how the ſame may be retaled to gain a required Sum by the whole.

Examp. A Merchant buys ℔ 3780 Nett of Cotton-Wool, for 126 *l.* what may he ſell the ſame for *per* ℔. to gain *l.* 15 : 15 : by the Sale?

Rule. Add *l.* 15 : 15 : — to the Coſt, and the Sum is *l.* 141 : 15 —. Then ſay, If ℔ 3780 coſt me *l.* 141 : 15 : —, what will 1 ℔ coſt?

℔. *l.* *s.* ℔.
3780 : 142 : 15 : : 1

Divide the Pence in 141 *l.* 15 *s.* by 3780, and the Quotient is 9 *d.* which it muſt be ſold for *per* Pound, to gain *l.* 15 : 15 : in all.

Caſe 2.] Admit I bought 15 Bales of Linnen at *l.* 17 : 15 : *per* Bale, how much muſt I ſell it for *per* Bale, to gain 10 *per Cent.* by the Sale?

Add { *l.* 266 : 5 : 0
      { *l.* 26 : 12 : 6

The Value of the Linnen, and 10 *per Cent.* — — — } *l.* 292 : 17 : 6

You ſee, by Practice, the Value of the Bales is *l.* 266 : 5 : —

Then as 100 to 10 : : ſo *l.* 266 : 5 to *l.* 26 : 12 : 6; which is the Advance at 10 *per Cent.* which added as above, makes *l.* 292 : 17 : 6.

Bales 15 at 17 *l.* 15 *s.* each.
17
105
15
l. 255 at 17 *l.*
7 : 10 at 10.
3 : 15 at 5.
l. 266 : 5 Val. of the Bales.

l. *l.* *l.* *s.* *l.*
100. 10. : : 266 : 5. $26\frac{625}{1000}$.

Bales. *l.* *s.* *d.* Bale. *l.* *s.* *d.*
15. 292 : 17 : 6 : : 1. 19 : 10 : 6

Then

Then as 15 Bales to *l.* 292 : 17 : 6 : : so is 1 Bale to *l.* 19 : 10 : 6; which is the Answer: for 15 Bales at *l.* 19 : 10 : 6, is *l.* 292 : 17 : 6, the Cost and Interest proposed.

Case 3.] A Merchant bought Broad-Cloth at *l.* 20 *per* Piece ready Money, and sold it again for *l.* 25 : 10 : — *per* Piece, to be paid at the end of six Months: What does he gain *per Cent.* in 12 Months? Answer, *l.* 55 *per Cent.*

First say, As 20 the Cost to *l.* 5 : 10 : — the Gain : :
So is *l.* 100 to *l.* 27 : 10 : — the Gain in six Months.

Secondly say, As six Months to *l.* 27 : 10 : —
So is 12 Months to *l.* 55 the Gain *per Cent. per Ann.*

Case 4.] *In Case of Loss* you must deduct instead of adding. Thus a Merchant bought 20 Bags, containing 6020 ℔ Nett of Cotton for *l.* 301; but being demaged by Sea, he is willing to lose 10 *per Cent.* What must he sell it for *per* Pound, to lose that Sum?

1st, You see by the first Proportion, that the Loss by the whole will be *l.* 30 : 2 : —; which deducted from the Cost, there remains *l.* 270 : 18. therefore as 6020 ℔ Cotton is to *l.* 270 : 18 : : so is 1 ℔ of Cotton to 10 *d.* 3$\frac{44}{215}$, the Answer.

l.	*l.*	*l.*	*l.* Loss.	
100.	10. : :	301.	30 : 2 : —	
	Deduct	30 : 2 : —		
	Remains	*l.* 270 : 18 : —		
			d.	*qr.*
℔ 6020.	*l.* 270 : 18 : — : :	℔ 1.	10 :	3$\frac{44}{215}$

This is sufficient to shew the Arithmetical Work of Loss and Gain: How the Account thereof (or of Profit and Loss) is kept in Books of Accounts, is shewn in my *Merchant's Magazine.* But this Book is not intended to be a Treatise of Merchandize, but to teach all kind of Arithmetic.

SECT. VI. *FELLOWSHIP.*

FELLOWSHIP is the Application of the Rule of Proportion, shewing what share of Gain or Loss every Merchant of those who trade in company shall have, or bear in proportion to his share of the general Stock; and consequently by this Rule also those who have underwritten Policies know how to settle their Averages, in case part of what is insured be lost, proportionably to what each has subscribed.

So

So that this may be called the Rule of Distribution, or (in case of Loss) of Contribution, (of the Profits obtained, or of Loss sustained.)

Quest. 1.] Two Men let to freight a Ship, wherein the first hath $\frac{9}{16}$, the other 7; and when the Voyage was performed, and all Charges deducted, the Nett Profits are found *l.* 480: What must each have?

Rule.] In this Case divide the whole Gain by the Number of Shares, and multiply the Quotient by each Man's particular Share, and the Products are the Answers, thus:

Or thus,

$$\left.\begin{matrix}16.\,480::9.\,270\\16.\,480::7.\,210\end{matrix}\right\}\text{2d way.}\qquad 16)\,480\,(30\ \text{by 9 is} = l.\,270 = \text{1st.}\ \ \text{by 7 is} = l.\,210\ \text{the 2d.}$$

0

480 Sum. Proof *l.* 480 Sum.

Quest. 2.] Five Merchants trade in Company; *A.* put in 1173 *l.* *B.* 800, *C.* 977, *D.* 562, and *E.* 1000 *l.* and having each continued his Share first put in Stock till the Profits were computed and a Dividend made, they find they had gained by Trading 25312 *l.* What must each have?

This is best done by the Rule of Proportion, as you see: For as the whole Stock is to the whole Gain :: so is every one's Share in the Stock to his Share in the Gain.

Sum or whole Stock.	The whole Gain.	Each Man's Stock.	Each Man's Share of the Gain.
4512.	25312 ::	1173.	*A.* $6580\frac{2016}{4512}$
4512.	25312 ::	800.	*B.* $4487\frac{4226}{4512}$
4512.	25312 ::	977.	*C.* $5480\frac{4064}{4512}$
4512.	25312 ::	562.	*D.* $3152\frac{3520}{4512}$
4512.	25312 ::	1000.	*E.* $5609\frac{4192}{4512}$

The Sum for Proof, 25312

The Sum of the Fractions is 4, as *per* Addition of Vulgar Fractions, *Chap.* II. *Sect.* 2. *Head.* 3. And the Aggregate of each Man's Profit makes the whole Gain; which proves the Truth of the Rule and Operations.

Quest. 3.] Three Men underwrite a Policy of Insurance, *viz.*

A. is contented with the Insurance to pay in case of Loss, *l.* 1000
B. is contented with the Insurance for ——— ——— 750
C. for ——— ——— ——— ——— ——— 550

Now it happen'd, that being closely pursued by Pirates, they were obliged to lighten the Ship, by

Sum *l.* 2300

throwing

throwing over-board Goods to the Value of *l.* 450; what must each Subscriber bear of that Loss, in proportion to what he has insured?

The Proportion is as			
the whole Sum insured	*l.*	*l.* Loss. *l.*	
is to the whole Loss	2300.	450 : : 1000.	$195\frac{15}{23}$ = *A*'s Loss.
l. 450 : so is the Sum	2300.	450 : : 750.	$146\frac{17}{23}$ = *B*'s Loss.
each Insurer subscribed	2300.	450 : : 550.	$107\frac{14}{23}$ = *C*'s Loss.
to his Proportion of the			
Loss, as appears by the		Sum for Proof, *l.* 450	
Operation in the Margin.			

Or if you would know what the Loss is *per Cent.* to make the Average; this is only done by one single direct Proportion thus, 2300. 450 : : 100. $19\frac{13}{23}$, which is the Rate *per Cent.* to be paid, as is proved.

by multi-	*A*'s 10 Hund. multipl. by the Rate $19\frac{13}{23}$ gives	*l.* $195\frac{15}{23}$
plying the	*B*'s $7\frac{1}{2}$ Hund. by the Rate $19\frac{13}{23}$ gives ——	*l.* $146\frac{17}{23}$
100 *l.* that	*C*'s $4\frac{1}{2}$ by the Rate $19\frac{13}{23}$ produceth ——	*l.* $107\frac{14}{23}$
each insu-		

red by the Rate *per Cent.* as in the Margin.

Quest. 4.] Three Men bought 120 Acres of Land for *l.* 2400; for which *A.* was to pay $\frac{1}{2}$, *B.* a 3d, and *C.* a 4th of the Charge, and were to have the like Proportion of the Land: how much must each pay, and have of the Land, according to the true Intent and Meaning of their Contract? For to think that $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, will make but one, is absurd, because a 3d is more than a 4th, and $\frac{1}{2}$ and $\frac{2}{4}$ makes 1. The Answer is found as follows.

$\frac{1}{2}$ of Acres 120 is	60; and of *l.* 2400 =	1200
$\frac{1}{3}$ is ——	40; and ——	*l.* 800
$\frac{1}{4}$ is ——	30; and ——	*l.* 600
	Acres 130	Sums *l.* 2600
	Too much by 10	and —— *l.* 200

Therefore I say, If 130 abate 10, and *l.* 2600 must abate *l.* 200; what must each abate, to answer the intended Contract? Thus;

Acres.	Acres.		*l.*	
130. 10 : : 60.	$4\frac{8}{13}$	and if *l.* 2600. 200 : : 1200.	$92\frac{8}{26}$	= *A*'s Ded.
130. 10 : : 40.	$3\frac{1}{13}$ ——	*l.* 2600. 200 : : 800.	$61\frac{14}{26}$	= *B*'s
130. 10 : : 30.	$2\frac{4}{13}$ ——	*l.* 2600. 200 : : 600.	$46\frac{4}{26}$	= *C*'s
	Acres 10		Sums *l.* 200	

So

So that the Deductions of the respective Acres and Pounds being made (as *per* the Rules in Substraction of Fractions) from their apparent Shares, above,

The Answer is,
- *A.* must have Acres $55\frac{5}{13}$, and pay *l.* $1107\frac{18}{26}$.
- *B.* must have Acres $36\frac{12}{13}$, and pay *l.* $738\frac{12}{26}$.
- *C.* must have Acres $27\frac{9}{13}$, and pay *l.* $553\frac{22}{26}$.

Acres 120 Sums *l.* 2400 Proof.

Or the Analogies (or Proportions) above, may be done from the Parts each is to have, by reducing the Fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, into a common Denominator; as $\frac{6}{12}$, $\frac{4}{12}$, and $\frac{3}{12}$: For

As the Sum of the Numerators 13, is to 10, and to 200:
So is each Numerator 6, 4, and 3, to the Proportion that each is to deduct.

Acres. *l.*

Thus as 13. 10 : : 6. $4\frac{8}{13}$ And as 13. 200 : : 6. $92\frac{4}{13}$.
13. 10 : : 4. $3\frac{1}{13}$ And as 13. 200 : : 4. $61\frac{7}{13}$.
13. 10 : : 3. $2\frac{4}{13}$ And as 13. 200 : : 3. $46\frac{2}{13}$.

When Time is considered in Partnership, You must multiply the Money in Stock by the Time it continued therein, and work with the Products as by the Rules above, and as in the Example following.

Quest. 5.] *A.* and *B.* trade in Partnership for one Year. *A.* put into Stock *l.* 200, and at 4 Months end he withdrew *l.* 40, and at 9 Months end put in *l.* 170. *B.* put in at first *l.* 150, and at 7 Months end *l.* 200 more, and at the end of 10 Months he takes out *l.* 160: they gain 700, what must each have of it?

A's *l.* 200 : 4 Months is 800
l. 160 : 5 Months = 800
l. 330 : 3 Months = 990
Sum of these Products 2590

B's *l.* 150 : 7 Months = 1050
l. 350 : 3 Months = 1050
l. 190 : 2 Months = 380
Sum of these Products 2480

Sum Total = 5070

Then 5070. 700 : : 2590. $357\frac{301}{507}$ = *A*'s Profit.
5070. 700 : : 2480. $342\frac{206}{507}$ *B*'s. Sum *l.* 700 = the Proof.

Quest. 6.] A Ship's Company take a Prize worth 6000 *l.* Now the Captor having on board the following Officers, Midshipmen, and

and Sailors, who are each to participate of the Prize-Money according to his Pay and Time entred on board; which was thus.

8 Officers entered on board 10 Mon. at 50 *s. per* Mo.		What must
10 Midshipmen — — — 9 Mon. at 35 *s. per* Mo.		each have of
150 Sailors — — — — 8 Mon. at 25 *s. per* Mo.		the Prize?

Mon.

8 multip. by 10, and that Prod. by *l.* 2 : 10, produceth	*l.* 200 : 00 : 00
10 mult. by 9, and that —— by 1 : 15, gives ——	157 : 10 : 00
150 —— by 8, and that —— by 1 : 5, is — —	1500 : 00 : 00
Sum——	1857 : 10 : 00

Then I say,

As $1857\frac{1}{2}$. 6000 : : 200. *l.* $646\frac{110}{3715}$ To be divided equally among the Officers.

$1857\frac{1}{2}$. 6000 : : 157 : 10. $508\frac{2780}{3715}$, Among the Midshipmen.

$1857\frac{1}{2}$. 6000 : : 1500. $4845\frac{1825}{3715}$, Among the Sailors.

Sum for Proof = *l.* 6000

SECT. VII. *Equation of Payments.*

BY this Rule or Part of the Application of Arithmetic, Men discover, when several Payments are due at different Times, how to fix upon one certain Time when the whole may be paid at once, without Loss to the Debtor or Creditor.

The General and true Rule is,

As the Total of the Sums payable: is to a Unit (or 1 Month)
So is the Sum of the Products made of each Sum in its respective Time,
To the true Time when the whole ought to be paid.

There is a certain Gentleman, who makes some Figure in these kind of Sciences, and he denies the Truth of this Rule; which I shall prove under my second Question to be just, notwithstanding his Pretence of its being erroneous.

Quest. 1.] *A.* is indebted to *B. l.* 100, to be paid at the end of three Months, also *l.* 200 to be paid at the end of 4 Months, and *l.* 300 to be paid at the end of 5 Months: Now to prevent the trouble of many Meetings, they agree to have but one Payment of the three Sums at one time; the Question is, when that must be, without loss to either *A.* or *B.*

According to the foregoing Rule,

l. 100 multiplied	by 3 Months, produceth	300	}	Sum 2600 of the Products.
200 ———	by 4 — — — — —	800		
300 ———	by 5 — — — — —	1500		
l. 600 Sum				

l. *Mon.* *l.* *Mon.*
Then I say, 600. 1 :: 2600. $4\frac{1}{3}$.

The Time sought is $4\frac{1}{3}$ Months; at the end of which time, if the *l.* 600 be paid, neither Party will sustain Loss, as I shall prove by and by.

Quest. 2.] *A.* oweth *B.* 5 Sums of Money to be paid at 5 Payments, *viz.*

l. 100 ready Money	At what time must the whole be paid without Loss to *A.* or *B?*
200 at the end of $\frac{1}{2}$ a Month, or $\frac{1}{24}$ of a Year,	
300 at 1 Month's end,	
400 at 3 Months end,	
500 at 6 Months end,	Answ. $3\frac{1}{15}$ Months.

l. 100 ready Money.

200 multiplied by $\frac{1}{2}$ a Month	= *l.* 100
300 by 1 Month — — —	= 300
400 by 3 Months — —	= 1200
500 by 6 Months — —	= 3000
1500 = Sum.	4600

Then as 1500. 1 :: 4600. $3\frac{1}{15}$ = the Answer.

Now that this is the true equated Time, will appear thus: For if the Interest of the respective Sums for their Times be equal to that of the whole for the equated Time, I think the Truth of the Answer cannot be denied.

Now you'll find *l.* 100 ready Money Interest to be *l.*	0 : 0 : 0
Interest of *l.* 200 for $\frac{1}{2}$ a Month at 5 *per Cent.* is	00 : 8 : 4
300 for 1 Month is — — —	1 : 5 : 00
400 for 3 Months is — — —	5 : 00 : 00
500 for 6 Mon. (all at 5 *per Cent.*) is	12 : 10 : 00
the Sum =	19 : 3 : 4.

Note, The Inter. for Months I take from that of a Year proportionably.

And whoever will take the small pains to examine, will find, That the Interest of *l.* 1500 for $3\frac{1}{15}$ Months is the very same Sum. But no wonder he denies it, who denies the common Way of computing

puting simple Interest, tho' always practised by his Betters; and he may as well pretend that it ought to be computed by way of Compound Interest for every 2d in the Year, as object what he has advanced against the general Ways and Rules given by every body but himself, for working Questions both in Interest and Equation of Payments, which I have in the last example sufficiently proved to agree, tho' I never did see the Truth of Equation proved this way before; which I hope will be a Satisfaction to the Ingenious, as well also the Novelty of the last Question, &c.

And as a farther Proof to shew the Agreement of other Rules to the first, (or common one above) and that it is universally true for any Sum; I will find the equated time under the first Question, without taking notice of the Sums of Money, only the Parts of any Sum, and of the Time when payable. As,

Parts of any Sum	*Payable at these Mon.*	*Products of these two.*	*Being Months*	
$\frac{1}{6}$ at	3	$\frac{3}{6}$	$\frac{1}{2}$	These three add.
$\frac{1}{3}$ at	4	$\frac{4}{3}$	$1\frac{1}{3}$	
$\frac{1}{2}$ at	5	$\frac{5}{2}$	$2\frac{1}{2}$	

Sum = the equated Time above = $4\frac{1}{3}$ as before.

And thus might the Equation of Time be reduced into Tables.

Quest. 3.] I lent my Friend *l.* 500 for 5 Months; for what time must he lend me *l.* 750, to recompence my Kindness to him? This is done as in the Margin.

And for Proof of this,

```
   l.   Mon.  l.
If 500.  5.  750
     5
   -----
75) 2500 (3⅓ Months for Answ.
      |
   -----
     25
```

Interest at 5 *per Cent.* of *l.* 500 : 5 Months is *l.* 10 : 8 : 4
of *l.* 750 : $3\frac{1}{3}$ Months *l.* 10 : 8 : 4

SECT. VIII. *BARTER.*

B*Arter or Commutation* is a Rule among Traders, whereby they do, by considering the Price of their Goods, whether as for ready Money, or advanced in Barter, so proportion the Rates and Quantities, as to know how much of one Specie may be exchanged for any quantity of another kind of Commodity. And all this is

 discovered

discovered by that Golden Rule of Single Direct Proportion, as appears by the following Examples.

Quest. 1.] *A.* hath 1752 Ells of Linnen at 2 *s.* 9 *d. per* Ell; *B.* has Cheese at 31 *s. per C.* How much Cheese must *B.* give *A.* for his Linnen? See the Operation.

In this Case it will be necessary to know the Value of *A*'s Linnen only in Shillings thus,

1st, $\left.\begin{array}{r}1752\\2\end{array}\right\}$ Mult.

3504 at 2 *s.*
¼ that = 876 at 6 *d.*
½ that = 438 at 3 *d.*

Shill. = 4818 Sum.

2dly, Then say, If 31 *s.* buy 1 *C.* of Cheese, what will 4818 *s.* buy? Answer, $155\frac{11}{31}$.

s.	*C.Cheese.*	*s.*	
31.	1 : :	4818	*C.*

31) 4818 ($155\frac{11}{31}$
171
168
13

So that *B.* must give *A.* $155\frac{11}{31}$ *C.* of Cheese for his Linnen.

Quest. 2.] *A.* has 52 Dozen of Hats (or 624) which he values at 2 *s.* 6 *d.* ready Money, But in Barter expects 2 *s.* 9 *d. per* Hat. *B.* has Cotton at 10 *d. per* Pound ready Money. How much Cotton must he give for the Hats, at a Price advanced in Barter proportionably to *A*'s Advance in Barter?

1st say, If *A*'s 2 *s.* 6 *d.* advances 3 *d.* what does *B*'s 10 *d.* advance?

d. *d.* *d.* *d.*
30. 3 : : 10. 1 = *B*'s advance on a Pound of Cotton; so it is 11 *d. per* Pound in Barter.

2dly, The Value of 624 Hats at 2 *s* 9 *d.* is *d.* 20592.

d. ℔ *Cott.* *d.* ℔ *Cott.*
3dly say, If 11. 1 : : 20592. 1872 for Answer.

And for Proof of this, 624 Hats at 2 *s.* 9 *d.* each is *l.* 85 : 16 : 00
and so is 1872 ℔ of Cotton at 11 *d. per* ℔ *l.* 85 : 16 : 00

Quest. 3] Two Merchants have various kinds of Goods to Barter: *A.* has *Indian* Silk 735 Yards at 8 *s.* 6 *d.* but in Barter will have 10 *s.*

Canes 532 at 3 *s.* but will in Barter have 3 *s.* 4 *d.*

Muslin 16 Pieces at *l.* 4, but will have in Barter 4 *l.* 10 *s.*

B. has Scarlet Cloath at *l.* 1 *per* Yard — —
Glass Manufacture at *l.* 0 : 1 : 8 *per* Pound, } Ready Money.
Ditto finer at ——— 2 : 4 *per* Pound,

How

How many Yards of Cloth, and Pounds of each kind of Glafs, (of each a like Number) muft *B.* give *A.* advancing his Goods proportionably alfo in Barter? To anfwer this Queftion,

1ft, See what *A*'s Goods amount to at his bartering Price thus:

735 Yards at 10 *s.* each, is *l.* 367 : 10 : 00	}	Sum *l.* 528 : 3 : 4
532 Canes at 3 *s.* 4 *d.* —— *l.* 88 : 13 : 4	}	
Muflin 16 Pieces at *l.* 4 : 10 : 00 *l.* 72 : 00 : 00	}	

2dly, The Difference between the Sums of *A*'s ready Money and bartering Price, is 11 *s.* 10 *d.* therefore what *B.* muft advance in proportion is thus:

As the Sum of the Ready-Money Rate of one of each of *A*'s Goods, *l.* 4 : 11 : 6

Is to 11 *s.* 10 *d.* the Sum advanced upon one of each in Barter:

So is *B*'s *l.* 1 : 4 : 00 the Sum of one of each of his Goods,

To 3 *s.* 1 *d.* $\frac{270}{1098}$ *B*'s Advance in Barter; which muft therefore be 27 *s.* 1 *d.* $\frac{270}{1098}$ for the Price of 1 of each of his Goods.

3dly fay, If 27 *s.* 1 *d.* $\frac{272}{1098}$ buy one of each of *B*'s Commodities, what will *l.* 528 : 3 : 4, the Value of *A*'s Goods, buy of each of *B*? It will ftand thus;

$$\begin{array}{llll} l. \quad s. & & l. \quad s. \quad d. & \\ 27 : 1\frac{270}{1098}. & 1 :: & 528 : 3 : 4. & 389\frac{26280}{35712} \end{array}$$

So that the Price of *A*'s Goods will buy $389\frac{26280}{35712}$ Yards of *B*'s Cloth, and as many Pounds, of each fort of his Glafs.

And for Proof of this you'll find that $389\frac{26280}{35712}$ at 27 *s.* $1\frac{270}{1098}$ *d.* each, (being a Set of *B*'s Goods, or 1 of each) will amount to *l.* 528 : 3 : 4, the Value of *A*'s Goods, according to his advanced Price in Barter.

Or you may find the Price in Barter that *B.* ought to rate each fort of his Goods: for, B's *Advance in Barter.*

	l. *s.* *d.*	*s.* *d.*	*s.*	*s.* *d.*	
As *A*'s	4 : 11 : 6.	11 : 10 ::	20. to	2 : $7\frac{42}{1098}$	on Cloth.
	4 : 11 : 6.	11 : 10 ::	1 : 8. to	0 : $2\frac{644}{1098}$	on Glafs.
	4 : 11 : 6.	11 : 10 ::	2 : 4. to	0 : $3\frac{682}{1098}$	on ditto.

And by thefe particular Prices of *B*'s Goods, you may not only prove the Truth of the foregoing Operation, by feeing what

	s. *d.*	
$389\frac{26280}{35712}$ will amount to at	22 : $7\frac{42}{1098}$ *per* Yard of Cloth,	Whofe 3 Products
	1 : $10\frac{644}{1098}$ *per* ℔ of Glafs,	added together, will
	2 : $7\frac{682}{1049}$ *per* ℔ ditto fine,	make *l.* 528 : 3 : 4.

But if you were not to deliver an equal Number of each, but according to any Proportion, you may now eafily do the fame.

And

And thus I have given the Reader a farther Account of the Usefulness and Extent of this Rule of Barter, &c. than has been done by any Author before, to make it agreeable to the other parts of the Book foregoing; all which I doubt not but will be gratefully accepted by the Ingenious Reader.

SECT. IX. *EXCHANGE.*

AS the former or last Section exhibits Rules for the Bartering of one Commodity for another; so this shews how to exchange Money for Money, or, in the way of Negotiation, Money for Bills, &c. And therefore this is the proper place where Rules for Exchange ought to be inserted, as being another kind of Barter.

I shall not here trouble the Reader with an unnecessary and uncertain Account of the Value of foreign Coin of all Countries, (about which most differ) because they who have Money to remit, must be governed by the Course of Exchange, and it is notorious that does rise and fall according as foreign Trade is influenced by several Circumstances relating to this or that Country; only I shall give the Weight and Value of the four several sorts of Pieces of Eight, because they are current in most places.

	Weight. *dw. gr.*	*True Value,* *s. d.*
The Piece of 8 *Pillar*	17 : 12	4 : 6¼
Piece of 8 *Seville*	17 : 12	4 : 6
Piece of 8 *Mexico*	17 : 12	4 : 6
Piece of 8 *Peru*	17 : 12	4 : 5

And for the other Denomination of foreign Coin at Places which have Exchange with *England*, they are,

1. The Pound *Flemish* 33 *s.* 4 *d.* in Value 1 Pound *Sterling*; in which Denomination *London* exchanges with *Amsterdam*, *Rotterdam*, *Antwerp*, and *Hamburgh*.

2. In Pieces of 8 *Mexico* for *English* Pence, *London* exchanges with *Madrid*, *Cadiz*, *Genoa*, *Leghorn*.

3. For Ducats (one being in real Value 4 *s.* 4. *d.* ½ *Sterling*) *London* exchanges with *Venice*.

4. For the *French* Crown (in Value 4 *s.* 6 *d. Sterling.*) *London* exchanges with *Paris*.

5. For the Mill-rea of 6 *s.* 8 *d.* ½ *Sterling*, *London* exchanges with *Oporto* in *Spain*, and *Lisbon* in *Portugal*.

But

But the *Course of Exchange* is sometimes higher and lower than the Rates above, which are called *the Par of Exchange.*

6. I shall next shew the Nature of Exchanging by Bill; and 2dly, how to cast up Bills.

The first I cannot do more effectually than by giving the Form of a Foreign Bill, and that is thus:

Rotterdam, *April* 10. 1716. for *l.* 1272 : 13 : 4 *Flemish* at 33 *s.* 4 *d. per* Pound *Sterl.*

AT Usance pay this my first of Exchange unto Mr Edward Jones, *or his Order, Twelve Hundred Seventy-two Pounds Thirteen Shillings and Four Pence* Flemish; *Exchange at Thirty-three Shillings and Four Pence* per *Pound* Sterl. *for the Value received of Mr* John Hall, *and account it to*

To Messieurs Andrea *and* Jean Varelst *Merchants in* London.

Your Humble Servant,
Herman Vanderstagen.

Hence 'tis plain, that *Hall* pays the Money in *Holland*, (and is called the Remitter) to *Vanderstagen* (who is the Drawer) drawing his Bill on the *Varelsts* (his Correspondents at *London*) to pay the Value to *Jones* at *London*, who is *Hall*'s Correspondent.

And thus there are, you see, four Persons concerned in a Bill of Exchange, *viz.* the Remitter, the Drawer, he that pays the Bill, and he to whom it is paid. *Note, Usance is the Time between any Day of one Month to the same Day of the next.*

7. The next thing is to shew how this Bill, *&c.* is converted into Pounds, *&c. Sterling* Money, which is the principal thing intended by the Rule of Exchange as 'tis here placed; *i. e.* to shew how to reduce one Country's Coin to another.

```
                 l.    s.  d.
         Flemish 1272 : 13 : 4
                   20
  s. d.          ------
  33 : 4.        25453
  12               12
Pen.Flem. 400)   305440 (763 l.
                 ---
                 25
                 14
                   240 rests l.
                    20 s. mult.
            400) 4800 (12 s.
                 0
```

A shorter Way thus:

```
            l. 1272 : 13 : 4 Flem.
                           3 mult.
            ---------------
Divide by 5) 3818 : 00 : 0
   Answer = l. 763 : 12 : 0 Sterl.
```

So

So that in *l.* 1272 : 13 : 4 *Flemish*, there are *l.* 763 : 12 : 00 *Sterl.* And thus *Dutch* Money may be reduced to *English* at any Rate *per* Pound *Sterling*; but in the above, of 33 *s.* 4 *d.* *Flemish per* Pound, the Work is performed much shorter, as above toward the Right-Hand Margin.

8. And if a Bill is drawn from *Lisbon* of Mill-Reas 1432 at 6 *s.* 10 *d.* $\frac{5}{8}$ *Sterling per* Mill-Rea; how much *English* Money is that Bill?

Mill-rea.	*s.* *d.*	*Mill-reas.*	
1.	6 : 10$\frac{5}{8}$: :	1432	1432 at 6 *s.* 10 *d.* $\frac{5}{8}$

By Practice, $\frac{1}{3}$ = 477 : 6 : 8
and $\frac{1}{120}$ = 11 : 18 : 8
And 1432 multiplied by 5, and divided by 8 = 3 : 14 : 7

Answer, *Sterling* = *l.* 492 : 19 : 11

9. Now suppose the *Dutch* Bill above be endorsed and sent to *Legborn*, at 56 *d. Sterling per* Piece of Eight; and if it be again endorsed at *Legborn*, and remitted to *Amsterdam*, the Exchange at 93 $\frac{1}{3}$ Pence *Flemish per* Piece of Eight: How many Pieces of Eight must be paid for the Bill at *Legborn*, and how many Pounds *Flemish* at *Amsterdam*, according to this Course of Exchange?

1st, In *l.* 763 : 12 in the Bill, there are 183264 Pence *Sterling*; which divided by 56, gives 3272$\frac{4}{7}$ Pieces of Eight.

2dly, 3272$\frac{4}{7}$ being multiplied by 93$\frac{1}{3}$ Pence *Flemish* for each Piece of Eight, or $\frac{22908}{7}$ by $\frac{280}{3}$ produceth $\frac{6414240}{21}$ Pence *Flemish*; and being divided by 12, and then by 20, gives Pounds *Flemish* *l.* 1272 : 13 : 4 as above, which proves the Truth of the whole Work.

10. But these Questions will be so easy to those who have proceeded gradually to learn thus far, that I need not enlarge much farther on this Rule of Exchange; what has been observed being sufficient to shew how either the Coins, or Weights and Measures of one Country, are reduced to those of any other.

For in 32755 *Flemish* Ells there are 19653 *English*; for a *Flemish* Ell is $\frac{3}{5}$ of an *English*, or $\frac{6}{10}$ rather, because it saves the Work of Division

32755
6
196530

11. A *Dutchman* sells 29380 *Flemish* Ells of *Holland* Duck, to an *Englishman*, a *Spaniard*, an *Italian*, and a *Portuguese*, who are to have each a like quantity in their own Country Measure; how much must each have therein?

The

The *Dutch* in *English* Ells are 17628, and

17628 Ells divided by 4, gives 4407 Ells *English* each, and

Ells.	*Answer.*
4407 is for the *Briton* — — — — — — —	4407 Ells.
4407 for the *Spaniard* at $\frac{7}{10}$ Canes *per* Ell *English*	3084$\frac{9}{10}$ Canes.
4407 for the *Italian* at 2 Braces *per* Ell *English.*	8814 Braces.
4407 for the *Portuguese* at 1 Vare *per* Ell is — —	4407 Vares.

This may be proved several ways, which I leave to the Reader's Judgment.

SECT. X. *ALLIGATION.*

ALLIGATION may be called *The Rule of Mixture*, or of *compounding Ingredients*, because it teaches how to mix several Species of Simples according to any Intent or Design proposed. It is either Medial or Alternate.

Alligation Medial shews what the mean Price of a Pound, Ounce, *&c.* is worth, when several Quantities of several Values are mix'd together, *&c.* as *per* the Cases following.

Alligation Alternate shews how much of various kinds of Simples may be taken to make up any assigned Quantity of a Compound, which shall be worth a Price proposed.

Of Medial Alligation.

Case 1.] A Goldsmith hath Gold 12 ℥ at 4 *l. per* ℥ ; 8 ℥ at *l.* 4 : 5 ; 3 ℥ at *l.* 4 : 6 : 8 ; and 9 ℥ at *l.* 4 : 13 : 4 *per* Ounce : what is an Ounce worth, suppose these be all melted down together? Answer, *l.* 4 : 7 : 5½.

Rule.] Multiply each Quantity given by the Price ; then by direct Proportion,

As the Sum of the Quantities given
Is to the Sum of the said Products ;
So is one Ounce of the Mixture
To its Value. *See the Work following.*

Ounces Gold		*The Price of* 1 ℥.		
12	multiplied by	*l.* 4 : 00 : 0	the Product is	*l.* 48
8	——— by	4 : 5 : 0	produceth ——	34
3	——— by	4 : 6 : 8	produceth ——	13
9	——— by	4 : 13 : 4	produceth ——	42
32 Sum.				137 Sum.

Then

℥. *l.* ℥. *l.*
Then ſay, 32. 137 : : 1. 4$\frac{9}{32}$, or to *l.* 4 : 5 : 7½.

And by the ſame Rule the Value of any other Quantity of that Compoſition is found: as ſuppoſe 7 in the laſt Example is worth 29$\frac{31}{32}$, for as ℥. *l.* ℥. *l.*
32. 137 : : 7. 29$\frac{31}{32}$.

Caſe 2.] To increaſe or diminiſh a Compound proportionably, by knowing the ſeveral Quantities of the Simples in the Compoſition.

Rule. As the Sum of the particular Quantities of the Compound given

Is to the whole Quantity propoſed to be augmented or leſſened;

So is each particular Quantity in the given Compound

To the due proportion required of that Specie, Fineneſs, *&c.*

Example. I would augment the Compound in the laſt Caſe to 48 ℥, that is, I would add 16 to the 32; how much muſt I take of each ſimple Ingredient? See the Operation.

		Anſwer.
12	Then as 32. 16 : : 12.	℥ 6
8	32. 16 : : 8.	4
3	32. 16 : : 3.	1½
9	32. 16 : : 9.	4½
Sum = 32		
		Sum = 16 Proof to add.

So that I muſt have 18 ℥ of *l.* 4 *per* ℥.
12 ℥ of *l.* 4 : 5 : —
4½ of *l.* 4 : 6 : 8, and
13½ of *l.* 4 : 13 : 4.

48 Sum for Proof, in the whole.

Caſe 3.] Having the Simples of any Compound given, to find how much of each kind of ſimple Ingredient is in any part of that Compoſition.

Rule. As the Total of the Compoſition

Is to the Quantity of any Simple in that Compoſition:

So is the Total Quantity propoſed to be proportionably compounded,

To the Quantity of each Simple to be in that propoſed Quantity.

Example

Example. I would know how much of each Ingredient (or Price of Gold mentioned in the first Case) is in a Pound or 12 ℥ of the 32, being the Compound given? See the Operation in the Margin.

Answer.

32. 12 : :	12 ℥	$4\frac{1}{2}$	of *l.* 4. *per* ℥.	
32. 12 : :	8.	3	of	4 : 5 : —
32. 12 : :	3.	$1\frac{1}{8}$	of	4 : 6 : 8
32. 12 : :	9.	$3\frac{3}{8}$	of	4 : 13 : 4

℥ 12 Sum Proof.

Case 4.] The Total of the Compound of two Simples, with the Total Value of that Composition, and the Value of a Unit of each simple being given; to find the Quantiy of each simple Ingredient in the Composition.

Rule. Multiply the Total Quantity of the Composition, (here 20) by the lesser Price of the Unit (here 4), then deduct the Product from the Total Value of the Composition (here 82), and divide the Remainer by the Difference in Value of a Unit of the two Simples given (as here 5*s.* or $\frac{1}{4}$ of a Pound) and the Quotient is the Quantity of the higher-prized Simple (here 8) whose Complement to 20 is 12: so that the Answer is 12 ℥ of *l.* 4 *per* Ounce, and 8 ℥ of *l.* 4 : 5 *per* Ounce. This Canon I discover'd by Algebra, as appears in the Solution of Questions by various Positions.

Gold at *l.* 4 *per* ℥.
Ditto at *l.* 4 : 5

Total of the Composition } = 20 ℥. Total Value *l.* 82

4	80
80	*l.* $\frac{1}{4}$) 2 (8

Case 5.] To find the Quantities of each simple Ingredient (when those Simples are more than 2 in Number) contained in a Composition, by having the Totals of the Quantity compounded, and of the Value; and also the Value of a Unit of each simple Ingredient given, as

℥ of Gold at *l.* 4 *per* ℥ =
℥ ditto at *l.* $4\frac{1}{4}$ =
℥ ditto at *l.* $4\frac{1}{3}$ =
℥ ditto at *l.* $4\frac{2}{3}$ =

Total of the Composition = 32 ℥. Total Value *l.* 137

Rule. To these kind of Questions, as in those of Alligation Alternate, various Answers may be given, and yet all true. You may best

best do them by 2 at a time, as in the last Case. I suppose the 2 first 15 of the total Mixture, and 63 of the total Value, and so I find 3 at *l.* 4, and 12 at $4\frac{1}{4}$. Then the rest of the total Compound is 17, and of the Value 74; which, according to the 2 later Prices, gives 16 at $4\frac{1}{3}$, and 1 at $4\frac{2}{3}$.

But *note*, That you must so discreetly divide the total Quantity, and Value, that when the Product of the 1st in 1 of the 2 Prices is taken from the later, the Remainer may not be so much as (when divided by the Difference of the Prices) will give a Quotient so great as that part of the total Quantity of the Ingredient which you fix'd upon, or supposed. See the Operation above.

1st 2 = 15	and = 63
4	less — 60 deduct.
Product = 60	$\frac{1}{4}$) 3 (12 at $4\frac{1}{4}$.
2dly 15	63 deduct.
$4\frac{1}{4}$	$63\frac{3}{4}$
Product = $63\frac{3}{4}$	$\frac{1}{4}$) $\frac{3}{4}$ ($\frac{12}{4}$ = 3 at 4.
the 2d 2 = 17	and 74
$4\frac{1}{3}$	$73\frac{2}{3}$ deduct.
Product = $73\frac{2}{3}$	$\frac{1}{3}$) $\frac{1}{3}$ (1 at $4\frac{2}{3}$.
lastly — — 17	74 deduct.
$4\frac{2}{3}$	$79\frac{1}{3}$
$79\frac{1}{3}$	$\frac{1}{3}$) $5\frac{1}{3}$ ($\frac{48}{3}$ or 16 at $4\frac{1}{3}$

II. *Alligation Alternate.*

Quest. 1. A Farmer hath 4 sorts of Wheat, *viz.* 5 *s.* 6 *s.* 7 *s.* and 7 *s.* 6 *d. per* Bushel; and he is minded to mix so much of each sort, as will make 64 Bushels worth 6 *s.* 6 *d. per* Bushel: how much of each sort must he take?

	Prices.	Differences.	
	s.	*s.* : *d.*	
	5	1 : 0	
s. *d.*	6	0 : 6	the 1st
Mean Price 6 : 6	—		Way.
	7	0 : 6	
	7:6	1 : 6	
		3 : 6 = Sum.	

Having placed the Prices as you see, and the mean Price; take the Difference between the mean Price 6 *s.* 6 *d.* and 5 *s.* (the 1st Price) which is 1 *s.* 6 *d.* this you must put down (in the 1st Way) against 7 *s.* 6 *d.* (because bigger than the mean Price.)

Price.) Then go on, and put the Difference between 6 *s.* 6 *d.* the mean Price, and 6 *s.* (the 2d Price) which is 6 *d.* against the Price (7 *s.*) because bigger than the mean Price. Then put the Difference between 6 *s.* 6 *d.* and 7 *s.* (the 3d Price) against 6 *s.* because that is less than the mean Price. Lastly, the Difference between 6 *s.* 6 *d.* and 7 *s.* 6 *d.* is 1 *s.* which put against 5, the first Price. And thus having put the Differences between the mean Price and those less than it against the Prices bigger than the mean Price; and the Differences between the mean Price and those greater than it, right against those that are lesser alternately;

A Second Way.

	s.	*s.* *d.*
	5	0 : 6
s. *d.*	6	1 : 0
6 : 6 —		
	7	1 : 6
	7 : 6	0 : 6
	Sum =	3 : 6

Third Way.

	s.	*s.* *d.*	Sums. *s.* *d.*
	5	0 : 6 ; 1 *s.*	1 : 6
s. *d.*	6	0 : 6 ; 1	1 : 6
6 : 6 —			
	7	1 : 6 ; 6 *d.*	2 : 0
	7 : 6	1 : 6 ; 6 *d.*	2 : 0
		Sum Total =	7 : 0

2dly, Sum up the Differences, which you see is 3 *s.* 6 *d.*

3dly, Say by the single Rule of Proportion Direct,

	The Sum of the Differences. *s.* *d.*	Bushels the whole Mixture.	The Differences. *d.*	Bushels required.
As	3 : 6.	64 : :	12.	$18\frac{12}{42}$ of that of 5 *s.* *per* Bushel.
	3 : 6.	64 : :	6.	$9\frac{6}{42}$ of that of 6 *s.* *per* Bushel.
	3 : 6.	64 : :	6.	$9\frac{6}{42}$ of that of 7 *s.* *per* Bushel.
	3 : 6.	64 : :	18.	$27\frac{18}{42}$ of that of 7 *s.* 6*d.* *per* Bush.

Sum (or Proof) 64 being the whole Mixture.

The Proportions by the second Way of placing the Differences are thus:

s. *d.*		*d.*	
3 : 6.	64 : :	6.	$9\frac{6}{42}$ Bushels of 5 *s.* *per* Bushel.
3 : 6.	64 : :	12.	$18\frac{12}{42}$ Bushels of 6 *s.*
3 : 6.	64 : :	18.	$27\frac{18}{42}$ Bushels of 7 *s.*
3 : 6.	64 : :	6.	$9\frac{6}{42}$ Bushels of 7 *s.* 6 *d.*

64 Sum for Proof.

And

And the Proportions by the third Way of placing the Differences are still various. Thus,

	s.	d.		d.		
As	7 or	84.	64 : :	18.	$13\frac{60}{84}$	Bushels at 5 s. *per* Bushel.
		84.	64 : :	18.	$13\frac{60}{84}$	Bushels at 6 s.
		84.	64 : :	24.	$18\frac{24}{84}$	Bushels at 7 s.
		84.	64 : :	24.	$18\frac{24}{84}$	Bushels at 7 s. 6 d.
					64	Sum as before.

In the 2d Way you see that the Difference between the mean Price and 5 s. is placed against the Price 7 s. and 6 s. against 7 s. 6 d. 7 s. against 5 s. and between the 6 s. 6 d. and 7 s. 6 d. against 6 s.

And in the 3d Variety,

	s. d.	s.		s. s. d.
The Difference between ——	6 : 6 and	5 6 7 7 : 6	is placed against the Prices ——	7 and 7 : 6 7 and 7 : 6 5 and 6 5 and 6

So that the Difference between the mean Price and those less than it are placed against all those greater than it; and the Difference between the mean Price and those greater are placed against all those that are lesser than it. Then the Sum of the Differences in each Line are added together in the Column next the right hand; as 6 d. and 1 s. in the middle Column is 1 s. 6 d. in the third Column; and 1 s. 6 d. and 6 d. is 2 s. So the Sum of that third Column is 7 s. and thus ariseth the Numbers in porportion, in the third Way or Variety.

Hence you see, that there are three different Answers to one Question, and yet they are all true, as fully giving what is required in the Demand; whence (as well as from placing the Differences in each) it may justly be called *Alternate Alligation.* And for a thorow and intire Proof of this, I shall shew that 64 Bushels at the mean Price 6 s. 6 d. *per* Bushel, is (in the last or most abstruse, tho' best Way of the three Varities) the same Amount as each of the Quantities exhibited for Answers, being cast up at the Prices given, and added together, amount to. Thus,

64 Bushels

64 Bushels { the whole to be mix'd at the mean Price 6 *s.* 6 *d.* amount to —— } *l.* 20 : 16 : 0

	So also $13\frac{60}{84}$	Bush. of 5 *s.* *per* Bush. comes to		*l.* 3 : 8 $\frac{48}{84}$
The Clear	$13\frac{60}{84}$	—— of 6 *s.* ——	——	*l.* 4 : 2 $\frac{24}{84}$
Proof of the	$18\frac{24}{84}$	—— of 7 *s.* ——	——	*l.* 6 : 8
Operation.	$18\frac{24}{84}$	—— of 7 *s.* 6 *d.* ——		*l.* 6 : $17\frac{12}{84}$
	64 Sum of the Sorts.			Sum = *l.* 20 : 16 = Total Value *ut supra.*

I shall give one other Example, where there is only one Price lesser than the mean Price; and this I shall do according to the Method of the third Variety foregoing, that being the best, as making the equallest Mixture: where it is required to know much of 2, 5, 9, and 17 must be taken to make 100 of, or worth 4. See the Operation in the Margin.

	Prices given.	*Differences of the Prices and the mean Price.*		*Sum of each Line of Differ.*
4	2 *s.*	1, 5, 13,	=	19
= the mean Price.	5	2	=	2
	9	2	=	2
	17	2	=	2
		Sum of all the Differences		25

And this is proved as before, thus:

76 at 2	=	152	
8 at 5	=	40	
8 at 9	=	72	
8 at 17	=	136	
Sum	=	400	= 100 at 4.

Answer.

Now 25. 100 : : 19. 76 at 2 *s.*
25. 100 : : 2. 8 at 5
25. 100 : : 2. 8 at 9
25. 100 : : 2. 8 at 17

Note, That if the Prices given were ever so many, the Method is the same.

SECT.

SECT. XI. *The Rule of Falſe.*

THIS is ſo called not becauſe it requires any more Conjuration, or is more difficult to comprehend or perform than ſome of the Rules preceeding; but it is properly called *The Rule of Falſe Suppoſition*, becauſe by ſuppoſing Numbers to be the Anſwers to Queſtions which are not really ſo, but are feigned or ſuppoſed at pleaſure, we do by ſuch falſe or fictitious ones diſcover the true Numbers required.

An Example or two will illuſtrate this, and I ſhall not inſiſt farther, becauſe the Solution of an eaſy ſimple Equation in Algebra anſwers not only any Queſtion in this Rule, but alſo gives at the ſame time a Canon whereby any Queſtion of the like kind is much more eaſily and ſpeedily reſolved.

Queſt. 1.] Three Merchants built a Ship, which coſt *l.* 1600. *A.* paid a Sum not known, *B.* paid double to *A.* within *l.* 50, *C.* paid as much as *A.* and *B.* wanting *l.* 100: What did each pay of the Coſt?

	Operat. Colum. *l.*	Suppoſitions.	Errors.
1ſt, Suppoſe *A.* paid ——— ——— ———	200	200	
Then, according to the Queſtion, *B.* paid	350		
And *C.* paid ——— ——— ———	450		
Sum	1000		
But it ſhould have been *l.* 1600, therefore the Error is ——— ——— ———		———	600
2d, Suppoſe *A.* paid ——— ———	250	250	
Then it follows that *B.* paid ——— ———	450		
And *C.* paid, (according to the Queſtion)	600		
Sum	1300		
Which is therefore ſtill too little by ———		———	300
Then multiplying the 1ſt Poſition by the 2d Error gives ——— ———	60000		
And the 2d ſuppoſe in the 1ſt Error, gives	150000		
The Difference of the Product is =	90000		
Which divided by the Differ. of the Errors (300)			
The Anſwer is, that *A.* put in ——— ———	300		
Then *B.* put in ———	550		
And *C.* ——— ———	750		
Sum for Proof	1600		

Here you see, that the Suppositions and Errors being multiplied, you divide the Difference of the Products by the Difference of the Errors, which you must always do when the Errors are both Surplusages, or both Deficiencies of the supposed Numbers.

But if the one Error is caused by supposing too much, the other by supposing too little; then you must divide the Sum of the said Products by the Sum of the Errors, and the Quotient is the Answer, as in the next Example appears: which Rule you may retain in mind by this Distich;

When Errors are not both of the same kind,
To add the Products, as the Errors, mind:
But if they're both too small, or both too much,
Substraction must be us'd in Cases such.

Quest. 2.] Admit a Church hath a Choir (or Chancel) 40 foot long; and that the Ground taken up by the Belfrey is $\frac{1}{4}$ of the Chancel, and $\frac{1}{6}$ of the Nave, or Body; the Nave (or Body of the Church) is 3 times the Length of the Belfrey, and $\frac{3}{4}$ the Chancel: how long is the whole Church within the Walls, and every Part of it?

First find the Length of the Nave or main Body of the Church, for then the rest are discovered, thus:	*The Operation.*	*Suppositions.*	*Errors.*
1. Suppose the Nave be — — — — —		150	
$\frac{1}{6}$ of that is — — — —	25		
$\frac{1}{4}$ of the Chancel — — —	10		
Sum, is the Belfrey — —	35		
Then the Nave (according to the Question) is 3 times that — — — } =	105		
And $\frac{3}{4}$ of the Chancel, *viz.* — —	30		
Sum is but — —	135		
Which being less than 150 supposed, the Error is —		——	15
2. Suppose the Nave be — — — — — — —		102	
$\frac{1}{6}$ of that is — — — —	17		
$\frac{1}{4}$ of the Chancel is — — —	10		
The Sum or Belfrey is — —	27		
3 times that Belfrey is — — —	81		
$\frac{3}{4}$ of the Chancel is — — — —	30		
Sum is — —	111		
Which being more than the 102 supposed, the Error is		——	9

Now the Product of 150 by 9 is — — — 1350
And of the 102 by the Error 15, gives 1530

Which 2 Products I am to add according to the Rule, because the 1st Error was the effect of supposing too much, the 2d of supposing too little; and the Sum is — — — — — } 2880

That divided by the Sum of the Errors = 24 gives the Length of the Nave of the Church } = 120

And the Chancel being = 40
And the Belfrey $\frac{1}{6}$ of 120 and $\frac{1}{4}$ of 40 is = 30

The Length of the whole Church is = 190

And you'll find these 3 Dimensions (120, 40, and 30) to answer in all respects what is proposed in the Question, which is a Proof of the Work.

And thus you have a clear and perspicuous Method of solving Questions in this Rule of False Position; which may suffice till we come to Algebra, where the Reader will find a great Variety of Problems answered the best and shortest way, consistent with demonstrative Plainness.

The Canon for answering Questions of like nature with the 1st, is this: Divide the whole Value of the Ship, more twice what *B.* abateth of paying double to *A.* more what *C.* abateth of paying as much as *A.* and *B.* by 6, and the Quotient is what *A.* paid. And this Canon holds, vary the Numbers as you please, keeping to the Words.

The Canon answering Questions of like nature with the 2d, is, Whatsoever Length the Chancel of the Church is, if that be multiplied by 3, it gives the length of the Body of the Church proportioned as in the Question, as 3 times 40 is 120. But if you suppose the Chancel 60, then is the Nave 180, and the Belfrey 45, in all 285. For Proof $\frac{1}{6}$ of the Nave, and $\frac{1}{4}$ of the Chancel 60, is 45 = the Belfrey; and 3 times that (or 135) added to $\frac{1}{4}$ of the Chancel, is 180.

Single Position.

But there are some Questions answered by one single Supposition, with the help of the Rule of Proportion. As for

Example.

Example.] Three Men build a Houſe, which coſt *l.* 300. *A.* paid a Sum unknown, *B.* paid twice as much, and *C.* paid 3 times ſo much; what did each pay of the *l.* 300?

I ſuppoſe *A. l.* 40, then *B.* muſt pay 80, and *C.* = 120; the Sum of which is but *l.* 240, inſtead of *l.* 300. Then I ſay,

If 240 doth ariſe from ſuppoſing 40,
What Number will 300 be the Reſult of?
The Anſwer is 50; for

240. 40 : : 300. 50

Now that 50 is the Sum that *A.* paid, may be proved thus:

$$\left.\begin{array}{r} A. = 50 \\ B.\text{ paid} = 100 \\ C.\text{ paid} = 150 \end{array}\right\}\text{Sum} = 300 \text{ for Proof.}$$

Here ends VULGAR ARITHMETIC.

CHAP. III.

DECIMAL ARITHMETIC.

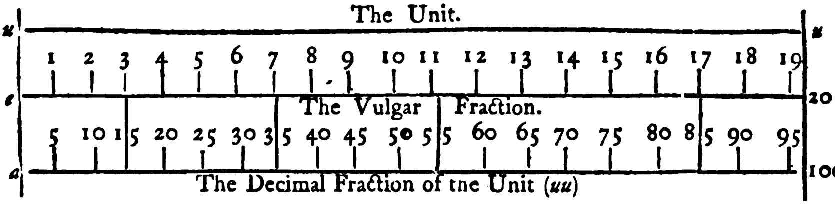
SECT. I. *Notation and Numeration.*

THIS kind of Arithmetic takes its Name from the Nature of the Denominator, which is always 10, or ſome Power of 10; in which only it differs from a Vulgar Fraction: for as that has any promiſcuous Number for its Denominator, ſo a Decimal Fraction hath always 10, 100, 1000, 10000, &c. for its Denominator.

2. Hence 'tis eaſy (the Denominators of Decimals being ſo few in compariſon of Vulgar, and ſo certain) to expreſs a Decimal Fraction without its Denominator, by ſeparating ſo many places of the Numerator, as the Denominator hath of Cyphers, by a Point: Thus $\frac{5}{10}$ is wrote .5; $\frac{25}{100}$ is .25; $\frac{75}{1000}$ is .075, &c. And if (as in the laſt) there be not ſo many places in the Numerator as there are Cyphers in its Denominator, you muſt make up that Number by placing a Cypher or Cyphers towards the left hand of the Numerator.

 3. The

3. The Nature of a Decimal Fraction will farther appear by what follows:



The uppermost Line (*uu*) is a Unit; the Line (*e* 20) is of Vulgar Fractions of that Unit, whose common Denominator is 20; and the Line (*a*, 100) is of Decimal Fractions of the Line (*uu*) whose Denominator is 100.

And here the Decimal Fraction $\frac{15}{100}$ or .15 is equal to the Vulgar Fraction $\frac{3}{20}$; .35 is equal to $\frac{7}{20}$; .85 equal to $\frac{17}{20}$, &c.

4. But that you may be the better able to read or write down any Decimal Fraction properly, take the following Table; the Words at the head of which shew how to read the Numerator, and those toward the right hand, to read the Denominator.

Millions.			Thousand								
1 Hundred	Twenty	Three	4 Hundred	Fifty	Six	7 Hundred	Eighty	Nine			
.0 1	2	3	4	5	6	7	8	9	Ten Thousand Million		Parts.
.0	2	3	4	5	6	7	8	9	Thousand Million		
	.2	3	4	5	6	7	8	9	Hundred Million		
		.3	4	5	6	7	8	9	Ten Million		
			.0	5	6	7	8	9	Million		
				.0	0	7	8	9	Hundred Thousand		
					.0	0	0	9	Ten Thousand.		
						.7	8	9	Thousand		
							.8	9	Hundred		
								.9	Tenth		

5. There

5. There is not, nor can be, any Fraction invented, so easy to work, because so like to an intire Number, as the Decimal; for there is the same Increase of the Places Values in these as in Integers, as appears by the following Table, from the One Hundred Millionth Part of a Unit to 100000000 of Units, as 111111111.11111111; and thus every place towards the left hand is ten times that towards the right.

$\frac{1}{100000000}$ Parts; 10 times that, is
$\frac{1}{10000000}$ Parts; 10 times that, is
$\frac{1}{1000000}$ Parts; 10 times that, is
$\frac{1}{100000}$ Parts; 10 times that, is
$\frac{1}{10000}$ Parts; 10 times that, is
$\frac{1}{1000}$ Parts; 10 times that, is
$\frac{1}{100}$ Parts; 10 times that, is
$\frac{1}{10}$ Parts; 10 times that, is
1 or a Unit; 10 times that, is
10 or 10 Units; 10 times that, is
100; 10 times that, is
1000; or 10 times that, is
10000; 10 times that, is
100000; 10 times that, is
1000000; 10 times that, is
10000000; 10 times that, is
100000000; &c.

6. Decimals are produced from Vulgar Fractions, they being the Quotients arising by dividing the Numerator, (with Cyphers annexed toward the right hand) by the Denominator. Hence it follows, that some Decimals are *Compleat*, and others *Infinite*.

7. I call that a Compleat Decimal, when nothing remains of the said Division by which the Decimal is made. And to distinguish a Compleat Decimal, you may put a Period after the last Figure towards the right hand thus .0125. and .00375.; and those not having such Mark, are supposed to be Infinite.

The more Places an Infinite Decimal consisteth of, the more it expresseth the Truth.

I call that an Infinite Decimal, which is no Aliquot Part of the Dividend by the Divisor that produceth such Decimal, but something will always remain in dividing.

Infinite Decimals are composed either of uncertain Digits, as .146129, &c. or of those certain and known, as .333, &c. .44444, &c. The former of these ought to have places more or less, according to the Use that is to be made of them, (for which I have given a Table to direct at the beginning of Division, *Sect.* 6. following.) But Infinite Decimals composed of certain Digits, may for shortness

ſhortneſs be wrote thus: The Decimal of $\frac{1}{3}$ which is .33333333, *&c.* may be .3*r* that is 3 *repeated infinitely*; or if a Digit repeat after others are in the Quotient, they will ſtand thus:

.83 *r* 1 which ſhews that 1 (or the 3) is only repeated.

.285714285714, *&c.* hath 6 repeating, wrote thus .285714 *r* 6 And ſhews that 6 of the Digits next the left hand would be repeated by carrying on the Diviſion *ad infinitum.* And theſe by ſome are called circulating Numbers, all which kinds of Decimals are produced by Diviſion thus:

Diviſors.	Digits divided.	Decimals ariſing.	Diviſors.	Digits divided.	Decimals ariſing.
9	8.0	.8 *r*	7	6.000000	.857142 *r* 6
	7.0	.7 *r*		5.000000	.714285 *r* 6
	6.0	.6 *r*		4.000000	.571428 *r* 6
	5.0	.5 *r*		1.000000	.142857 *r* 6
	1.0	.1 *r*	6	5.00	.83 *r* 1
8	7.000	.875.		4.	.6 *r*
	5.000	.625.		1.00	.16 *r* 1
	3.000	.375.	5	4.0	.8.
	1.000	.125.		3.0	.6.

A mixt Number (or one compoſed of a whole Number and Decimal) is thus written 365.125.; 1728.34617, and 252.6*r*, &c.

And this ſhall ſuffice to ſhew the Nature of Decimals, and how to read and write any. I ſhall next paſs on to Reduction, and the reſt of the Rules; inſerting only ſuch Examples as are likely to accrue in Practice; and avoiding all thoſe that are impertinent, and tend to perplex the Reader, and make this Part of Arithmetic abſtruſe and tedious, which in reality is moſt eaſy and obvious.

SECT. II. *Reduction of Decimals.*

IT is almoſt as improper to treat of Addition and the other Rules before this of Reduction in Fractions, as to teach Reduction before the Parts of ſingle Arithmetic in Integers: for what can be more irregular, than to ſhew how to add, ſubſtract, *&c.* thoſe Numbers, which you neither know the Production of, nor how to diſcover

discover what Parts of Coin, Weight, *&c.* they represent or include? Therefore

I shall, as is usual, as well as justly and methodically done, in the next place shew, how to reduce Vulgar Fractions of inferior Denominations to Decimals of superior, considered as Integers of those Fractions; and, by way of Proof thereof, give Rules and Examples to find the Values of those Decimals or any other.

I. *To reduce Vulgar Fractions into Decimals.*

A Decimal of 4 places, nay generally of 3, is sufficient, when that Decimal is not to be multiplied by any Number: Therefore the Denominator of a Decimal of 4 places being (as shewed before) 10000, the Proportion will be for any Fraction in general, to find the Numerator of such a Decimal;

As the Denominator of the Fraction given
Is to the Numerator of that Vulgar Fraction::
So is the Denominator of the Decimal (admit 10000)
To its Numerator or the Decimal required.

Example.] What is the Decimal of $\frac{27}{39}$?

Here it may be observed, *Rule* = As 39. 27 :: 10000. .6923 = the Answer.

39) 2700000 (.6923
.....
360
90
120
30

1st, That a Cypher next the right hand of my Decimal may be omitted, because the same place in the Denominator is a Cypher also: Thus $\frac{1390}{10000}$ is $\frac{139|0}{1000|0}$, and $\frac{13000}{100000}$ is $\frac{13|000}{100|000}$: that is, $\frac{139}{1000}$ is equal to $\frac{1390}{10000}$, and $\frac{13}{100}$ to $\frac{13000}{100000}$, in their lowest Terms.

2dly, *Note for a General Rule, That you must always set off with a Point so many places of the Quotient for your Decimal, as the Number of Cyphers in the proposed Denominator (according to the beginning of the last Sect). And if there be not so many Figures in the Quotient, as there are Cyphers in the Denominator given, then that Number must be made up by putting Cyphers towards the left hand next the Point in the Quotient or Decimal.*

3. Hence it follows, That *to reduce Coin, Weight, Measure,* &c. *to the Decimals* of an higher Denomination, may easily be done, by first

first representing them in Vulgar Fractions, and then reducing those Vulgar to Decimal, as above.

So to reduce $3\frac{1}{4}d.$ to the Decimal of a Pound, I consider that this is 13 Farthings, and that a Farthing is $\frac{1}{960}$ of a Pound: therefore 13 *qrs.* is $\frac{13}{960}$ of a Pound. So that

960. 13 : : 10000. .0135 = the Decimal sought.

Hereupon multiplying and dividing, I find the Quotient 135; but because there are 4 Cyphers in the Denominator (or 10000) given, therefore I must have 4 places towards the right hand of the Point for the Decimal, which 4th place I make up with a Cypher, as you see; and the Converse of this Proportion reduceth any Decimal to a Vulgar Fraction; or one Vulgar Fraction may be reduced to any other, whose Numerator or Denominator is given. For

135, *&c.* 10000 : : 13. 960 or 10000. 135, *&c.* : : 960. 13

Or if you would reduce the given Vulgar Fraction to another whose Denominator is 12372, the Proportion holds thus:

960. 13 : : 12372. $167\frac{516}{12372}$

So that $\frac{167}{12372}$ is near equal to $\frac{13}{960}$ or .0135

Which may suffice to shew how to reduce any one Fraction to another and consequently a Vulgar to another whose Denominator is 10, 100, *&c.* *i. e.* any Vulgar to a Decimal.

And this puts me in mind of a new kind of Decimal that might be contrived, which would not only admit of being expressed in one Line as the common Decimal; but whereas that Decimal saves much of Division (which is its Excellency) so this new one would save much trouble of Multiplication: and that is, by reducing all Vulgar Fractions to one whose Numerator is 10, 100, 1000, *&c.* And the Numerator might be shewn, by pointing over the Figure in the Denominator, where the 1 or place next the left hand of the Numerator would fall: thus $\frac{10}{1472}$ is wrote $14\dot{7}2$: $\frac{100}{1472}$ is $1\dot{4}72$; and $\frac{1000}{1472}$ $\dot{1}472$. But I shall leave this Notion to be improved by the Ingenious, if thought worth their while; and beg the Reader's pardon for this *small* Digression, which there is not room in this Book to inlarge on.

II. *To*

II. *To reduce Weight, &c. to Decimals, as* per *Rules Parag.* 2, 3, &c. *above.*

	qrs.	*lb.*					*Decimals required.*
1. In Averdupois	3 :	20,	is $\frac{104}{112}$ *C.*	And 112.	104 ::	10000.	.9285
		18,	is $\frac{18}{112}$ *C.*	And 112.	18 ::	10000.	.1607
	oz.	*dw. gr.*					
2. In *Troy* Weight	11 :	12 : 10,	is $\frac{5578}{5760}$.	And 5760.	5578 ::	10000.	.9684
	Gall.	*Pint.*					
3. In Liquid Measure this Vulgar Fraction of a Beer-Barrel	1 :	3 :	is $\frac{11}{288}$.	And 288.	11 ::	10000.	.0382
4. In Dry Measure 1 Pint is $\frac{1}{512}$ of a Qr.				And 512.	1 ::	10000.	.0019
5. In Long Meas. 1 Yard is $\frac{1}{1760}$ of a Mile.				And 1760.	1 ::	100000	.00056

And thus I have fully shewed the Fundamentals of making Decimals. I shall proceed in the third place,

III. *To find the Values of Decimals.*

Note, That .9 of any lowest Denomination, or .999 of an highest, may be taken for a Unit, as in the last of the 3 first Examples following, *&c.*

To find the Values (as in these Examples) you must multiply as in the 2d, 4th, 6th, and 8th Columns.

Value.	*Decimals of a L. Sterl. as*	*Value.*	*Decimals of 1 C. wt. as*	*Value.*	*Decimals of 1 lb Troy, as*	*Value.*	*Decimals of a Barrel of Beer, as*
	.0135 by 20 *s.*		.9285 by 4 *qrs.*		.9684 by 12		.0382 by 4
	.2700 by 12 *d.*	*Qrs.* 3	.7140 by 7 *l.*	℥ 11	.6208 by 20		.1528 *Firk.* by 9 *Gal.*
Pen. 3	.2400 by 4 *qrs*	4	.9980 by 4 *l.*	*dw.* 12	.4160 by 6	*Gal.* 1	.3752 8 *Pints.*
or 1 *Farth.*	.9600	lb 19 or 20 *Pound.*	.9920	2	.4960 by 4	*Pints* 3	.0016
				grs. 9 or 10	.9840		
1	2	3	4	5	6	7	8

And cut off as many Places from the right hand of each Product, as there are in the Decimal given. These prove the Examples above of the same Denominations.

Note, That in the 2d Example I multiply .7140 by 7, and that Product by 4, inſtead of multiplying .7140 by 28, the Pounds in a Quarter of an Hundred. And in the 3d Example, becauſe I cannot multiply .4160 by 24, as by 1 Figure, therefore I firſt multiply it by 6, and that Product by 4; which Methods produce the ſame thing as multiplying .7140 by 28, and .4160 by 24, only there is Addition ſaved in each Example.

IV. *A ſecond Way to find the Decimal of any thing, deduced from the four laſt Examples.*

From a due Conſideration of the laſt Examples, you may, by a direct contrary way of Operation, find the Decimals of any Denomination, here dividing, whereas there you multiplied. For inſtance,

Divide by	*To reduce to a Decimal of a Pound.*	*Quotients or Anſwers.*	*Divide by*	*To reduce to a Decimal of 1 C.*	*Quotients or Anſwers.*	*Divide by*	*To reduce to a Decimal of l. Troy.*	*Quotients or Anſwers.*
	l. 0	.0135416 *r* 1 = 3d *qu.*		*C.* 0	.92857 = 3d *qu.*		*l.*	.9684 = 4 *quo.*
20	*s.* 0	.27083 *r* 1 = 2d *quo.*	4	*qrs* 3	.71428 = 2d *qu.*	12	*oz.* 11	.62083 *r* 1 = 3 *q.*
12	*Pence* 3	.25 = 1ſt *quote.*	7	5	.= 1ſt *quote.*	20	*dw.* 12	.416 *r* 1 = 2d *q.*
4	*Farth.* 1		4	*lb.* 20		6	2	.5 = 1ſt *quote.*
						4	*gr.* 10	

1. Having put down your Denominations that you would reduce into the Decimal of an higher, as you ſee in theſe Examples, begin at the loweſt to divide by the proper Diviſor for reducing into the next ſuperior Denomination, putting, or imagining, Cyphers to ſtand in the Decimal places toward the right hand. But,

2. You will obſerve from theſe Examples, that the mixt Numbers, as 3.25 Pence, 3.7428 Quarters of a *C.* are the Dividends, and that the Decimals are the Quotients: Thus I firſt divide 1 Farthing or 1.00 by 4, and put the Quote to the right hand of the 3 Pence for the Decimal of a Penny; then I divide 3.25 by 12, *&c.* But,

3. In the ſecond Example, inſtead of dividing ℔ 20 by 28, I divide it at twice by 4, and that Quote by 7; and alſo in the third Example, inſtead of dividing by 24 Grains, I (in this Caſe putting down only the Quotient, and doing the reſt mentally) divide by the Digits 4 and 6, which amounts to the ſame thing.

V. *Some*

V. *Some brief Ways of Reduction of Decimals.*

There are in many Instances much shorter Ways both of making Decimals and of finding their Value.

1. As to the first, any even Number of Shillings under 20 are reduced to the Decimal of a Pound, by taking half thereof; and thus you see in the Margin.

The Decimal of 6 *s.* is		.3. *l.*
	12 ——	.6.
	14 ——	.7.
	18 ——	.9.

2. Any odd Number of Shillings under 20 are made Decimals of a Pound by putting a Cypher towards the right, and then taking the half, as in the Margin.

s.	*l.*
3.0 =	.15
9.0 =	.45
13.0 =	.65
17.0 =	.85

3. From hence it is plain, that any Number of Shillings may have their Decimals put down as soon as named, by taking half the even Number; and for the 1 *s.* (in an odd Number) you see that it is always 5 in the 2d place from the Point.

4. *To put any Pence and Farthings down in the Decimal of a Pound, consider what Farthings they make, and put them down in the 2d and 3d places from the Point: But after the Farthings are 24 or more, add 1 to the third place in the Decimal. And thus* 19 s. 7 d. ¾ *is put down in a Decimal immediately* .982; *for the* 19 s. *is* .95, *and the* 7¾ *is* 31 *Farthings, more* 1 *because* 1 *must be added at every* 24, *is* 32; *which added to the* 2d *and* 3d *places from the Point in* .95, *the Sum is* .982. And,

5. If you desire to be so exact to have a 4th place from the Point, you may make the Figure in the 4th place 1 more than a 3d of what Farthings are in the Farthings given, when they do not exceed 23. As in the 3d Example of the 4 in the Margin, for 16 *s.* I put 8; for 9 *d.* ½ or 38 Farthings I put 39 in the 2d and 3d places, because 38 is 13 above 25. Then for the 3d Place I take ⅓ of 13, which is 4; to which I add 1, and the Sum is 5 for the 4th place from the Point.

s.		*d.*	*May be put in Decimals.*
8	:	4½ ——	.4187
13	:	2¼ ——	.6594
16	:	9½ ——	.8395
19	:	11¼ ——	.9989

So also in the 4th Example for the 19 *s.* I put .95, for the 6 *d.* of the 11¼, I put 25, (that is, 1 added at 6 *d.*) lastly, for the 5 *d.* ¼ (the rest of the 11¼, or 23 *qrs.*) I put ⅓ of 24, more 1.

For *note*, That when the Farthings given are under, or so much as they are more than 24, you must take a third part of that Sum they are next to, which can be divided without a Remainer; as in the last, I take $\frac{1}{3}$ of 24, because 23 is nearer 24 than it is to 21.

And thus the Decimal of a Pound may be wrote to 4 places at once, or mentally computed, in a fifth part of the time it can be found by the nearer of the Ways above, and that near enough the Truth, unless it be required to be multiplied, as is before mentioned, and as in the Table at the beginning of Division.

6. *The Value also of any Decimal of a Pound may be found by Inspection: For if the place next the Point be doubled, it gives the Shillings, to which add* 1 s. *so often as the 2d place is 5 or more, for the reason under the 2d Rule above; then call what is under 5 (or more than 5) in the 2d place so many Tens of Farthings, and the Digit in the 3d place so many Units; and as often as they are 25, make them less by* 1.

l.		*s.*	*d.*	*q.*
.912	=	18 :	2 :	$3\frac{1}{2}$
.123	=		2 :	$5\frac{1}{2}$
.754	=	15 :	1	
.8766	=	17 :	$6\frac{1}{8}$	
.9877	=	19 :	9	

Or more accurately thus; tho' the Rule above may be thought near enough, as not erring a Farthing:

If the Farthings under or above 5 in the 2d place from the Point, and those in the 3d place be — — — —	6, and under	19, deduct	$\frac{1}{2}$ a Farthing.
	19, ———	33, ———	1
	33, ———	45, ———	$1\frac{1}{2}$
	45 and upward	———	2

And so often as the 4th place is 6 or more, you may add half a Farthing.

And after this manner you may read Decimals in Shillings, Pence, *&c.* as fast as if they were wrote so; as I have examined many a thousand, having the Species read to me looking on the Decimals, which if continued to ever so many places, they alter not the Rules above.

Note, That these Allowances or Deductions are made because the Rule supposeth 1000 Farthings in a Pound *Sterling*, whereas there are but 960; *therefore* if 1000 Farthings is 40 more than the Truth, 500 is 20 more, 250 is 10, or 25 is 1 too much. And it is sufficient if done to a Farthing, which Decimals may be valued without regarding the 4th place,

7. *To find the Value of the Decimal of a Foot in Length by Inspection.*

Rule. *For every* 10*th of a Foot reckon as many Inches and as many Quarters, under* .5, (*which is known* 6 *Inches.*) 2dly, *For every* 2 *in the* 2*ds place add* 1 *quarter of an Inch.* And this will not err a quarter

quarter of an Inch in the generality of Decimals, if in any.

	In.	*qr.*
Here in the 2d Example .2 according to the Rule is	2 :	2
and .08 is half so many qrs. =	1 :	—
Sum =	3 :	2

Feet.	*Inch.*	*qr.*	
.37 =	4 :	2	*Examples.*
.28 =	3 :	2	
.73 =	8 :	3	
.92 =	11 :	1	

And in the 4th Example,

.9 ; .5 I say is ———— ————	6 :	—
.4 is 4 *inch.* 4 *qr.* or	5 :	—
.02 is ———— ————	:	1
Sum =	11 :	1

Note, That in adding half, in the 2ds place, if odd, you may omit the odd, and take half the next lesser even Number, as in .07 and .03 I take only 3 *qrs.* and 1.

The immediate writing down any Inches and Quarters in the Decimal of a Foot is done so easily by the Rule above inverted, that there is no occasion for Examples.

8. *To find the Value of the Decimal of a Gallon by Inspection.*

Rule.] *Multiply the Digit in the Primes place of the Decimal by 8, and the Digit next the left hand of the 2 Digits in the Product is so many Pints; then add the Digit next the right hand of the Product to that in the 2ds place of the Decimal given, and multiply that Sum by 4, and the place next the left hand of the Product are Quarters of Pints.* Which is done with about a third of the trouble in the common Way; and gives the Answer accurately enough.

Gall.	*Pints.*	*qrs.*	*Gall.*	*Pints.*	*qrs.*	*Gall.*	*Pints.*	*qrs.*
.19 =	1 :	2	.64 =	5 :	—	.34 =	2 :	3
.28 =	2 :	1	.73 =	5 :	3	.71 =	5 :	2
.37 =	3 :	—	.82 =	6 :	2	.66 =	5 :	1
.46 =	3 :	3	.91 =	7 :	1	.88 =	7 :	—
.55 =	4 :	2	.11 =	— :	3	.99 =	8 :	—

Thus in .66, 8 times 6 is 48, the 4 are Pints, and the 8 of the Product added to 6 in the 2ds place is 14; 4 times 14 is 56; the 5 are Quarters of Pints or $1\frac{1}{4}$ Pints, which with the 4 is $5\frac{1}{4}$ Pints, &c.

9. *To*

9. *To find the Value of the Decimal of a Barrel of Beer very briefly, and accurately.*

Rule.] *Multiply the given Decimal by 4, cut off from the Product 1 less than the Decimal places given; then substract every Digit in the Product from its right hand Digit, and the Remainer is the Value sought in Gallons and Parts.* Thus in the 3d Example, the Product of .9876 by 4 is 39.504 (cutting but 3 off, according to the Rule): Then I begin with 4, and take that from (0) which I suppose next it towards the right hand, that is, from 10, rests 6; 1 borrow'd and 0 (in Tens place) is 1 from 4 (in Units place) there rests 3; 5 from 10 rests 5; 1 borrow'd and 9 is 10 from 15 rests 5; 1 borrow'd and 3 (next the left hand, Column 2.) is 4, from 9 rests 5, and 0 from the said 3 rests 3; which said Rests are the Answers in Column 3, *viz.* 35.5536.

Examples.		*Answers.*	*Examples.*		*Answ.*
Barrels.	*Products.*	*Gallons.*	*Barrels.*	*Products*	*Gallons.*
.1234	4.936	4.4424	.2 *r*	8.8 *r*	8.
.5678	22.712	20.4408	.5 *r*	22.2 *r*	20.
.9876	39.504	35.5536	.7 *r*	31.1 *r*	28.
.5432	21.728	19.5552	.9 *r*	39.9 *r*	36.

10. The 2 short Rules above being observed, the Decimal of a Barrel is found in Gallons, Pints, and Quarters of a Pint, very concisely thus:

Example 1.	*Example* 2.
Barrels Beer .19765	*Barrels Beer* .8764
7.9060	35.056
Rests Gallons 7.11540	*Rests Gallons* 31.5504
Pints 0 : 3.6 { *Quarters of a Pint and Parts.*	*Pints* = 4 : 2 *quarters Pints.*

I have contrived these 2 last for the Use of his Majesty's Officers of Excise, where they may almost by Inspection see the Value of the Decimal of a Barrel, to the 10th part of a Pint.

11. *A most concise and easy Way to find the Value of the Decimal of a Pound Averdupois Weight exactly.*

Rule 1.] This may be done by this Rule: Multiply the Decimal given by 3, and cut off the Product 2 places fewer than are in the Decimal given; from that Product substract a 7th of itself; then take a 16 of the Remainer, (which you may do, only putting down

down the Quotient, which are Ounces; and the Remainer (less 1) are Drams, *per* this Example.

2. But the Rule to be insisted upon as more brief and accurate, is this: Multiply the given Number by 8, and double the Product of every 2 Digits as you go on, and the Product is Ounces and Parts. Then multiply in like manner the Parts of an Ounce, and the Product is Drams and Parts.

.924
277.2
39.6
237.6
℥ 14 : 12 ʒ.

	Example 1.	Example 2.	Example 3.
Given =	℔ .924	℔ .567	℔ .234
Answer { Ounces =	14.784	Oun. = 9.072	Oun. = 3.744
Answer { Drams =	12.544	Dr. = 1.152	Dr. = 11.904

Or if it be thought burthensome to the Memory to multiply by 8, &c .as *per* the Rule above, it will be more easily done thus:

Rule 3.] *Multiply the Decimal given by* 6, *and after you have added the Tens carried (as usual) add thereto the Digits next towards the right hand of that which you last multiplied, and put down what is above* 10 *(as commonly): But when you have multiplied the Digit next the left hand, add the Tens carried to that Digit.* For instance, in the first Example above say 6 times 4 is 24, put down 4, and carry 2; 6 times 2 (the next Digit) is 12, and 2 carry'd is 14, and the 4 in Units place is 18, put down the 8, and carry 1; 6 times 9 is 54, and 1 carry'd is 55, and 2 (the middle Digit) is 57; put down 7, and add the 5 carry'd to the 9 last multiplied, and that makes 14: which I take to be the most concise and easy Method that this thing is capable of, unless as by the following

Rule 4.] Take the Primes place for so many Ounces, Halfs and Tenths of an Ounce; Seconds place for so many 2.5 Drams; and Thirds place for so many Quarters of a Dram.

Example. In .924 ℔ .9 is 9 ℥, 9 Halfs, and .9 = ℥ 14 : 6 ʒ. 2 in Seconds place is 2 times 2.5 or 5 ʒ; and 4 in Thirds place is 1 ʒ = 14 ℥ 12 ʒ.

12. *The most easy and short Way of finding the Value of a Decimal of* 1 ℔ *Troy Weight in* ℥, dw. *and* grs.

1st, Multiply the Decimal given by 2, and add what is carry'd, and the Digit next the right hand (as in the third Rule) gives the Ounces.

2dly, Twice Primes place in the next Decimal of Ounces is so many Penny-Weight, and every 5 in the Seconds place is 1 more.

3dly,

3*dly*, What the Seconds place is under or above 5, and the Thirds place; take half thereof for Grains. But if they are 25 or upward, deduct 1; if 38 or upward, deduct 2. And if the Digit in Thirds place be an odd one, you may deduct only the odd one at 25, and 1 and the odd one at 38, &c.

Examples.	℥. *dw.* *gr.*
℔ .5642 or 6 : 15 : 10	
℥ 6.7704 do this by Inspection.	
℔ .9782 or 11 : 14 : 18	*dw.* *gr.*
℥ 11.7384 by Inspection is	14 : 18

SECT. III. *Addition of Decimals.*

THIS Rule has no more difficulty than that of one Denomination in whole Numbers: observing,

1. To place the Decimal Places next the Point, or the several Places next the left hand, and the Points themselves, &c. one under another, as in the Examples.

2. If the Decimals to be added exceed not 5 in number, it is sufficient if they consist of 5 places where the Sum is not to be multiplied by any thing; but where it is to be multiplied, you must observe the Rules given in the beginning of Division, *Sect.* 6. of this Chapter. And if Decimals of a Pound *Troy* are to be added, the Decimals to be added ought to consist of at least 6 places: But if the Decimals be compleat, it is no matter how few the places be.

Example 1.

.925.
.0775.
.5.
.25.
.75.
.05.
.125.
Sum 2.6775.

Example 2.

Decimals of a Pound Sterl.		*Value in Specie.*
.9.	———	18 *s.* — *d.*
.05.	———	1 : —
.125.	———	2 : 6
.75.	———	15 : —
.775.	———	15 : 6
.65.	———	13 : —
.825.	———	16 : 6
.15.	———	3 : —
Sum 4.225.	=	*l.* 4 : 4 : 6 Sum Proof.

3. But

3. But because the Decimals that occur in Practice are generally, if not always promiscuous, that is, neither all compleat Decimals, nor infinite, (certain or uncertain) therefore Examples of such mixt will be the most useful.

Example 3. *Of a mixt Number.*	*Example 4.* *Promiscuous Decimals.*	*Example 5.* *Promiscuous Decimals, &c.*
198765.	.17565	.9666666
19876.5.	.6 *r*	.12345
1987.65.	.72939	7.6777777
198.765.	.5.	4.00276
19.8765.	.17234	.9879879
1.98765.	.97548	.12729
	.9 *r*	.9.
Sum 220849.77915.	.87654	14.78593 Sum.
	.25.	or 14.7859324 *r* 3
	.09842	
	Sum=5.44447 or = 5.444486 *r* 1	

By the 3 first Examples you see how easy the Addition of Decimals is, when placed as they ought to be, and so many are cut off for Decimals in the Sum, as are the greatest Number of places of those given; which is sufficient for the understanding of this Rule.

But because some have made a difficulty where there was none, by talking much of repeating Numbers; I have given the two last Examples, to shew that the common Method is accurate enough, without taking Notice of repeating Digits as differing from others.

In Example 4. there being 2 Decimals consisting of repeating Digits, I put down only those next the Point, and sum up them, 1st saying 9 and 6 is 15, and 1 carry'd (supposing the 9 and 6 which repeat were placed 2 places more to the right hand than 5 places, the greatest Number given) is 6: which I put down as you see in the lowest Line, and say 1 and 2 is 3, and 4 is 7, and 8 is 15, and 4 is 19, and 9 is 28, and 5 is 33, and (coming downward again at every Series you add upward, with the repeating Digits) 6 is 39, and 9 is 48; put down 8 and carry 4 to the next Series, which makes it 29, and the 6 and 9 repeating is 44; put down 4, and carry 4 to the third Series, which makes it 39, and the 6 and 9 Repeaters is 54; put down 4, and so proceed in like manner to the Digits next the Points, where the 6 and 9 are added in Course, as being there placed; so the Sum

is 5.444486 *r* 1. And if you had filled only 5 places each with the 2 Digits that repeat, and had added them as the 3 former Examples, the Sum would have been the same, wanting only .000016, which supposing even the Decimal of a Pound *Troy*, is less than a 10th of a Grain. And in the 5th Example the Difference between the Sums having regard to the repeated Digits, and the Sum the common way is so inconsiderable, as appears by the 2 Sums; not worth notice, if the 2d Rule of this Section be observed.

I have demonstrated in the 2d Example, by the Addition of the Specie answering the Decimals, that the Rules for adding Decimals are right: I shall here shew *that the Rule for adding these is agreeable to that for Vulgar Fractions.* For example, in adding $\frac{365}{1000}$ to $\frac{96}{100}$, the common Denominator is 100000, the Numerators 36500 and 96000, the Sum of which is 132500; so the Answer, cutting off the Cyphers as useless, is $\frac{1325}{1000}$: and dividing the Numerator by the Denominator, that is, cutting off 3 Figures, the Answer is 1.325: But the superfluous trouble of putting down the Cyphers being omitted, the Work of Addition of Decimals is as above, and as *per* Margin.

$$\begin{array}{r} .365 \\ .96 \\ \hline 1.325 \end{array}$$

SECT. IV. *Substraction of Decimals.*

THERE is no difference between the Method of this and Intire Numbers, observing to place the Point of the Subtrahend exactly under that of the greater Decimal. Two or three Examples will suffice to shew it.

Example 1.	*Example* 2.	*Example* 3.
From .9876	From 98.76	From 9876.
Take .87654321	Take .08765	Take .1234
Remains = .11105679	Remains = 98.67235	Rem. = 9875.8766

Example 4.

From 123.45
Take 97.

Rests = 26.45

Example 5. *Averdupois Weight.*

			lb	℥	ʒ
From	*l.* 1.0976	— or from	1 :	1 :	8.9856
Take	.987	— or take	0 :	15 :	12.672
Difference	.1106	— — or =	0 :	1 :	12.3136

The

The Reason of the Work of Substraction of Decimals is the same as is said of Addition, only substracting instead of adding the Numerators; agreeing exactly with the Deduction of Vulgar Fractions, in which Method if the Cyphers be neglected in the Results, the Method falls just into that of Substraction of Decimals, where the cutting off from the Sums, Remainers, Products, &c. is the same as dividing by a Unit with Cyphers (which is the Nature of the Denominators of Decimals) as is shewed in Division of Intire Numbers.

SECT. V. *Multiplication of Decimals.*

THERE is no Difference between the Operations here and by Integers, but observe this Rule:

After the Work is over, you must set off so many Figures towards the right hand of the Product, as you have Decimals in both the Factors.

But if so many places are not in the Product (as it will happen when you multiply Decimals of small Value) then you must make up that Number by placing Cyphers towards the left hand of the Product next the Point.

Example 1.
A Decimal by an Integer.

```
   .012345.
       932
   -------
     24690
    37035
  111105
  --------
 11.505540
```

Example 2.
A mixt by a mixt Number.

```
     9.37241.
     25.324.
   ----------
     3748964
    1874482
   2811723
  4686205
 1874482
 ------------
 237.34691084
```

Example 3.
A mixt Number by a Decimal.

```
   36.252.
   .00032.
   -------
    72504
  108756
  --------
 .01160064
```

Example 4.
A Decimal by a Decimal.

```
     .12564 r 1
     .00009
 ---------------
 .0000113079 r 1
```

Note, That in the fourth Example, because the 4 is repeated *ad infinitum*, therefore I say, 9 times 4 is 36, and 3 (which would be carried if you actually put down another 4) is 39; put down 9 and carry 3.

Now

Now if you had put down and multiplied 100 Fours of those repeated, so many Nines would also be repeated in the Product; but for brevity sake I only put down one of each with an *r*.

To multiply mixt Numbers, Decimals, &c. and to have only so many Decimals in the Product as you assign: And how to avoid all unnecessary Figures in such Operations.

Case 1. A mixt Number by a mixt, as 1.234504 by 9.2123. and to have only 4 Decimals in the Product.

```
   1.234504
   3212.9. = the Multiplier inverted.
 ----------
  111105
    2469
     123
      25
       4
 ----------
  11.3726 = the Product.
```

Case 2. A Decimal by a whole Number, as .1234504 by 92123, and to have 3 places in the Product Decimals.

```
  .1234504
     32129 = the multiplier inverted.
 ----------
  11110536
    246901
     12345
      2469
       370
 ----------
  11372.621 = the Product.
```

Case 3. A Decimal by a Decimal, and to have only 4 Decimal places in the Product; as .12345 by .92123

```
    .12345
  .32129. = the Multiplier inverted.
 --------
  1111
    25
     1
 --------
  .1137 = Product required.
```

1. To

1. To perform these Operations, it is plain that the Multiplier is inverted, and Units place put next the left hand, &c. when you place it down to multiply.

2. The due placing of it is the next thing, which is to put Units place of the intire part under that place in the Decimals of the Multiplicand, which answers that place next the right hand of the Decimals you would have in the Product: As in the first Example, because I would have 4 places Decimal in the Product, therefore I put 9 (the Units given) under the 4th place from the Point in the Multiplicand. And in the 2d Example, I put 3 (the Units place of the Multiplier given) under the 3d Decimal of the Multiplicand; and in the 3d Example, because I would have 4 Decimals in the Product, I put the Digit next the Point in the Multiplier under the 3d place of the Multiplicand, because there is no intire Number to put under the 4th place; and place all the other Figures of the Multipliers in a reverse Order, as *per* the Example.

3. Begin to multiply the Figure next the right hand of the Multiplier, when placed as *per* Rules above, into the Figure standing over it, &c. as in the common Way; observing at the same time what Tens would be carry'd if you multiplied the next Figure or two toward the right hand, and adding such Tens and half Tens or upward as Ten, and so proceed with each Figure in the Multiplier that hath another standing over it, (omitting the rest) and place the 1st Product of each Figure, *viz.* what the same is above ten or tens Units, under the Units of the first Line, &c. and not as in common Multiplication; which Lines add up, and the Aggregate is the Product required. The 3 Examples above make it plain, and that much Trouble and many superfluous Figures are prevented.

Case 4. *But it may sometimes happen*, That, as in the following Example, so many places are not in the Sum as you proposed for Decimals: in which case add 1 or more Cyphers.

Thus to multiply .12345 by .234, see the Work in the Margin, to have 4 Decimal places in the Product; where the making of 13 Digits are saved.

```
.12345.
 432.
 ------
 247
  37
   5
 ------
.0289 = Product.
```

Case 5. *Also it may happen*, That there are not so many Decimal places in the Multiplicand as you propose shall be in the Multiplier; so that you cannot proceed as *per* Rule 2. above: in this Case you may sup-

ply the place by Points: thus, to multiply 32.34. by 7.93245 see the Work in the Margin, to bring out 4 places Decimal.

These are all the Variety I could think of as to this matter, which are mostly not extant before that I know of: And the following Compendiums are intirely my own Contrivance; and if any of them are extant, I can assure the Reader I borrow'd nothing therefrom.

```
 32.34. .
 54239.7
 --------
 22638. .
 . 29106.
    9702
     647
     129
      16
 --------
256.5354
```

New COMPENDIUMS in Multiplication.

1. To multiply by any Factor under 20 at once.

```
Example   3.4567        The common Way  3.4567
     by     .19.                          .19.
          ------                       -------
         .656773 Product.               311103
                                       34567
                        The Probation  .656773
```

2. To multiply by any Factor between 100 and 110 (exclusive) at once.

```
Example   34567         The common Way  34567
           .108                          .108
         ------                       -------
       3733.236 Product.               276536
                                      34567
                                      -------
                              Prob. = 3733.236
```

3. To multiply by any Factor between 111 and 119 (inclusive) at once.

```
Example  .34567         The common Way .34567
           11.7.                         11.7.
         ------                       -------
       4.044339 Product.               241969
                                      34567
                                     34567
                                     -------
                              Prob. = 4.044339
```

4. To

4. To multiply by any Number between 1111 and 1119 (inclusive) at once.

Example 345 67
11.14.
385076.38 = Product

The common Way = 345 67
11.14.
138268
34567.
34567
34567
Prob. = 385076.38

5. To multiply by a Factor between 11111 and 11119 (inclusive) putting only the Product.

Example 345 67
111.16.
3842467.72 = Product.

The common Way = 345 67
111.16.
207402
34567
34567
34567
34567
Prob. = 3842467.72

6. To multiply by a Factor between 20 and 100 exclusive.

Example 3.4567.
65
224.68855 = Product.

The common Way = 3.4567
by ..65
172835
207402
Prob. = 224.68855.

7. To multiply by a Factor consisting of a Cypher between two Digits.

Example .34567
605
209.13035 = Product

The common Way = .34567
605
172835
207402
Prob. = 209.13035

8. To

8. To multiply by a Factor consisting of 2, &c. Cyphers between two Digits.

Example		*The common Way*	
	34567		34567
	6005		6005
Product =	207574835		172835
			207402
		Prob. =	207574835

9. To multiply by a Factor consisting of so many Cyphers between two Digits, as there are Places in the Multiplicand.

	34567
	6000005
Product =	207402172835

10. To multiply by any Factor between 100 and 200; only putting down the Product.

Example		*The common Way*	
	23456		23456
	154		154
Product =	3612224		93824
			117280
			23456
		Prob. =	3612224

11. To multiply by a Factor consisting of a Unit between any 2 Digits.

Example		*The common Way* =	
	23456		23456
	416		416
Product =	9757696		140736
			23456
			93824
		Prob. =	9757696

Thus far by only putting down the Figures of the Product.

12. From

12. From what is above, it is plain that any Number may be multiplied by a Factor, being the Square of any in the Propositions above, or the Rectangle of any two of them, by only making 1 Line besides the Product. For knowing that 1235 is the Product of 65 by 19, multiply for example 34567 by 19 (as *per* Case 1.) produceth 656773; and that by 65 (as *per* Case 6.) produceth 42690245, the same as 34567. by 1235.

Example 1.	*Example* 2.
34567	34567
19	11116
656773	384246772
65	6005
42690245	Prod.= 2307401865860

	or 34567
	by 66751580
	276536
	172835
	34567
	172835
Examp. 2.	241969
proved.	207402
	207402
Proof =	2307401865860

The Factor 66751580 being the Rectangle of the 2 towards the left hand; therefore 34567 is multiplied by 66751580, by only making 9 Figures besides the Product; which is 32 fewer than must be put down and added in the common way.

13. Or if a Factor or Multiplier consist of the Figures in any two of the former Digits, &c. put down in course; the same may be performed as follows:

12.3456732.
111166005

741357675660 = the multip^d. by 6005.
13723450329 12 = the multiplied by 11116.

= 1372419168.6795660 = Product.

By this Method there are put down and added 38 Figures fewer than in the common way.

See also the Proof of the last Example in the Book save one, where 9 Places are multiplied by, and only 5 Lines made.

The common Way 12.3456732.

111166005
617283660
740740392
740740392
123456732
123456732
123456732
123456732
Proof = 1372419168.6795660

14. To multiply by a whole Number consisting of any Digit repeated any number of times, as suppose by 7 repeated 8 times.

12345678932	*Common Way* 1234.5678932.
77777777	77777777
86419752524	86419752524
. . . .	86419752524
	86419752524
96021946288669 4164 Product.	86419752524
	86419752524
	86419752524
	86419752524
	86419752524
	Proof = 96021946288669 4164

Note, This Invention is so much shorter than the common Way, that in this Example there are 77 Figures fewer.

15. To multiply by a Decimal consisting of any Digit repeated *ad infinitum,* very easily, briefly, and accurately.

12345678932
.77777777, &c.
86419752524
Product 9602194724.8 *r* 1

Note, This is done with 84 Figures fewer than the common Way, and is about 100 nearer the Truth.

In this Case cut one place more from the Product than the Decimal Places in the Multiplicand, for Decimals of the Product; or so many more than one as there are Cyphers before the repeated Digit of the Multiplier.

16. To multiply by a mixt Number, consisting of any Number of Digits repeated.

Example.	*For Proof of this.*
as 12345678932	12345678932
by = 7777.7777, &c.	by 7777.
by 7 = 86419752524	= 96012345054164
	and by .7, &c. = 9602194724.8
Prod. = 96021947248888.8 *r* 1	Sum = 96021947248888.8 *r* 1

Directions concerning the 16 Compendiums above.

As to *Compend.* 1.] It is performed as in common Multiplication by 9, only adding the Digit toward the right hand of every one that you multiply by the 9; and when you have multiplied the last in order, add what you carry to that Digit next the left hand, and that gives the 2 last in the Product.

Compend. 2.] Work as in the last, only with this difference, that you must add the 7 in Units place to the Product of 8 by 5, and the 6 to the Product of 8 and 4, *&c.* Lastly, add what you carry to the 4, is 7, and then put down the 3 next the left hand.

Compend 3.] When there is 1 in Units and 1 in Tens place, besides a place Decimal, or 114, 116, 117, *&c.* you must add to each Product of every Digit multiplied by 7, the 2 next Digits standing towards the right hand, when there are so many. As to the Product of 6 by 7, I add the 7 next the 6; and then say, 7 times 5 is 35, and 5 carried is 40, and 6 is 46, and 7 is 53; put down 3, and carry 5, *&c.* Lastly, when you have multiplied the 3 by 7, and added the 4 and 5, add the 3 carried to the 3 and 4 is 10, and 1 carried and 3 is 4.

Compend 4.] This is performed as the last, only adding 3 Figures (when you have so many) standing toward the right hand of every Digit which you multiply by the 4. And at last the 3 carried is added to 3, 4 and 5 is 15, and the 1 carried (the 5 being put in the Product) added to the 3 and 4 make 8, which put down, and the 3, as you see in the Example.

Compend. 5.] Whereas here you have 4 Units towards the left hand, you must add 4 Figures (when you have so many) standing towards the right hand of every Digit you multiply by 6. And what is carried must be added to the 4 places next the left hand of the Multiplicand at last, and then 3 and then 2, and then put the 3 in the Multiplicand down.

Compend 6.] Say, 5 times 7 is 35, 5 and carry 3; 5 times 6 is 30, and 3 is 33, and 6 times 7 makes 75; put down 5, and carry 7, and say 5 times 5 is 25, and 7 is 32, and 6 times 6 makes 68; put down 8, and carry 6, and say 5 times 4 is 20, and 6 is 26, and 6 times 5 makes 56; put down the 6 and carry 5, and say 5 times 3 is 15, and 5 carried is 20, and 6 times 4 makes 44; put down 4, and say 6 times 3 is 18, and 4 is 22.

Compend. 7.] In this Example, because there is a Cypher between, you must multiply the two first as usual, and to the Product of the 5 in the 3d add what is carried, and the Product of the 6 in the

first or Units place of the Multiplicand. Then to the Product of 5 in the 4th add what is carried, and the Product of 6 in the 2d or 6, *&c.* and lastly multiply 4, then 3 by the 6, and put the Product down.

Compend 8. Multiply as usual to multiply the 4, then say 5 times 4 is 20, and 2 is 22, and 6 times the 7 is 64, 4 and carry 6; 5 times 3 is 15, and 6 is 21, and 6 times the 6 (or 36) makes 57; put down the 7, and multiply the 5, 4 and 3 by the 6, adding the 5 carried.

Compend. 9.] In this and the like Cases there is no more difficulty than in multiplying by a single Digit; for the Number of Cyphers being equal to the places in the Multiplicand, the Product by the 6 falls just to the left hand of the Product by the 5. But if the Product of the Digit next the left hand by that in Units place, and what is carried be less than 10, then a Cypher must be put down between the two Products.

Compend. 10.] This is performed by considering the Nature of the Operations of the first and 6th Compendiums. For 4 times 6 is 24, 4 and carry 2; 4 times 5 is 20, and 2 carried and 5 times 6 makes 52, 2 and carry 5; 4 times 4 is 16, and 5 is 21, and 5 times 5 is 46, and 6 in Units place makes 52, 2 and carry 5; 4 times 3 is 12, and 5 is 17, and 5 times 4 makes 37, and 5 (in Tens place of the Multiplicand) is 42, 2 and carry 4: 4 times 2 is 8, and 4 is 12, and 5 times 3 makes 27, and 4 (in Hundreds place) is 31; put down 1 and carry 3. Then (having multiplied all by the first Figure) I say 5 times 2 is 10, and 3 carried is 13, and 3 in Thousands place is 16; 1 time 2 is 2, and 1 carried is 3.

Compend. 11.] This is performed as the last, only instead of adding the Digit next the right hand, *&c.* to the Product of the second Figure in the Multiplier, you must here add it to the Product of the first Figure, because the 1 stands next that Digit; and then you keep to the 1st and 6th Compendiums, as in the last.

Compend 12.] By this Compendium is shewn how to multiply by 66751580, or other large Number produced by the Multiplication of 2 or Square of one known Number, by only making 1 Line besides the Product. The Example is that of the 5th and 8th.

Compend. 13.] This is likewise an Example shewing how the Work of Multiplication by large Numbers may be shorten'd, by dividing such large Number into 2, 3, *&c.* such Numbers as fall within some of the abovesaid Rules: as 111166005 I divide into two Multipliers, 11116 and 6005, *&c.* done at twice.

Compend.

Compend. 14.] This Compendium is performed, firſt, by multiplying the given Number by one of the repeated Digits, as in the Example above the Product is 86419752524, from which the general Product is found thus; Add that Product from the right hand toward the left, and ſay 4 and 2 is 6, and 5 is 11, (put down the 4, 6 and 1, and for every 10 you arrive at, put 1 to the Sum more than what is wrote down) and 1 is 12, and 2 is 14, and 5 is 19, and 7 is 26, put down the 4, 9 and 6; then adding 1 more for the 2d 10, I ſay, and 1 is 27, and 9 is 36, and 1 (for that 3d 10) is 37, and 1 next the 9 is 38; ſo I put down the 6 and 8.

Then becauſe I have added as many places as are in the Multiplier, I leave out Units place (4) and add from 2 in Tens place to 4, the 3d Figure from the left hand incluſive, which with the 3 carried (part of the ſaid 38) makes another 38; put down that 8, and carry 3, ſo adding that to 5, &c. to 6, the 2d Figure from the left hand (leaving out the 2 firſt Figures next the right hand) that Sum is 42; I put down the 2, and carry 4, adding that to 2, (the 4th from the right hand, leaving out the 524) and ſo forward to 8, next the left hand incluſive, the Sum is 46: I put down the 6, and add the 4 to the Figures between 5 and 8 incluſive, (leaving out the 2524) and the Sum is 44; I put down 4, and add 4 to the Figures between 7 and 8 incluſive, (leaving out the 52524, and ſo every time I leave out 1 more) as 4 and 7 is 11, and 9 is 20, &c. to 8 makes 39; put down the 9, and add 3 to 86419, as they ſtand from the right to left, as before, and the Sum is 31: put down 1, and add 3 to 8641, as taught before, and the Sum is 22; put down the 2, and add 2 to the 864 as aforeſaid, and the Sum is 20; 0, and carry 2 and 6 is 8, and 8 is 16; put down 6, and ſay 1 and 8 is 9: ſo have you the Anſwer only by eaſy Additions.

Compend. 15.] If the repeated Figure be a Decimal for the Multiplier, I multiply as before to gain the 86419752524; whence the Product is found, firſt by adding that whole Line from the right to the left hand, which makes 53: I add the 5 to the 3 is 8, which I put down for the repeating Digit of the Product. Then add the 5 carried to 2 (in Tens place) and that to the reſt of the Line towards the left hand makes 54; put down the 4, and carry 5 to the 5, (in Hundreds place) &c. adding all toward the left hand as before, leaving out 1 toward the right for every Digit put in the Sum: which is a moſt eaſy and brief Way of anſwering all Queſtions of that kind.

Compend.

Compend. 16.] If the Multiplier be a repeated Digit and a mixt Number, and the Multiplicand an intire Number, you have nothing to do more than in the laſt Compendium: only to put ſo many of the repeated Digits toward the right hand of the integral part of the Product, as there are places in the integral part of the Multiplier, as in the Example 16. above.

A ſecond Example of the 16th Compendium, to multiply 1234 by 333.3333333, &c. or 333.3 *r*. See the Operation.

Multiply 1234
by — 333.33333333, &c.
3702

411333.3 *r* = Product.

Proved thus —— 1234
as *per* 15th by .3 .3 *r*
is = 411.3 *r* 1
and by 333 = 410922

Sum = Proof = 411333.3 *r*

In this (as the laſt) having multiplied the 1234 by 3, it produces 3702, which I add from the right hand towards the left, and the Sum is 12; ſo 2 and the 1 is 3 = the repeating Digit, and 1 carried added to 0, 7 and 3 is 11: I put 1 down, and carry 1 and 7 is 8, and 3 is 11; put 1 down, and carry 1 to 3 makes 411.3 *r*; and becauſe there are 3 places Integers of the Multiplier, I therefore put 3 places of the repeating Figure towards the right hand of the 411, makes 411333.3 *r* the true Anſwer, as by the firſt Example and Rules for working the ſame under the laſt mentioned Compendium.

Or if there be Decimal places in the Multiplicand: you may omit ſo many of the repeating Integers in the Product: as in the above Suppoſition 1.234, then the Product will be 411.3 *r* as *per* the Rule under the 15th Compendium.

17. Or if it ſhould be ſuppoſed (tho' ſuch a thing does not often happen) that the Decimal Digits repeat in both the Factors: then work as under the 15th Compendium, and to that Reſult add the Product by the next, &c. Digits, putting down what would be carried if the Decimal Digit were really repeated.

Example 3.6 *r*
4.3 *r*
Product = 110

Sum = 1.22 *r*
Product = 14.6 *r*

Sum = Anſwer = 15.8 *r* 1

Proof $3\frac{2}{3} = \frac{11}{3}$

$4\frac{1}{3} = \frac{13}{3}$

and $\frac{11}{3}$ by $\frac{13}{3} = \frac{143}{9} = 15.8\,r\,1$

18. But

18. But if the repeating Digits in both Factors begin after the Primes place, (altho' the Case may rarely happen) you may proceed as in the following Example.

To multiply 27.16 *r* 1 by 5.23 *r* 1.

27.1666666, &c.
5.23333333, &c.

.8150 = Product by ,03 in 27.16 *r* 1

.905 *r* 1 = Sum of the last Line, as *per* 15th, &c. Compend.
5.433 *r* 1 = 27.16 *r* 1 in .2
135.833 *r* 1 = 27.16 *r* 1 in 5

142.172 *r* 1 = the true Product, or Sum of the three last Lines.

But if the Product of 3 by 6 and what is carried had not amounted to even Tens (as here 20) then the Digit in Units place of the 1st Product will repeat, and must be so considered in adding them thus

27.17 *r*
by 5.23 *r*

.8153 *r* 1

.90592 *r* 3
5.435 *r* 1
135.8 *r* 1

142.23037 *r* 3

In this Case I suppose 3 repeated so often in .8153 (as .81533) till by adding, it brings out the .90592, and shews the repeating of the 592.

I need say no more to illustrate the last Example; and having now finished what I thought might be of real use in Operations of Multiplication, whether by Decimals or Integers; I shall not trouble my Reader with useless as well as troublesome and tedious Speculations. But for the Solution of all Questions in Practice, as of Concrete Numbers, I refer to the third Rule in the next Sect. (6.) for making your Decimal longer or shorter, as occasion requires, according to the Greatness of the Number by which such Decimal is to be multiplied: for which purpose a Table is there inserted, to direct on almost all Occasions.

These

These 18 Compendiums, as well as the 3 or 4 Cases preceeding them, are new to me; tho' I am since my writing them informed, that one or two of the first and most easy of these Compendiums are in Mr *Leyburn*'s Arithmetic; however that be, I have no reason to omit any, as being my own Contrivance.

These Operations, altho' at first they may seem difficult, and to burthen the Memory; yet I can assure the Reader, that by Use they will become as easy as common Multiplication, (as I know by Experience) and that they very much abbreviate the Operations, cannot be deny'd: nay, many of these new Compendiums are not only shorter, but easier and more accurate than the common Way. And I doubt not but these, and those many in Reduction of Decimals, being new and great Improvements, will be accordingly received by the Ingenious.

That Multiplication of Decimals is done by the same Rule as Vulgar Fractions.

To instance in multiplying .126 by .74: According to the Rule of Vulgar Fractions, the Product of the Numerators is 9324, of the Denominators 100000, the new Fraction $\frac{9324}{100000}$ as a Vulgar; or as a Decimal, by the Rule in Notation, .09324, that is, 9324 being divided by 100000, quotes .09324.

$$\left.\begin{array}{r} .126 \\ .74 \end{array}\right\} \text{Numerators.}$$

$$\overline{9324} \ \text{Product.}$$

$$\text{Denominators} \left\{\begin{array}{r} 1000 \\ 100 \end{array}\right.$$

$$\text{Product} = 100000$$

that is $\frac{9324}{100000}$ as Vulgar.
or .09324 as Decimal.

Hence appears *wherein the Preference of the Decimal Fraction consists:* As,

1. In the easy Multiplication of the Denominators, which is only putting the Cyphers of the one toward the right hand of the other.

2. In that the Decimal can be expressed without its Denominator. And,

3. In Case of finding the Value of the Decimal Fraction, the dividing by the Denominator is done by Inspection, which in Vulgar Fractions often requires a tedious Operation, &c.

SECT. VI. *Division of Decimals.*

THIS Rule is the same, as to the Operation, with Division of Intire Numbers; but it must be observed,

1. *To know how many Cyphers to add to the Dividend, that so the Quotient may have a competent Number of Decimal Places,* take this

Rule. Consider how many Decimals will suffice to be in your Quotient, (upon the Foundation of the Rule of Decimals, as *per* the little Table following the next Rule) and add that Number to the Decimal places in the Divisor, and then make the Decimal places in the Dividend equal to that Sum, by putting Cyphers toward the right hand, if need be.

2. *To know (when the Work of Division is ended) how many Places of the Quotient are Decimal.*

Rule. The difference between the Decimal places in the Dividend and Divisor, are the Decimal places toward the right hand of the Quotient.

But if so many Places be not in the Quote as the said difference is, you must make up that Number by prefixing Cyphers towards the left hand, as *per* the Rule in Reduction.

I could here give another Rule for this purpose, but it not being so intelligible and useful in all Cases, the inserting of it may only serve to hinder the Learner in his Progress: I therefore omit it.

A few Examples will enable the Reader to apply the Rules above, and fully to understand Division of Decimals; as in the 9 Cases following. But first observe,

3. The Number of Places in a Decimal sufficient, when to be multiplied as followeth.

In the Decimal of l. 1. Sterling.		*In the Decimal of 1 Shilling.*		*In the Decimal of 1 lb. Troy.*		*In the Decimal of 1 lb. Averdu.*		*In the Decimal of 1 Barrel Beer.*		
Places sufficient.	*When to be multiplied by these or under.*	*Places sufficient.*	*When to be multiplied by these or under.*	*Places sufficient.*	*When to be multiplied by these or under.*	*Places sufficient.*	*When to be multiplied by these or under.*	*Places sufficient.*	*When to be multiplied by these or under.*	
9	1000000	9	20000000	9	190000	9	4000000	9	3800000	*Without losing a Unit of the least Denomination.*
8	100000	8	2000000	8	19000	8	400000	8	380000	
7	10000	7	200000	7	1900	7	40000	7	38000	
6	1000	6	20000	6	190	6	4000	6	3800	
5	100	5	2000	5	19	5	400	5	380	
4	10	4	200	4	1.9	4	40	4	38	Vid. *also Case* 5. *above.*
3	1.0	3	20			3	4.0	3	3.8	
		2	2.0							

I have inserted this Table, that you may not run your Decimal to more places than are absolutely necessary.

To apply the first and second Rules above.

Example 1. To divide a mixt Number by a mixt, as 237.34691084 by 25.324.

25.324)237.34691084(9.37241.
······
94309
183371
61030
103828
25324
0

Example 2. To divide 11.50554 by by 932, so that six places Decimal may be in the Product.

932)11.505540 (.012345 = Quotient as *per* the two Rules above, adding one Cypher to the Dividend.
·····
2185
3215
4194
4660
0

Example 3. To divide .1234567 by .321, a Decimal by a Decimal, and to have seven Decimal Places in the Quote.

Here according to the first Rule above, to have 7 places in the Quotient, I must have 10 in the Dividend, which I make the other places up by adding 3 Cyphers.

2. According to the 2d Rule, I point off 7 Decimal places in the Quote, because that is the difference between those places in the Dividend and Divisor.

.321) .1234567000 (.3846003.
·······
2715
1476
1927
1000
37

Example 4. To divide a Decimal by a whole Number, as .463 by 3214, and to have 8 Decimal places in the Quotient.

3214). 46300000 (.00014405
····
14160
13040
18400
2130

Example 5. To divide a whole by a mixt Number, so as to have 9 places in the Decimal Part of the Quote; as 9 by 3.214.

3.214) 9.000000000000 (2.800248911
·········· = the Quote
25720
8000
15720
28640
29280
3550
3260
46 Rest.

Example

Example 6. To divide a whole Number by a whole, and to have 10 Decimal Places in the Quotient; as 5 by 365.

365) 5.0000000000 (.0136986301
1350
2550
3600
3150
2300
1100
0500
135

Example 7. To divide a mixt Number by a Decimal; as 1.25999 by .002: and to to have 4 Decimal Places in the Quotient.

.002) 1.2599900 (629.9950

Half the Dividend gives the Figures of the Quote.

Example 8. To divide a whole Number by a Decimal, as 999 by .00013; and to have only 3 Decimal places in the Quote.

.00013) 999.00000000 (7684615.384
89
110
60
80
20
70
50
110
60
8 Rests

Example 9. To divide a Decimal by a mixt Number, as .5 by 7.5; and to have 2 places Decimal in the Quotient: which are sufficient in case of Decimals of a Gallon, and the Quotient not required to be multiplied by any thing; as follows.

7.5) .500 (.06 r
50

These are all the Varieties that can happen in Division of Decimals: And the Exemplification of the two Rules first above, appear at one View thus.

The 9 Cases.	*Divisors.*	*Dividends given.*	*Cyphers added as per Rule* 1. *above.*	*Quotients; having Decimals, as per Rule* 2.
1	25.324	237.34691084		9.37241.
2	932.	11.50554	0	.012345.
3	.321	.1234567	000	.3846003
4	3214.	.463	00000	.00014405
5	3.214	9.	000000000000	2.800248911
6	365.	5.	0000000000	.0136986301
7	.002	1.25999	00	629.9950
8	.00013	999.	00000000	7684615.384
9	7.5	.5	00	.06r1

Of Decimals by way of Vulgar Fractions.

To work any Question so as to give the Answer exquisitely (without omitting any thing) by Decimals, as well as by Vulgar Fractions. [Note, *This is my own Thought, as the rest of what is new in Decimals is.*]

It has been one great Objection against Decimals, that where the Decimals in Quotients happen not to be compleat, but infinite, the Answer cannot be exhibited so accurately, as to be truly said to be the whole Truth: But this is easily solved, by considering the Nature of the Remainers in the dividing and making a Fraction of that and the Divisor.

So that the Proof of this depends both upon Vulgar and Decimal Fractions worked promiscuously, or of Decimal worked by way of Vulgar Fractions.

To instance in Example 3. foregoing, the true Quote (as in the Margin below is proved) is $= .3846003\frac{.0000000037}{.321}$: For the 37 is what remained in dividing the last part of the Dividend; and besides the 2 places next the right hand thereof, there being 8 more Decimal places towards the left hand, therefore so many Cyphers are in the Numerator of that part expressed by way of Vulgar Fractions, which must always be observed.

The Truth of this appears by reducing this mixt Number to one Fraction: for the Product of .3846003 multiplied by the Denominator .321, produceth .1234566963; to which adding the Numerator, the Sum is .1234567: which placed over the Denominator

tor .321, gives the Fraction answerable to the first proposed Question in Example 3.

$$\begin{array}{r} .3846003 \\ .321 \\ \hline \text{add}\left\{\begin{array}{r} .1234566963 \\ .0000000037 \end{array}\right. \\ \hline \text{Sum } .1234567000 \end{array}$$

or $\frac{.1234567}{.321}$ for Proof.

And by the same Rules the Quote or Answer *in the 4th Example* is to be expressed thus .00014405 $\frac{.0000233}{.3214}$. Which if reduced into a single Fraction, as before, is $\frac{463}{3214}$.

So likewise the accurate Answer or Quotient *in Example* 5. is .280024891 $\frac{.00000000000046}{3.214}$, reducible to $\frac{2}{3.21}$.

In Example 6. the exact Quote is consequently $=$ .0136986301 $\frac{.0000000135}{365}$: or by the Rules above $\frac{5}{365}$ in a simple Fraction for Proof.

In Example 8. The precise Quotient is 7684615.384 $\frac{.0000008}{.33313}$. Or by reducing the 2 sorts of Fractions to one Fraction, the whole will stand thus 7684615: $\frac{.00005}{.33313}$ without any loss; which (for Proof) is reducible to .00 $\frac{992}{.3}$.

Lastly, *In Example* 9. The true Quote is .0666 *r* 1, or rather .06 $\frac{.5}{7.5}$; whose Value (if the Fraction of a Pound) is known to be 16 *d*. We will try if it will prove so by the Method shew'd above.

The 50 remaining (by the Rule relating to the third Example) is .050, there being 1 more place Decimal towards the left hand besides the 2 places in the Remainer: so that the .06 $\frac{.5}{7.5}$ includes the whole Answer without any Loss, as *per* the Work in the Margin; and is reduced to $\frac{.5}{7.5}$, whose Value, as *per* Vulgar Fractions, having respect to the Points, as in Multiplication and Division of Decimals, is 1 *s*. 4 *d*.

The new Fract. $= \frac{.5}{7.5}$ *l*.

$$\begin{array}{r} 7.5 \\ .06 \end{array}\Big\}\text{mult.}$$

$$\begin{array}{r} .450 \\ .05 \end{array}\Big\}\text{add.}$$

$$\begin{array}{r} \hline .500 \text{ Sum.} \end{array}$$

The Value is thus found, as of Vulgar;

$$\begin{array}{r} .5 \\ 20 \\ \hline 7.5)\ 10.0\ (1\ s. \\ \hline 2.5 \\ 12 \\ \hline 7.5)\ 30.0\ (4 \text{ Pence} \\ \hline 0 \end{array}$$

3. *Note*, That to divide any Decimal, Whole, or Mixt Number by a Unit with Cyphers, is only to remove the Point so many places towards the left hand, as there are Cyphers. Thus,

If

If 10 divide 98765.4 the Quote is		9876.54
100 divide it,	— — — —	987.654
1000	— — — — —	98.7654
10000	— — — — —	9.87654
100000	— — — — —	.987654
1000000	— — — — —	.0987654

4. Or if 60 divide 98765.4 the Quote is		1646.09
600	— — — — —	164.609
6000	— — — — —	16.4609
60000	— — — — —	1.64609
600000	— — — — —	.164609

dividing only by the 6, and cutting of so many from the Quote (besides the Decimal Places in the Dividend) as there are Cyphers in the Divisor.

5. *That Division of Decimals is performed as that of Vulgar Fractions.*

As if $\frac{1234567}{10000000}$ be divided by $\frac{321}{1000}$, the Quote will be $\frac{1234567000}{3210000000}$ equal to the Decimal Quote .3846003, by dividing the Numerator by 321, and cutting as many from the Quotient as there are Cyphers to the right of 321, as *per* Margin.

$\frac{321}{1000}$) $\frac{1234567}{10000000}$ ($\frac{1234567000}{3210000000}$.

321) 1234567000 (.3846003

SECT. VII. *The Rule of Proportion by Decimals.*

THIS being the same as in whole Numbers, observing Multiplication and Division of Decimals for the proper Rules of pointing out the Products and Quotients; I need only give one Example.

What is the Interest of *l.* 325 : 15 : 0 for a Year at 4 *l.* 10 *s. per Cent. per Annum?*

The Answer is *l.* 14.65775, as *per* Compend. 6, *&c.* or *l.* 14 : 13 : 1¼, by the 6th Rule of the Valuation of Decimals.

100. 4.5 : : 325.75
4.5

l. 14.65775 Answer.

And this puts me in mind of

SECT.

SECT. VIII. *The Use of Decimals in the calculating Interest Simple and Compound.*

I. SIMPLE Intereſt is the Intereſt of the Principal only, for the Time propoſed.

II. The true Diſcount of Money is in proportion to its Principal: As 5 is to 105 (at 5 *per Cent.*) or as 6 to 106 at 6 *per Cent.* &c. So that the Diſcount of any Sum for any time is leſs than the Intereſt for that time, and conſequently the Diſcount of any Sum at any rate is found for a Year by this Proportion:

As 100 and the Rate: is to the Rate: ſo any Sum to the Diſcount for the ſame time.

III. Compound Intereſt is the Intereſt of the Principal and of the Intereſt put together, (the later not being paid when due, but becomes Principal.)

1. *As to Simple Intereſt*, there are Laws in moſt Countries to aſcertain it, that no one ſhall take more, under the penalty of forfeiting the Principal, &c. as now ours in 5 *per Cent.* for what is lent for the future.

To find the Simple Intereſt of any Sum for a Year.

(1.) What is the Intereſt of *l.* 364 : 17 : $5\frac{1}{4}$ at 5 *per Cent*?

	As 100. 5 :: 364 : 17 : $5\frac{1}{4}$	
	5	Anſw. *l.* 18 : 4 : $10\frac{1}{2}$
Multiply the whole by 5, and divide by 100 continually, as you ſee here.	*l.* 18.24 : 7 : $2\frac{1}{4}$.	
	20	
	s. 4.87	
	12	
	d. 10.46	
	4	
	qr. 1.85	

(2.) But if the Shillings, &c. are eaſily by Inſpection reducible to a Compleat Decimal, the beſt way is to do ſuch Queſtions decimally.

Example.

Example. What is the Intereſt of *l.* 364 : 18 : 0 for a Year, at 5 *per Cent?*

See the Operation in the Margin, the Anſwer being *l.* 18 : 4 : 10$\frac{1}{4}$.

100. 5 : : 364.9
5
―――
18.245 Anſwer.

(3.) If the Intereſt of any Sum be required for any Number of Days (or part of a Year) 'tis beſt done by Decimals.

Example. Suppoſe the Intereſt of the Principal in the laſt Caſe is required only for 145 Days:

1ſt ſay, 100. 5 : : 364.9
5
―――
the Intereſt = 18.245 for 1 Year.

Days. *l.* *Days.*
2dly, 365. 18.245 : : 145
145
――― *l.*
365)2645.525(7.248 $\frac{005}{365}$ = Anſwer.
· · · ·
―――
905
―――
1752
―――
2925
―――
5

2. And conſequently the Intereſt of the ſaid *l.* 364 : 17 : 11$\frac{1}{4}$ for 314 Days is found, reducing the odd Money to Decimals, as *per* the ſecond Rule in Reduction, which is *l.* .871875 : Therefore

100. 5 : : 364.871875. 18.2435937 = Intereſt for a Year.
Days. *l.* *Days.* *l.*
365. 18.2435937 : : 314. 15.694 = the Anſwer.

3. By the ſame Rule with the two laſt, the Intereſt of *l.* 1 for one Day at any rate is found, and they are Multipliers for finding the Simple Intereſt of any Number of Pounds for any Days:

$\begin{cases} \text{For 100. 5 : : 1. .05} \\ \text{And as 365. .05 : : 1. .000136, \&c.} \end{cases}$

Thus

Thus 1 *l.* for 1 Day at	4 *per Cent.* = .00010958904	}	*Factors for finding the Interest for Days.*
	5 *per Cent.* = .0001369863		
	6 *per Cent.* = .00016438356		
	7 *per Cent.* = .00019178082		
	8 *per Cent.* = .00021917808		

These Numbers may be thus used: Suppose I would know the Interest of *l.* 400 for 297 Days at 5 *per Cent.* I multiply .0001369863 by 297, and the Product is the Interest of *l.* 1 for 297 Days; which Product multiplied by *l.* 400, gives the Interest of *l.* 400 for 297 Days = the Answer, *l.* 16 : 5 : 5 ¼.

4. If you divide 100 by the Rate of Interest, it gives the Years Purchase in Fee of Lands Value in many places. And consequently if 100 be divided by the Years Purchase that the generality of Lands are valued at, the Quote gives the proportionable Rate of Interest. Which is a Rule that many go by, to know whether Interest is too high or low, by observing how Lands sell: It happens right with us at present, for 5 *per Cent.* agrees with 20 Years Purchase of Land.

5. It is easy to find the Simple Interest of any Sum for any Number of Years, as suppose *l.* 100 : 15, for 7 Years at 5 *per Cent.* say

100. 5 : : 100.75. 5.0375 = Interest for 1 Year.
7 mult.

35.2625 = Answer, or *l.* 35 : 5 : 3

6. And consequently for the Interest of a Sum under *l.* 1, as 15 *s.*

100. 5 : : .75. .0375 for 1 Year; or .2625 *l.* for 7 Years.

7. Finding the Interest of *l.* 732 for 5 Years to be *l.* 183, what is that *per Cent. per Ann.?* I first divide *l.* 183 by 5 Years, and the Quote is *l.* 36.6. Then say, As 732. 36.6. : : 100. 5 *per Cent.* Answer.

Here followeth Tables of Interest, Simple, Compound, and Discount, with their Calculation and Use.

A TABLE *of* SIMPLE INTEREST *at* 4 *per Cent.*

Principal.	1 Day.			2 Days.			3 Days.			4 Days.			5 Days.			6 Days.			7 Days.			8 Days.		
l.	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*
1	*l.*000109589																				1			1
2									1			1			1			1			1			2
3						1			1			1			1			2			2			2
4						1			1			2			2			3			3			3
5			1			1			2			2			3			3		1	0		1	0
6			1			2			2			3			3		1	0		1	0		1	1
7			1			2			2			3		1	0		1	0		1	1		1	2
8			1			2			2			3		1	0		1	1		1	2		1	3
9			1			2			3		1	0		1	1		1	2		1	3		2	0
10			1			2			3		1	0		1	1		1	2		1	3		2	1
20			2		1	0		1	2		2	0		2	2		3	1		3	3		4	1
30			3		1	2		2	1		3	1		4	0		4	2		5	2		6	1
40		1	0		2	0		3	1		4	1		5	1		6	1		7	1		8	2
50		1	1		2	2		4	0		5	1		6	2		7	3		2	1		10	2
60		1	2		3	1		4	3		6	1		7	3		9	2		11	0	1	00	2
70		1	3		3	3		5	2		7	1		9	1		11	0	1	00	3	1	2	3
80		2	0		4	1		6	1		8	2		10	2	1	00	2	1	2	3	1	5	0
90		2	1		4	3		7	0		9	2	1	00	0	1	2	1	1	4	2	1	7	0
100		2	2		5	1		7	3		10	2	1	1	0	1	3	3	1	6	2	1	9	0
200		5	1		10	2	1	3	3	1	9	0	2	2	1	2	7	2	3	00	3	3	6	0
300		7	3	1	3	3	1	11	3	2	7	2	3	3	2	3	11	2	4	7	1	5	3	1
400		10	2	1	9	0	2	7	2	3	6	0	4	4	2	5	3	1	6	1	2	7	00	0
500	1	1	0	2	2	1	3	3	2	4	4	3	5	5	3	6	7	0	7	8	0	8	9	1
600	1	3	3	2	7	2	3	11	2	5	3	1	6	7	0	7	10	3	9	2	2	10	6	1
700	1	6	1	3	00	3	4	7	1	6	1	2	7	8	0	9	2	2	10	8	3	12	3	1
800	1	9	0	3	6	0	5	3	1	7	00	0	8	9	1	10	6	1	12	3	1	14	00	1
900	1	11	3	3	11	2	5	11	0	7	10	3	9	10	2	11	10	0	13	9	3	15	9	2
1000	2	2	1	4	4	2	6	7	0	8	9	1	10	11	2	13	1	3	15	4	1	17	6	2
ditto	*l.* 109589			.219178			.328767			.438356			.547945			.657534			.767123			.8767120		

SIMPLE INTEREST *at* 4 *per Cent.*

Principal.	9 Days.			10 Days.				20 Days.				30 Days.				40 Days.				50 Days.				60 Days.			
l.	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			1				1				2				3			1	0			1	1			1	2
2			2				2			1	0			1	2			2	0			2	2			3	1
3			3				3			1	2			2	1			3	1			4	0			4	3
4		1	0			1	0			2	1			3	1			4	1			5	1			6	1
5		1	1			1	1			2	3			4	0			5	1			6	2			7	3
6		1	2			1	2			3	1			4	3			6	1			7	3			9	2
7		1	3			1	3			3	3			5	2			7	1			9	1			11	1
8		2	0			2	1			4	1			6	1			8	2			10	2		1	00	2
9		2	1			2	2			4	3			7	0			9	2		1	00	0		1	2	1
10		2	2			2	3			5	1			7	3			10	2		1	1	0		1	3	3
20		4	3			5	1			10	2		1	3	3		1	9	0		2	2	1		2	7	2
30		7	0			7	3		1	3	3		1	11	3		2	7	2		3	3	2		3	11	2
40		9	2			10	2		1	9	0		2	7	2		3	6	0		4	4	3		5	3	1
50	1	00	0		1	1	0		2	2	1		3	3	2		4	4	3		5	5	3		6	7	0
60	1	2	1		1	3	3		2	7	2		3	11	2		5	3	1		6	7	0		7	10	3
70	1	4	2		1	6	2		3	00	3		4	7	1		6	1	2		7	8	0		9	2	2
80	1	7	0		1	9	0		3	6	0		5	3	1		7	00	0		8	9	1		10	6	1
90	1	9	1		1	11	3		3	11	2		5	11	0		7	10	3		9	10	2		11	10	0
100	1	11	3		2	2	1		4	4	3		6	7	0		8	9	1		10	11	2		13	1	3
200	3	11	2		4	4	3		8	9	1		13	1	3		17	6	2	1	1	11	0	1	6	3	3
300	5	11	0		6	7	0		13	1	3		19	8	3	1	6	3	3	1	12	10	2	1	19	5	2
400	7	10	3		8	9	1		17	6	2	1	6	3	3	1	15	00	3	2	3	10	0	2	12	7	1
500	9	10	2		10	11	2	1	1	11	0	1	12	10	2	2	3	10	0	2	14	9	2	3	5	9	0
600	11	10	0		13	1	3	1	6	3	3	1	19	5	2	2	12	7	1	3	5	9	0	3	18	11	0
700	13	9	2		15	4	0	1	10	8	1	2	6	00	1	3	1	4	2	3	16	8	2	4	12	00	2
800	15	9	2		17	6	2	1	15	00	3	2	12	7	1	3	10	1	2	4	7	8	0	5	5	2	2
900	17	9	0		19	8	3	1	19	5	2	2	19	2	0	3	18	11	0	4	18	7	2	5	18	4	1
1000	19	8	3	1	1	11	0	2	3	10	0	3	5	9	0	4	7	8	0	5	9	7	0	6	11	6	0
ditto	*l.*986301			1.09589				2.191781				3.287671				4.383562				5.479452				6.575342			

SIMPLE INTEREST *at* 4 *per Cent.*

Principal.	70 Days.				80 Days.				90 Days.			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			1	3			2	0			2	1
2			3	3			4	1			4	3
3			5	2			6	1			7	0
4			7	1			8	2			9	2
5			9	1			10	2		1	00	0
6			11	0		1	00	2		1	2	1
7		1	00	3		1	2	3		1	4	3
8		1	2	3		1	5	0		1	7	0
9		1	4	3		1	7	0		1	9	1
10		1	6	2		1	9	0		1	11	3
20		3	00	3		3	6	0		3	11	2
30		4	7	1		5	3	1		5	11	0
40		6	1	2		7	00	0		7	11	0
50		7	8	0		8	9	1		9	10	2
60		9	2	2		10	6	1		11	10	0
70		10	8	3		12	3	1		13	9	3
80		12	3	1		14	00	1		15	9	1
90		13	9	3		15	9	2		17	9	0
100		15	4	1		17	6	2		19	8	3
200	1	10	8	1	1	15	00	3	1	19	5	1
300	2	6	00	1	2	12	7	1	2	19	2	1
400	3	1	4	2	3	10	1	2	3	18	11	0
500	3	16	8	2	4	7	8	0	4	18	7	2
600	4	12	00	3	5	5	2	2	5	18	4	1
700	5	7	4	3	6	2	8	3	6	18	1	0
800	6	2	8	3	7	00	3	1	7	17	9	3
900	6	18	1	0	7	17	9	3	8	17	6	2
1000	7	13	5	0	8	15	4	1	9	17	3	1
ditto	*l.* 7.671233				8.767123				9.863014			

Principal.	100 Days.				110 Days.				120 Days.			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			2	2			3	0			3	1
2			5	1			5	3			6	1
3			7	3			8	3			9	3
4			10	2			11	3		1	00	2
5		1	1	1		1	2	2		1	3	3
6		1	3	3		1	5	2		1	7	0
7		1	6	2		1	8	1		1	10	0
8		1	9	0		1	11	1		2	1	1
9		1	11	3		2	2	0		2	4	2
10		2	00	0		2	5	0		2	7	2
20		4	4	3		4	10	0		5	3	1
30		6	7	0		7	2	3		7	10	3
40		8	9	1		9	7	3		10	6	1
50		10	11	2		12	00	2		13	1	3
60		13	1	3		14	5	2		15	9	2
70		15	4	1		16	10	2		18	4	3
80		17	6	1		19	3	2	1	1	00	2
90		19	8	3	1	1	8	1	1	3	8	0
100	1	1	11	0	1	4	1	1	1	6	3	3
200	2	3	10	0	2	8	2	2	2	12	7	1
300	3	5	9	0	3	12	4	0	3	18	11	0
400	4	7	8	0	4	16	5	1	5	5	2	2
500	5	9	7	0	6	00	6	2	6	11	6	0
600	6	11	6	0	7	4	7	3	7	17	9	3
700	7	13	5	0	8	8	9	1	9	4	1	1
800	8	15	4	0	9	12	10	2	10	10	5	0
900	9	17	3	0	10	16	11	3	11	16	8	2
1000	10	19	2	1	12	1	1	0	13	3	00	0
ditto	10.9589				12.054794				13.150684			

SIMPLE INTEREST *at 4 per Cent.*

Principal.	130 Days.				140 Days.				150 Days.				160 Days.				170 Days.				180 Days.			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			3	2			3	3			4	0			4	1			4	2			4	3
2			6	3			7	1			7	3			8	2			9	0			9	2
3			10	1			11	0		1	0	0		1	0	2		1	1	2		1	2	1
4		1	1	3		1	2	3		1	4	0		1	5	0		1	5	3		1	7	0
5		1	5	1		1	6	2		1	7	3		1	9	0		1	10	1		1	11	3
6		1	8	2		1	10	1		1	11	3		2	1	1		2	2	3		2	4	2
7		2	0	0		2	1	3		2	3	3		2	5	2		2	7	1		2	9	1
8		2	3	1		2	5	2		2	7	2		2	9	3		2	11	3		3	1	3
9		2	6	3		2	9	1		2	11	2		3	1	3		3	4	1		3	6	2
10		2	10	1		3	0	3		3	3	2		3	6	0		3	8	3		3	11	2
20		5	8	1		6	1	2		6	7	0		7	0	0		7	5	2		7	10	3
30		8	6	2		9	2	2		9	10	2		10	6	1		11	2	0		11	10	0
40		11	4	3		12	3	1		13	1	3		14	0	1		14	11	0		15	9	2
50		14	3	0		15	4	1		16	5	1		17	6	2		18	7	2		19	8	3
60		17	1	0		18	5	0		19	8	3	1	1	0	2	1	2	4	1	1	3	8	0
70		19	11	2	1	1	5	3	1	3	0	0	1	4	6	1	1	6	1	0	1	7	7	1
80	1	2	9	2	1	4	6	2	1	6	3	3	1	8	0	3	1	9	9	3	1	11	6	3
90	1	5	7	3	1	7	7	1	1	9	7	0	1	11	6	3	1	13	6	2	1	15	6	0
100	1	8	6	0	1	10	8	1	1	12	10	2	1	15	0	3	1	17	3	1	1	19	5	2
200	2	17	0	0	3	1	4	2	3	5	9	0	3	10	1	2	3	14	6	1	3	18	11	0
300	4	5	5	3	4	12	0	2	4	18	7	2	5	5	2	2	5	11	9	2	5	18	4	1
400	5	13	11	3	6	2	8	3	6	11	6	0	7	0	3	1	7	9	0	2	7	17	9	3
500	7	2	5	2	7	13	5	0	8	4	4	3	8	15	4	1	9	6	3	3	9	17	3	1
600	8	10	11	2	9	4	1	1	9	17	3	1	10	10	5	0	11	3	6	3	11	16	8	2
700	9	19	5	2	10	14	9	2	11	10	1	2	12	5	5	3	13	0	10	0	13	16	2	0
800	11	7	11	2	12	5	5	3	13	3	0	0	14	0	6	2	14	18	1	0	15	15	6	3
900	12	16	5	1	13	16	2	0	14	15	10	3	15	15	7	1	16	15	4	1	17	15	0	3
1000	14	4	11	1	15	6	10	1	16	8	9	1	17	10	8	1	18	12	7	1	19	14	6	1
Ditto *l.*	14.246574				15.342464				16.438354				17.534244				18.630134				19.726024			

SIMPLE INTEREST *at* 4 *per Cent.*

Principal.	190 *Days.*				200 *Days.*				210 *Days.*				220 *Days.*				230 *Days.*				240 *Days.*			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			5	0			5	1			5	2			5	3			6	0			6	1
2			10	0			10	2			11	0			11	3		1	0	0		1	0	2
3		1	3	0		1	3	3		1	4	3		1	5	2		1	6	0		1	7	0
4		1	8	0		1	9	0		1	10	0		1	11	1		2	0	0		2	1	1
5		2	1	0		2	2	1		2	3	3		2	5	0		2	6	1		2	7	2
6		2	6	0		2	7	2		2	9	1		2	10	3		3	0	1		3	1	3
7		2	11	0		3	0	3		3	2	3		3	4	2		3	6	1		3	8	1
8		3	4	0		3	6	0		3	8	1		3	10	1		4	0	1		4	2	2
9		3	9	0		3	11	2		4	1	3		4	4	0		4	6	2		4	8	3
10		4	2	0		4	4	3		4	7	1		4	10	0		5	0	2		5	3	1
20		8	4	0		8	9	1		9	2	2		9	7	3		10	1	0		10	6	1
30		12	6	0		13	1	3		13	9	2		14	5	3		15	1	2		15	9	1
40		16	7	3		17	6	2		18	5	0		19	3	2	1	0	2	0	1	1	0	2
50	1	0	10	0	1	1	11	0	1	3	0	0	1	4	1	1	1	5	2	2	1	6	3	3
60	1	5	0	0	1	6	3	3	1	7	7	1	1	8	11	1	1	10	3	0	1	11	6	3
70	1	9	1	3	1	10	8	1	1	12	2	2	1	13	9	0	1	15	3	2	1	16	10	0
80	1	13	4	3	1	15	0	3	1	16	10	0	1	18	7	0	2	0	4	0	2	2	1	0
90	1	17	5	3	1	19	5	2	2	1	5	0	2	3	4	3	2	5	4	2	2	7	4	1
100	2	1	7	3	2	3	10	0	2	6	0	1	2	8	2	2	2	10	5	0	2	12	7	1
200	4	3	3	2	4	7	8	0	4	12	0	2	4	16	5	1	5	0	10	0	5	5	2	2
300	6	4	11	1	6	11	6	0	6	18	1	0	7	4	7	3	7	11	2	3	7	17	9	3
400	8	6	6	3	8	15	4	0	9	4	1	1	9	12	10	2	10	1	7	3	10	10	5	0
500	10	8	2	2	10	19	2	0	11	10	1	2	12	1	1	0	12	12	0	3	13	3	0	0
600	12	9	10	2	13	3	0	0	13	16	2	0	14	9	3	3	15	2	5	2	15	15	7	1
700	14	11	6	0	15	6	10	1	16	2	2	1	16	17	6	2	17	12	10	2	18	8	2	2
800	16	13	1	3	17	10	8	1	18	8	2	2	19	5	9	0	20	3	3	2	21	0	10	0
900	18	14	9	2	19	14	6	1	20	14	3	0	21	13	11	3	22	13	8	1	23	13	5	0
1000	20	16	5	1	21	18	4	1	23	0	3	1	24	2	2	1	25	4	1	1	26	6	0	1
ditto	*l.* 20.821914				21.917805				23.013695				24.109585				25.205476				26.301366			

SIMPLE INTEREST *at* 4 *per Cent.*

Principal.	250 *Days.*				260 *Days.*				270 *Days.*				280 *Days.*				290 *Days.*				300 *Days.*			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			6	2			6	3			7	0			7	1			7	2			7	3
2		1	1	0		1	1	2		1	2	1		1	2	3		1	3	1		1	3	3
3		1	7	3		1	8	2		1	9	1		1	10	0		1	11	0		1	11	3
4		2	2	1		2	3	1		2	4	2		2	5	2		2	6	2		2	7	2
5		2	8	3		2	10	1		2	11	2		3	0	3		3	2	0		3	3	2
6		3	3	2		3	5	0		3	6	2		3	8	1		3	9	3		3	11	2
7		3	10	0		4	0	0		4	1	3		4	3	2		4	5	1		4	7	1
8		4	4	2		4	6	3		4	8	3		4	11	0		5	1	0		5	3	1
9		4	11	1		5	1	2		5	4	0		5	6	1		5	8	3		5	11	0
10		5	3	1		5	8	1		5	11	0		6	1	2		6	4	1		6	7	0
20		10	11	2		11	4	3		11	10	0		12	3	1		12	8	2		13	1	3
30		16	5	1		17	1	0		17	9	0		18	5	0		19	0	3		19	8	3
40	1	1	11	0	1	2	9	2	1	3	8	0	1	4	6	2	1	5	5	0	1	6	3	3
50	1	7	4	3	1	8	6	0	1	9	7	0	1	10	8	1	1	11	9	2	1	12	10	2
60	1	12	10	3	1	14	2	1	1	15	6	0	1	16	10	0	1	18	1	2	1	19	5	2
70	1	18	4	1	1	19	10	3	2	1	5	0	2	2	11	2	2	4	6	0	2	6	0	1
80	2	3	10	0	2	5	7	0	2	7	4	0	2	9	1	1	2	10	10	1	2	12	7	1
90	2	9	3	3	2	11	3	2	2	13	3	1	2	15	2	3	2	17	2	2	2	19	2	0
100	2	14	9	2	2	17	0	0	2	19	2	1	3	1	4	2	3	3	6	3	3	5	9	0
200	5	9	7	0	5	13	11	3	5	18	4	1	6	2	8	3	6	7	1	2	6	11	6	0
300	8	4	4	2	8	10	11	2	8	17	6	2	9	4	1	1	9	10	8	1	9	17	3	1
400	10	19	2	0	11	7	11	2	11	16	8	2	12	5	5	3	12	14	3	0	13	3	0	0
500	13	13	11	3	14	4	11	1	14	15	10	3	15	6	10	1	15	17	9	3	16	8	9	1
600	16	8	9	1	17	1	11	0	17	15	0	3	18	8	2	2	19	1	4	2	19	14	6	1
700	19	3	6	3	19	18	11	0	20	14	3	0	21	9	7	0	22	4	11	1	23	0	3	1
800	21	18	4	1	22	15	10	3	23	13	5	0	24	10	11	2	25	8	6	0	26	6	0	1
900	24	13	1	3	25	12	10	2	26	12	7	1	27	12	4	0	28	12	0	3	29	11	9	2
1000	27	7	11	2	28	9	10	1	29	11	9	2	30	13	8	1	31	15	7	1	32	17	6	2
ditto	*l.*27.397256				28.49315				29.58904				30.68493				31.78082				32.87671			

SIMPLE INTEREST *at* 4 *per Cent.*

Principal.	310 Days.				320 Days.				330 Days.				340 Days.				350 Days.				360 Days.				365 Days, or 1 Year.			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			8	1			8	2			8	3			9	0			9	1			9	2			9	3
2		1	4	1		1	5	0		1	5	2		1	6	0		1	6	2		1	7	0		1	7	1
3		2	0	2		2	1	1		2	2	0		2	2	3		2	3	3		2	4	2		2	5	0
4		2	8	2		2	9	3		2	10	3		2	11	3		3	0	3		3	1	3		3	2	2
5		3	4	3		3	6	0		3	7	1		3	8	3		3	10	0		3	11	2		4	0	0
6		4	0	3		4	2	2		4	4	0		4	5	3		4	7	1		4	8	3		4	9	3
7		4	9	0		4	11	0		5	0	3		5	2	2		5	4	2		5	6	1		5	7	1
8		5	5	1		5	7	1		5	9	2		5	11	2		6	1	2		6	3	3		6	5	0
9		6	1	1		6	3	3		6	6	0		6	8	2		6	11	0		7	1	1		7	2	2
10		6	9	2		7	0	0		7	2	3		7	5	2		7	8	0		7	10	3		8		
20		13	7	0		14	0	1		14	5	2		14	11	0		15	4	0		15	9	2		16		
30	1	0	4	3	1	1	0	2	1	1	8	1	1	2	4	1	1	3	0	0	1	3	8	0	1	4		
40	1	7	2	0	1	8	0	3	1	8	11	1	1	5	9	3	1	10	8	1	1	11	6	3	1	12		
50	1	13	11	3	1	15	0	3	1	16	2	0	1	17	3	1	1	18	4	1	1	19	5	2	2	0		
60	2	0	9	1	2	2	1	0	2	3	4	3	2	4	8	2	2	6	0	1	2	7	4	1	2	8		
70	2	7	6	3	2	9	1	1	2	10	7	2	2	12	2	0	2	13	8	1	2	15	2	3	2	16		
80	2	14	4	1	2	16	1	1	2	17	10	2	2	19	7	1	3	1	4	2	3	3	1	2	3	4		
90	3	1	1	3	3	3	1	2	3	5	1	1	3	7	0	3	3	9	0	2	3	11	0	0	3	12		
100	3	7	11	2	3	10	1	3	3	12	4	0	3	14	6	1	3	16	8	2	3	18	11	0	4			
200	6	15	10	3	7	0	3	1	7	4	7	3	7	9	0	2	7	13	5	0	7	17	9	3	8			
300	10	3	10	0	10	10	5	0	10	16	11	3	11	3	6	3	11	10	1	2	11	16	8	2	12			
400	13	11	9	2	14	0	6	2	14	9	4	3	14	18	1	0	15	6	10	1	15	15	7	1	16			
500	16	19	8	3	17	10	8	1	18	1	7	3	18	12	7	1	19	3	6	2	19	14	6	1	20			
600	20	7	8	0	21	0	10	0	21	13	11	3	22	7	1	2	23	0	3	1	23	13	5	0	24			
700	23	15	7	1	24	10	11	2	25	6	3	3	26	1	7	3	26	17	0	0	27	12	4	0	28			
800	27	3	6	3	28	1	1	1	28	18	7	2	29	16	2	0	30	13	8	1	31	11	2	3	32			
900	30	11	6	0	31	11	2	3	32	10	11	2	33	10	8	1	34	10	5	0	35	10	1	3	36			
1000	33	19	5	2	35	1	4	2	36	3	3	2	37	5	2	2	38	7	1	2	39	9	0	2	40			
ditto	*l.* 33.9726				35.06849				36.16438				7.260 27				38.35616				39.45206				40.0			

A TABLE *of* SIMPLE INTEREST *at* 5 *per Cent.*

Principal.	1 Day.			2 Days.			3 Days.			4 Days.			5 Days.			6 Days.			7 Days.			8 Days.			
l.	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1	*l.*00013699																				1				1
2												1			1			1			2				2
3									1			1			2			2			3				3
4						1			1			2			2			3		1	0			1	0
5						1			2			3			3		1	0		1	0			1	1
6						1			2			3		1	0		1	1		1	1			1	2
7						2			3		1	0		1	0		1	1		1	2			1	3
8			1			2			3		1	0		1	1		1	2		1	3			2	0
9			1			2			3		1	1		1	2		1	3		2	0			2	1
10			1			2		1	0		1	1		1	2		2	0		2	1			2	2
20			2		1	1		2	0		2	2		3	1		4	0		4	2			5	0
30		1	0		2	0		3	0		4	0		5	0		5	3		6	3			7	3
40		1	1		2	2		4	0		5	1		6	0		7	3		9	1			10	2
50		1	2		3	1		5	0		6	2		8	1		10	0		11	2		1	1	0
60		2	0		4	0		5	3		7	3		10	0		11	3	1	1	3		1	3	3
70		2	1		4	2		6	3		9	1		11	2	1	1	3	1	4	0		1	6	1
80		2	2		5	1		7	3		10	2	1	1	0	1	3	3	1	6	1		1	9	0
90		3	0		5	3		8	3		11	3	1	2	3	1	5	3	1	8	3		1	11	2
100		3	1		6	2		10	0	1	1	0	1	4	2	1	7	3	1	11	0		2	2	1
200		6	2	1	1	0	1	7	3	2	2	1	2	8	3	3	3	2	3	10	0		4	4	2
300		10	0	1	7	3	2	5	2	3	3	2	4	1	1	4	11	0	5	9	0		6	6	3
400	1	1	0	2	2	1	3	3	2	4	4	2	5	5	2	6	6	3	7	8	0		8	9	1
500	1	4	2	2	8	3	4	1	1	5	5	2	6	10	0	8	2	2	9	7	0		10	11	2
600	1	7	3	3	3	2	4	11	0	6	6	3	8	2	2	9	10	1	11	6	0		13	1	3
700	1	11	0	3	10	0	5	9	0	7	8	0	9	7	0	11	6	0	13	5	0		15	4	0
800	2	2	1	4	4	2	6	6	3	8	9	1	10	11	2	13	2	0	15	4	0		17	6	1
900	2	5	2	4	11	0	7	4	3	9	10	2	12	4	0	14	9	2	17	3	0		19	8	3
1000	2	8	3	5	5	3	8	2	2	10	11	2	13	8	1	16	5	1	19	2	0	1	1	11	0
ditto	*l.* ·1369			*l.* ·2739			*l.* ·4109			*l.* ·5479			*l.* ·6849			*l.* ·8219			*l.* ·9589			*l.* 1.0956			

SIMPLE INTEREST *at* 5 *per Cent.*

Principal.	9 Days.				10 Days.				20 Days.				30 Days.				40 Days.				50 Days.				60 Days.			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1				1				1				2			1	0			1	1			1	2			2	0
2				2				2			1	1			2	0			2	2			3	1			4	0
3				3			1	0			2	0			3	0			4	0			5	0			5	3
4			1	1			1	1			2	2			4	0			5	1			6	2			7	3
5			1	2			1	2			3	1			5	0			6	2			8	1			9	3
6			1	3			2	0			4	0			5	3			7	3			10	0			11	3
7			2	0			2	1			4	3			6	3			9	1			11	2		1	1	3
8			2	1			2	2			5	1			7	3			10	2		1	1	0		1	3	3
9			2	3			3	0			5	3			8	3			11	3		1	2	3		1	5	3
10			3	0			3	1			6	2			10	0		1	1	0		1	4	2		1	7	3
20			5	3			6	2		1	1	0		1	7	3		2	2	1		2	8	3		3	3	2
30			8	3			10	0		1	7	3		2	5	2		3	3	2		4	1	1		4	11	1
40			11	3		1	1	0		2	2	1		3	3	2		4	4	3		5	5	2		6	6	3
50		1	2	3		1	4	2		2	8	3		4	1	1		5	5	3		6	10	0		8	2	2
60		1	5	2		1	7	3		3	3	2		4	11	1		6	6	3		8	2	2		9	10	1
70		1	8	3		1	11	0		3	10	0		5	9	0		7	8	0		9	7	0		11	6	0
80		1	11	2		2	2	1		4	4	3		6	6	3		8	9	1		10	11	2		13	1	3
90		2	2	2		2	5	2		4	11	1		7	4	3		9	10	2		12	4	0		14	9	2
100		2	5	2		2	8	3		5	5	3		8	2	2		10	11	2		13	8	0		16	5	1
200		4	11	0		5	5	2		10	11	2		16	5	1	1	1	11	0	1	7	4	3	1	12	10	2
300		7	4	2		8	2	2		16	5	1	1	4	7	3	1	12	10	2	2	1	1	0	2	9	3	3
400		9	10	1		10	11	2	1	1	11	0	1	12	10	2	2	3	10	0	2	14	9	2	3	5	9	0
500		12	4	0		13	8	1	1	7	4	3	2	1	1	0	2	14	9	2	3	8	5	3	4	2	2	1
600		14	9	2		16	5	0	1	12	10	2	2	9	3	3	3	5	9	0	4	2	2	1	4	18	7	2
700		17	3	0		19	2	0	1	18	4	1	2	17	6	1	3	16	8	2	4	15	10	3	5	15	0	3
800		19	8	3	1	1	11	0	2	3	10	0	3	5	9	0	4	7	8	0	5	9	7	0	6	11	6	0
900	1	2	2	1	1	4	7	3	2	9	3	3	3	13	11	3	4	18	7	2	5	3	3	3	7	7	11	1
1000	1	4	7	3	1	7	4	3	2	14	9	2	4	2	2	1	5	9	7	0	6	16	11	3	8	4	4	2
ditto	*l.* 1.2328				*l.* 1.3698				*l.* 2.7396				*l.* 4.1094				*l.* 5.4792				*l.* 6.8490				*l.* 8.2188			

SIMPLE INTEREST *at 5 per Cent.*

Principal.	70 Days.				80 Days.				90 Days.				100 Days.				110 Days.				120 Days.			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			2	1			2	3			3	0			3	1			3	2			4	0
2			4	2			5	1			5	3			6	2			7	1			7	3
3			6	3			7	3			8	3			9	3			10	3			11	3
4			9	1			10	2			11	3		1	1	0		1	2	2		1	3	3
5			11	2		1	1	0		1	2	3		1	4	2		1	6	0		1	7	3
6		1	1	3		1	3	3		1	5	3		1	7	3		1	9	2		1	11	2
7		1	4	0		1	6	1		1	8	3		1	11	0		2	1	1		2	3	2
8		1	6	1		1	9	0		1	11	2		2	2	1		2	5	0		2	7	2
9		1	8	3		1	11	2		2	2	2		2	5	2		2	8	2		2	11	2
10		1	11	0		2	2	1		2	5	2		2	8	3		3	0	0		3	3	2
20		3	10	0		4	4	2		4	11	0		5	5	3		6	0	1		6	6	3
30		5	9	0		6	6	3		7	4	3		8	2	2		3	0	2		9	10	1
40		7	8	0		8	9	1		9	10	1		10	11	2		12	0	2		13	1	3
50		9	7	0		10	11	2		12	4	0		13	8	1		15	0	3		16	5	1
60		11	6	0		13	1	3		14	9	2		16	5	1		18	1	0		19	8	2
70		13	5	0		15	4	0		17	3	1		19	2	0	1	1	1	0	1	3	0	0
80		15	4	0		17	6	1		19	8	2	1	1	11	0	1	4	1	1	1	6	3	2
90		17	3	0		19	8	3	1	2	2	1	1	4	7	3	1	7	1	2	1	9	7	0
100		19	2	0	1	1	11	0	1	4	7	3	1	7	4	3	1	10	1	2	1	12	10	2
200	1	18	4	1	2	3	10	0	2	9	3	3	2	14	9	2	3	0	3	1	3	5	9	0
300	2	17	6	1	3	5	9	0	3	13	11	2	4	2	2	1	4	10	5	0	4	18	7	2
400	3	16	8	2	4	7	8	0	4	18	7	2	5	9	7	0	6	0	6	2	6	11	6	0
500	4	15	10	3	5	9	7	0	6	3	3	2	6	16	11	3	7	10	8	1	8	4	4	2
600	5	15	0	3	6	11	6	0	7	7	11	1	8	4	4	2	9	0	9	3	9	17	3	0
700	6	14	3	0	7	13	5	0	8	12	7	1	9	11	9	1	10	10	11	2	11	10	1	2
800	7	13	5	0	8	15	4	0	9	17	3	0	10	19	2	0	12	1	1	0	13	3	0	0
900	8	12	7	1	9	17	3	0	11	1	11	0	12	6	6	3	13	11	2	3	14	15	10	2
1000	9	11	9	1	10	19	2	2	12	6	6	3	13	13	11	2	15	1	4	1	16	8	9	0
ditto	*l.* 9.5889				*l.* 10.9587				*l.* 12.3286				*l.* 13.6984				*l.* 15.0682				*l.* 16.4379			

SIMPLE INTEREST *at 5 per Cent.*

Principal.	130 Days.				140 Days.				150 Days.				160 Days.				170 Days.				180 Days.			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			4	1			4	2			4	3			5	1			5	2			5	3
2			8	2			9	0			9	3			10	2			11	1			11	3
3		1	0	3		1	1	3		1	2	3		1	3	3		1	4	3		1	5	3
4		1	5	0		1	6	1		1	7	3		1	9	0		1	10	1		1	11	2
5		1	9	1		1	11	0		2	0	2		2	2	1		2	4	0		2	5	2
6		2	1	2		2	3	2		2	5	2		2	7	2		2	9	2		2	11	2
7		2	5	3		2	8	1		2	10	2		3	0	3		3	3	1		3	5	2
8		2	10	0		3	0	3		3	3	2		3	6	0		3	8	3		3	11	1
9		3	2	2		3	5	1		3	8	1		3	11	1		4	2	1		4	5	1
10		3	6	3		3	10	0		4	1	1		4	4	2		4	7	3		4	11	1
20		7	1	2		7	8	0		8	2	2		8	9	0		9	3	3		9	10	1
30		10	8	1		11	6	0		12	3	3		13	1	3		13	11	3		14	9	2
40		14	3	0		15	4	0		16	5	1		17	6	1		18	7	2		19	8	3
50		17	9	2		19	2	0	1	0	6	2	1	1	11	0	1	3	3	2	1	4	7	3
60	1	1	4	1	1	3	0	0	1	4	7	3	1	6	3	2	1	7	11	1	1	9	7	0
70	1	4	11	0	1	6	10	1	1	8	9	0	1	10	8	1	1	12	7	1	1	14	6	1
80	1	8	5	3	1	10	8	1	1	12	10	2	1	15	0	3	1	17	3	0	1	19	5	1
90	1	12	0	2	1	14	6	1	1	16	11	3	1	19	5	2	2	1	11	0	2	4	4	2
100	1	15	7	1	1	18	4	1	2	1	1	0	2	3	10	0	2	6	6	3	2	9	3	3
200	3	11	2	3	3	16	8	2	4	2	2	1	4	7	8	0	4	13	1	3	4	18	7	2
300	5	6	10	0	5	15	0	3	6	3	3	2	6	11	6	0	6	19	8	2	7	7	11	1
400	7	2	5	2	7	13	5	0	8	4	4	2	8	15	4	0	9	6	3	2	9	17	3	0
500	8	18	0	3	9	11	9	1	10	5	5	3	10	19	2	0	11	12	10	2	12	6	6	3
600	10	13	8	1	11	10	1	2	12	6	6	3	13	3	0	0	13	19	5	2	14	15	10	2
700	12	9	3	3	13	8	5	3	14	7	8	0	15	6	10	1	16	6	0	1	17	5	2	2
800	14	4	11	0	15	6	10	0	16	8	9	0	17	10	8	1	18	12	7	1	19	14	6	0
900	16	0	6	2	17	5	2	1	18	9	10	1	19	14	6	1	20	19	2	0	22	3	10	0
1000	17	16	1	3	19	3	6	2	20	10	11	2	21	18	4	1	23	5	9	0	24	13	1	3
ditto	*l.* 17.8078				*l.* 19.1776				*l.* 20.5479				*l.* 21.9177				*l.* 23.2875				*l.* 24.6573			

SIMPLE INTEREST at 5 per Cent.

Principal. l.	190 Days.				200 Days.				210 Days.				220 Days.				230 Days.				240 Days.			
	l.	s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.
1			6	1			6	2			7	0			7	1			7	2			7	3
2		1	0	2		1	1	0		1	1	3		1	2	2		1	3	0		1	3	3
3		1	6	3		1	7	3		1	8	3		1	9	2		1	10	3		1	11	3
4		2	1	0		2		1		2	3	2		2	5	0		2	6	1		2	7	2
5		2	7	1		2	2	3		2	10	2		3	0	1		3	1	3		3	3	2
6		3	1	2		3	3	2		3	5	2		3	7	2		3	9	1		3	11	1
7		3	7	3		3	10	0		4	0	1		4	2	2		4	5	0		4	7	1
8		4	2	0		4	4	2		4	7	1		4	9	3		5	0	2		5	3	0
9		4	8	1		4	11	1		5	2	1		5	5	0		5	8	0		5	11	0
10		5	2	2		5	5	3		5	9	0		6	0	1		6	3	2		6	7	0
20		10	5	0		10	11	2		11	6	0		12	0	2		12	7	1		13	1	3
30		15	7	1		16	5	1		17	3	0		18	1	0		18	10	3		19	8	2
40	1	0	9	3	1	1	11	0	1	3	0	0	1	4	1	1	1	5	2	2	1	6	3	2
50	1	6	0	1	1	7	4	3	1	8	9	0	1	10	1	3	1	11	6	0	1	12	10	2
60	1	11	2	3	1	12	10	2	1	14	6	1	1	16	2	0	1	17	9	2	1	19	5	2
70	1	16	5	1	1	18	4	1	2	0	3	1	2	2	2	1	2	4	1	1	2	6	0	1
80	2	1	7	3	2	3	10	0	2	6	0	1	2	8	2	2	2	10	5	0	2	12	7	1
90	2	6	10	1	2	9	3	3	2	11	9	1	2	14	3	0	2	16	8	2	2	19	2	0
100	2	12	0	2	2	14	9	2	2	17	6	2	3	0	3	1	3	3	0	0	3	5	9	0
200	5	4	1	1	5	9	7	0	5	15	0	3	6	0	6	2	6	6	0	1	6	11	6	0
300	7	16	2	0	8	4	4	3	8	12	7	1	9	0	9	3	9	9	0	2	9	17	3	0
400	10	8	2	2	10	19	2	1	11	10	1	1	12	1	1	1	12	12	0	2	13	3	0	0
500	13	0	3	1	13	13	11	3	14	7	8	0	15	1	4	2	15	15	0	3	16	8	9	1
600	15	12	4	0	16	8	9	1	17	5	2	2	18	1	7	3	18	18	1	0	19	14	6	1
700	18	4	4	2	19	3	6	3	20	2	8	3	21	1	11	0	22	1	1	0	23	0	3	1
800	20	16	5	1	21	18	4	1	23	0	8	1	24	2	2	1	25	4	1	1	26	6	0	1
900	23	8	5	3	24	13	1	3	25	17	9	2	27	2	5	2	28	7	1	2	29	11	9	1
1000	26	0	6	2	27	7	11	1	28	15	4	0	30	2	8	3	31	10	1	2	32	17	6	1
ditto	l. 26.0271				l. 27.3969				l. 28.7671				l. 30.1368				l. 31.5067				l. 32.8765			

SIMPLE INTEREST *at* 5 *per Cent.*

Principal.	250 Days.				260 Days.				270 Days.				280 Days.				290 Days.				300 Days.			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			8	1			8	2			8	3			9	1			9	2			9	3
2		1	4	2		1	5	0		1	5	3		1	6	2		1	7	0		1	7	3
3		2	0	3		2	1	3		2	2	2		2	3	2		2	4	2		2	5	2
4		2	8	3		2	10	1		2	11	2		3	0	3		3	2	1		3	3	2
5		3	5	1		3	6	3		3	8	1		3	10	0		3	11	3		4	1	1
6		4	1	1		4	3	1		4	5	1		4	7	1		4	9	0		4	11	1
7		4	9	2		4	11	3		5	2	1		5	4	2		5	6	3		5	9	0
8		5	5	3		5	8	2		5	11	0		6	1	2		6	4	1		6	7	0
9		6	2	0		6	5	0		6	7	3		6	11	0		7	1	3		7	4	3
10		6	10	0		7	1	2		7	4	3		7	8	0		7	11	1		8	2	2
20		13	8	2		14	3	0		14	9	2		15	4	0		15	10	3		16	5	1
30	1	0	6	2	1	1	4	2	1	2	2	1	1	3	0	0	1	3	10	0	1	4	8	0
40	1	7	4	3	1	8	5	3	1	9	7	0	1	10	8	1	1	11	9	1	1	12	10	2
50	1	14	3	0	1	15	7	1	1	16	11	3	1	18	4	1	1	19	8	3	2	1	1	0
60	2	1	1	0	2	2	8	3	2	4	4	2	2	6	0	1	2	7	8	0	2	9	3	3
70	2	7	11	1	2	9	10	1	2	11	9	1	2	13	8	2	2	15	7	1	2	17	6	2
80	2	14	9	2	2	16	11	3	2	19	2	0	3	1	4	1	3	3	6	3	3	5	9	0
90	3	1	7	3	3	4	1	1	3	6	7	0	3	9	0	2	3	11	6	0	3	13	11	3
100	3	8	5	3	3	11	2	3	3	13	11	3	3	16	8	2	3	19	5	2	4	2	2	1
200	6	16	11	3	7	2	5	2	7	7	11	1	7	13	5	0	7	18	10	3	8	4	4	2
300	10	5	5	3	10	13	8	2	11	1	11	0	11	10	1	2	11	18	4	1	12	6	6	3
400	13	13	11	2	14	4	11	1	14	15	10	2	15	6	10	1	15	17	9	2	16	8	9	0
500	17	2	5	2	17	16	2	0	18	9	10	1	19	3	6	3	16	17	3	0	20	10	11	2
600	20	10	11	2	21	7	4	3	22	3	10	0	23	0	3	1	23	16	8	2	24	13	1	3
700	23	19	5	1	24	18	7	2	25	17	9	2	26	16	11	3	27	16	2	0	28	15	4	0
800	27	7	11	1	28	9	10	1	29	11	9	1	30	13	8	1	31	15	7	1	32	17	6	1
900	30	16	5	1	32	1	1	0	33	5	9	0	34	10	5	0	35	15	0	3	36	19	8	3
1000	34	4	11	1	35	12	4	0	36	19	8	3	38	7	1	2	39	14	6	1	41	1	11	0
ditto	*l.* 34.2466				*l.* 35.6164				*l.* 36.9862				*l.* 38.3561				*l.* 39.7259				*l.* 41.0957			

SIMPLE INTEREST *at* 5 *per Cent.*

Principal. l.	310 Days. l. s. d. q.	320 Days. l. s. d. q.	330 Days. l. s. d. q.	340 Days. l. s. d. q.	350 Days. l. s. d. q.	360 Days. l. s. d. q.	365 Days, or 1 Year. l. s.
1	10 0	10 2	10 3	11 1	11 2	11 3	1
2	1 8 1	1 9 0	1 9 3	1 10 1	1 11 0	1 11 3	2
3	2 6 2	2 7 2	2 8 2	2 9 2	2 10 2	2 11 2	3
4	3 4 3	3 6 0	3 7 1	3 8 3	3 10 0	3 11 1	4
5	4 3 0	4 4 2	4 6 1	4 7 3	4 9 2	4 11 1	5
6	5 1 0	5 3 0	5 5 0	5 7 0	5 9 0	5 11 0	6
7	5 11 1	6 1 2	6 4 0	6 6 1	6 8 2	6 10 3	7
8	6 9 2	7 0 0	7 2 3	7 5 2	7 8 0	7 10 3	8
9	7 7 3	7 10 3	8 1 2	8 4 2	8 7 2	8 10 2	9
10	8 6 0	8 9 0	9 0 2	9 3 3	9 7 0	9 10 1	10
20	16 11 3	17 6 1	18 1 0	18 7 2	19 2 0	19 8 3	1 0
30	1 5 5 3	1 6 3 2	1 7 1 2	1 7 11 1	1 8 9 0	1 9 7 0	1 10
40	1 13 11 3	1 15 0 3	1 16 2 0	1 17 3 0	1 18 4 1	1 19 5 2	2 0
50	2 2 5 2	2 3 10 0	2 5 2 2	2 6 6 3	2 7 11 1	2 9 3 3	2 10
60	2 10 11 2	2 12 7 1	2 14 3 0	2 15 10 2	2 17 6 1	2 19 2 1	3 0
70	2 19 5 2	3 1 4 1	3 3 3 2	3 5 2 2	3 7 1 2	3 9 0 2	3 10
80	3 7 11 1	3 10 1 2	3 12 4 0	3 14 6 1	3 16 8 2	3 18 10 3	4 0
90	3 16 5 1	3 18 10 3	4 1 4 2	4 3 10 0	4 6 3 2	4 8 9 0	4 10
100	4 4 11 1	4 7 8 0	4 10 5 0	4 13 1 3	4 15 10 2	4 18 7 2	5
200	8 9 10 1	8 15 4 0	9 0 9 3	9 6 3 2	9 11 9 1	9 17 3 0	10
300	12 14 9 2	13 3 0 0	13 11 2 3	13 19 5 1	14 7 8 0	14 15 10 3	15
400	16 19 8 3	17 10 8 1	18 1 7 3	18 12 7 0	19 3 6 3	19 14 6 1	20
500	21 4 7 3	21 18 4 1	22 12 0 2	23 5 9 0	23 19 5 1	24 13 1 3	25
600	25 9 7 0	26 6 0 1	27 2 5 2	27 18 10 3	28 15 4 0	29 11 9 1	30
700	29 14 6 1	30 13 8 1	31 12 10 2	32 12 0 2	33 11 2 3	34 10 5 0	35
800	33 19 5 1	35 1 4 1	36 3 3 2	37 5 2 2	38 7 1 2	34 9 0 2	40
900	38 4 4 2	39 9 0 1	40 13 8 1	41 18 4 1	43 3 0 0	44 7 8 0	45
1000	42 9 3 3	43 16 8 2	45 4 1 1	46 11 6 0	47 18 10 3	49 6 3 2	50
ditto	*l.* 42.4655	*l.* 43.8353	*l.* 45.2051	*l.* 46.5749	*l.* 47.9447	*l.* 49.3150	*l.* 50

A TABLE shewing the Number of Days between any Day of one Month, and the same Day of any other.

Jan. 1, &c.	Febr.	March.	April.	May.	June.
Febr. 31.	*Marc.* 28.	*April* 31.	*May* 30.	*June* 31.	*July* 30.
Marc. 59.	*April* 59.	*May* 61.	*June* 61.	*July* 61.	*Aug.* 61.
April 90.	*May* 89.	*June* 92.	*July* 91.	*Aug.* 92.	*Sept.* 92.
May 120.	*June* 120.	*July* 122.	*Aug.* 122.	*Sept.* 123.	*Oct.* 122.
June 151.	*July* 150.	*Aug.* 153.	*Sept.* 153.	*Oct.* 153.	*Nov.* 153.
July 181.	*Aug.* 181.	*Sept.* 184.	*Oct.* 183.	*Nov.* 184.	*Dec.* 183.
Aug. 212.	*Sept.* 212.	*Oct.* 214.	*Nov.* 214.	*Dec.* 214.	*Jan.* 214.
Sept. 243.	*Oct.* 242.	*Nov.* 245.	*Dec.* 244.	*Jan.* 245.	*Feb.* 245.
Oct. 273.	*Nov.* 273.	*Dec.* 275.	*Jan.* 275.	*Feb.* 276.	*Mar.* 273.
Nov. 304.	*Dec.* 303.	*Jan.* 306.	*Feb.* 306.	*Mar.* 304.	*Apr.* 304.
Dec. 334.	*Jan.* 334.	*Feb.* 337.	*Mar.* 334.	*Apr.* 335.	*May* 334.
Jan. 365.	*Feb.* 365.	*Mar.* 365.	*Apr.* 365.	*May* 365.	*June* 365.

July.	August.	Septemb.	October.	Novemb.	Decemb.
Aug. 31.	*Sept.* 31.	*Octob.* 30.	*Nov.* 31.	*Dec.* 30.	*Jan.* 31.
Sept. 62.	*Octob.* 61.	*Nov.* 61.	*Dec.* 61.	*Jan.* 61.	*Febr.* 62.
Oct. 92.	*Nov.* 92.	*Dec.* 91.	*Jan.* 92.	*Febr.* 92.	*Mar.* 90.
Nov. 123.	*Dec.* 122.	*Jan.* 122.	*Feb.* 123.	*Mar.* 120.	*Apr.* 121.
Dec. 153.	*Jan.* 153.	*Feb.* 153.	*Mar.* 151.	*Apr.* 151.	*May* 151.
Jan. 184.	*Feb.* 184.	*Mar.* 181.	*Apr.* 182.	*May* 181.	*June* 182.
Feb. 215.	*Mar.* 212.	*Apr.* 212.	*May* 212.	*June* 212.	*July* 212.
Mar. 243.	*Apr.* 243.	*May* 242.	*June* 243.	*July* 242.	*Aug.* 243.
Apr. 274.	*May* 273.	*June* 273.	*July* 273.	*Aug.* 273.	*Sept.* 274.
May 304.	*June* 304.	*July* 303.	*Aug.* 304.	*Sept.* 304.	*Oct.* 304.
June 335.	*July* 334.	*Aug.* 334.	*Sept.* 335.	*Oct.* 334.	*Nov.* 335.
July 365.	*Aug.* 365.	*Sept.* 365.	*Octob.* 365.	*Nov.* 365.	*Dec.* 365.

The

The Use of the foregoing Tables.

1. This Table last above I shall first shew the Use of, because it is naturally in Use before that of Simple Interest.

Example.] How many Days is contained between the 19th of *November* and the 28th of *March* following?

Rule.] This Table shews, that from *November* 19, *&c.* to the same day of *March*, it is 120 Days. To which adding the Days that 28 exceeds 19, *viz.* 9, gives 129 Days for Answer. But in case the Days you reckon to, are not so many as those you reckon from; then substract the difference. As, to find the Days between the 28th of *November* and the 19th of *March*; *November* 28, *&c.* to the same Day of *March* is 120 Days (as before) from which deducting what 28 exceeds the 19th of *November*, and the Remainer is 111 Days the Answer.

The Use of the Table of Simple Interest, at 5 per Cent.

Example 1.] What is the Interest of *l.* 700 from the 5th of *July* to the 13th of *April* following, at 5 *per Cent?*

The Time is 282 Days, and against *l.* 700 under 280 Days, is } *l.* 26 : 16 : 11 ¼

And under 2 Days is 00 : 3 : 10

Answer, *l.* 27 : 00 : 9 ¼

Example 2.] To find the Interest of *l.* 470 from the 18th of *December* to the 5th of *April*, at 5 *per Cent.* The Time is 108 Days; and

	Days.				
l. 400,	under 100	= *l.* 5 :	9 :	7	
l. 70,	under 100	= 00 :	19 :	2	
l. 400,	under 8	= 00 :	8 :	9 ¼	
l. 70,	under 8	= 00 :	1 :	6	
Sum or Answer		= *l.* 6 :	19 :	00 ¼	

3. Or any more odd or larger Sums in Questions of this kind may be multiplied one in another, and the Product of the Days and Pounds by the ,00013699 at the beginning of the Table of 5 *per Cent.* for Answer.

4. If you would find the Interest of *l.* 4700 (or other Sum above 1000) for 108, *&c.* Days; under 100 Days and 8 Days against ditto you find the marginal Numbers, whose Sum being multiplied by 4.7 (= 4700) gives the Answer *l.* 69 : 10 : 7 ½.

Against ditto under 100 is 13.6984
under 8 is 1.0956

Sum *l.* 14.7940
Multiply by 4.7

Product Answer = 69.5318

Such

Such Perſons as are minded to have an Inſtrument whereby to find the Number of Days between any two in the Year, may ſee the Form of one, *Plate* A. *Fig.* 25, and 26. which being done in Braſs, and of a ſix-inch Radius, will moſt eaſily and accurately anſwer their End.

The Deſcription and Uſe of the Circles of Days and Months.
(Fig. 25, 26. Plate A.)

This conſiſts of a Circle (as *Fig.* 26.) divided into 365 Days, which is to turn round concentrically in a Circle, (*Fig.* 25.) divided into the Months in the Year, and each Month into its reſpective Days: ſo that the inner Circle naturally meaſures the Days between any two in any of the Months, in order to *find the Intereſt or Diſcount* for thoſe *Days.*

Example.] How many Days are contained between the 15th of *November* and the 2d of *May* following?

For Anſwer, turn the Hand in the inner Circle to the 15th of *November* in the outer Circle, and againſt the 2d of *May* in that outer, you will find in the inner Circle 168 Days, including the Day you reckon from, and excluding that which you reckon to.

Rules how by the two Tables of 4 *and* 5 per Cent. *above, to find the Intereſt at any Rate from* 1 *to* 10 per Cent. *incluſive, very eaſily and briefly.*

For Inſtance, *l.* 500 for 170 Days.

Rules.	*Examples.*	
For 1 *per Cent.* take a fourth of the Int. at 4 *per Cent.*	*l.* 9 : 6 : 3 : 3 of which ½ is	*l.* 2 : 6 : 6¼ Anſwer.
2 *per Cent.* take half the Intereſt at 4 *per Cent.*	*l.* 9 : 6 : 3 : 3 ; ½ is 4 : 13 : 1¼	Anſwer.
3 *per Cent.* is the Inter. at 4 *per Cen.* leſs a 4th thereof, as leſs ——	*l.* 2 : 6 : 6 : 3 = 6 : 19 : 9	Anſwer.
6 *per Cent.* is the Intereſt at 4 *per Cent.* more half thereof	9 : 6 : 3 : 3 more 4 : 13 : 1 : 3	or 13 : 19 : 5½ Anſw.
7 *per C.* take the Int. of 1 *p. C.* from the Int. of 8 *p.C.* found thus	as 8 *p.C.* is = 18 : 12 : 7 : 2 1 *p.C.* leſs = 2 : 6 : 6 : 3 gives	16 : 6 : 0¼ Anſw.

Rules

Rules. *Examples.*

For 8 *per Cent.* add the Int. at 4 *per Cent.* to itself } 18 : 12 : 7 : 2 = 18 : 12 : 7½ Anf.

4 *per Cent.* is 9 : . 6 : 3 : 3 }
9 *per Cent.* add the Int. at 4 and 5 *per C.* together } Sum 20 : 19 : 2¼ Anf.
5 *per Cent.* is 11 : 12 : 10 : 2 }

10 *per Cent.* add the Int. at 5 *per Cent.* to itself— } 11 : 12 : 10 : 2 }
more 11 : 12 : 10 : 2 } Sum 23 : 5 : 9 Anfw.

II. *The Use of Decimals in calculating Discount.*

This matter wants very much to be fet in a clear Light, for I have not yet feen it done; tho' I have not perhaps feen all that has been wrote upon it: But I know that a learned Author in Folio has intirely miftaken it.

1. The Difcount of Money is the Allowance made by the Creditor out of a Sum of Money due to him at the end of fome number of Days, in confideration of the prompt Payment of the Remainer by the Debtor.

2. That Sum paid down inftead of the Principal due hereafter, may properly be called *the prefent Worth*; in regard that if it were put out to Intereft for the Days that the Difcount is computed, it would amount to the Principal due at the end of thofe Days.

3. The Intereft for any time is more than the Difcount for that time; becaufe (fuppofe of *l.* 1 for 1 Day at 6 *per Cent.*) the Quotient muft be more when .016438 (= the Intereft of *l.* 100 for one Day) is divided only by 100, than when it is divided by *l.* 100.016438.

4. To find *the prefent Worth* of *l.* 1. due 1 Day hence at 6 *per Cent.*

	Days.	*l. Int.*		*Day.*	*l. Int.*
1*ft*,	As 365.	6	: :	1.	.016438

	l.	*l.*		*l.*	*l.* Anfwer.
2*dly*,	As 100.016438.	100	: :	1	.99983565

5. For the Difcount of *l.* 1 for 1 Day.

1*ft*,	As 365.	6	: :	1.	.016438
2*dly*,	As 100.016438.	.016438	: :	1	to .00016435

Here the Sum of the prefent Worth and Difcount makes up the Principal payable at time, as a Proof of the Truth of both. Sum Proof, *l.* 1.00000000

6. Another way to prove this, is to try whether the Amount of the present Worth for the time, will make up the Principal due at that time thus:

l.	*l.*	*l.*	
100.	100.016438 : :	99983565.	to *l.* 1.

Whence I find, that if *l.* 100 in 1 Day amount to 100.016438; then the said present Worth of 1 *l.* due at the end of 1 Day, will amount to *l.* 1: which is a second Proof of the true Computation of the present Worth and Discount above.

7. To illustrate this matter farther, I shall give another Example at large; which shall be to find the Discount of *l.* 1000 payable at the end of 90 Days at 6 *per Cent.*

See the Operation.

```
                                              Days. l.   Days.
                                         1st, 365. 6 : : 90
                                                             6
2dly, As 101.47945. 1.47945 : : 1000. 14.57882 | ―――――
                    1000                          365) 540. (1.47945
                   ――――――            l.               175.0 (=100,
101.47945) 1479.45000 (14.57882 Ans. |                 290.0 (for 90 days.
             46465550                                  345.0
              5873770.0                                165.0
               7997775.0                               190.0
                8942135.0
                 8237790.0
                  2194340
```

And for Proof 101.47945. 100 : : 1000. 985.42118 = pres. Worth.
Which added to the Discount = - - - - - 14.57882 = 1000
Or for Proof say, 100. 101.47945 : : 985.42118. 1000

8. *A New Way of Calculating Discount and Present Worth.*

But because there are two Divisions, and one of them very operose, I have Algebraically (as you may see in the Use of Algebra) contrived this Rule or Canon, which has but one, and that a shorter Division.

1. *For the present Worth*; Multiply the Days in a Year, the Principal given, and 100, in each other, for the Dividend. And add the Product of 365 by 100 to that of the Days multiplied in the Rate given, and the Sum is the Divisor: so the Quote arising is the Answer.

In

In the last Example, 365 in 100 is 36500, that in 1000 makes 36500000, without farther trouble; then 365 by 100 = 36500; more 90 Days in 6 (the Rate) is 540; and that Sum is = 37040, the Divisor. See the Work.

3704.0) 3650000.0 (985.42117 = the Answer, or present Worth.
31640
2008.0
1560.0
784.0
432.0
616.0
2456.0

2.] For *Discount*; Multiply the Rate in the Principal given, and that in the Days given is your Dividend. And the Product of 365 Days by 100 added to that of the Days, in the Rate given, is the Divisor. Thus the Principal in the Rate is —— —— —— = 6000
By the Days = 90

And 365 in 100 = 36500
More 90 in 6 = 540
Divisor = 37040 Sum.

3704.0) 54000.0 = *Dividend* (14.57883 = *Discount*, as *before*.
16960
2144.0
2920.0
3272.0
3088.0
1248.0

Note, By this last Method you use fewer Figures by above 40, and have not a third of the trouble.

9. In Dr *Harris's Lexicon* the Discount for one Day is asserted to be the 365th part of that for a Year: However this Mistake came, I know not; but his two Folio Pages of Table of Discount being made upon the same Principle, are likewise erroneous. And to prove this, it is sufficient from the foregoing due Calculation, That the Discount of *l.* 1 for 1 Day, at 6 *per Cent.* is .00016435, and not .0001550788, as the *Lexicon* makes it. And to pretend that multiplying the Discount of *l.* 1 for 1 Day, gives the Discount of one Pound for other Days by which you multiply, is a wrong Notion; because every Day's Discount of *l.* 1 differs, being less according as that Day is distant from 1, as appears plain from what follows.

This

This is a full Indication that one Day's Difcount at the beginning of the Year is much more than at the middle or end: and therefore were the laft Example above of the Difcount of *l.* 1000 for 90 Days to be done by the Tables in the *Lexicon*, it would be but — — — } *l.* 13 : 19 : 1¼
Whereas 'tis in truth by the Calculation above — — } 14 : 11 : 6¼

So that the faid *Lexicon-Tables* err in this Inftance 12 *s.* 5 *d.* And were the Days fewer, for which the Difcount is required, the Error would be proportionably greater.

Days.	*Difcount of* l. 1. *for those Days at* 6 per Cent.	*Differences or each Day's Difcount.*
1	.00016435	
2	.00032865	.00016430
3	.0004929	.00016425
4	.0006571	.0001642
180	.0287386	
181	.0288937	.0001551
182	.0290487	.0001550
183	.0292036	.0001549
362	.05616466	
363	.05631106	.00014642
364	.05645744	.00014638
365	.05660377	.00014633

Hence it may be inferred, That no Tables of Difcount can be ufed with Accuracy, but fuch as have the Difcount for every Day in the Year, becaufe every Day's Difcount differeth. And had I time, and room in this Book, I would oblige the Publick with a Table of Difcount for every Day, becaufe I know not of any true Table of Difcount extant. But in the mean time, the following will be better than any yet publifhed, and will be found accurate enough in Practice.

A TABLE

A TABLE *of* DISCOUNT *at* 4 *per Cent.* per Ann. *more accurate than any extant.*

Principal.	*1 Day.*			*2 Days.*			*3 Days.*			*4 Days.*			*5 Days.*			*6 Days.*			*7 Days.*			*8 Days.*		
l.	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*
1																					1			1
2									1			1			1			1			1			1
3						1			1			1			1			2			2			2
4						1			1			2			2			2			3			3
5						1			1			2			2			3		1	0		1	0
6			1			1			1			2			3		1	0		1	0		1	1
7			1			1			2			3		1	0		1	0		1	1		1	2
8			1			2			2			3		1	0		1	1		1	2		1	3
9			1			2			3		1	0		1	1		1	2		1	3		1	3
10			1			2			3		1	0		1	1		1	2		1	3		2	0
20			2		1	0		1	2		2	0		2	2		3	1		3	3		4	1
30			3		1	2		2	1		3	1		4	0		4	3		5	2		6	1
40		1	0		2	0		3	1		4	1		5	1		6	1		7	1		8	2
50		1	1		2	2		4	0		5	1		6	2		7	3		9	1		10	2
60		1	2		3	1		4	3		6	1		7	3		9	2		11	0	1	0	2
70		1	3		3	3		5	2		7	1		9	1		11	0	1	0	3	1	2	3
80		2	0		4	1		6	1		8	2		10	2	1	0	2	1	2	3	1	5	0
90		2	1		4	3		7	0		9	2	1	0	0	1	2	1	1	4	2	1	7	0
100		2	2		5	1		7	3		10	2	1	1	0	1	3	3	1	6	2	1	9	0
200		5	1		10	2	1	3	3	1	9	0	2	2	1	2	7	2	3	0	3	3	6	0
300		7	3	1	3	3	1	11	3	2	7	2	3	3	2	3	11	2	4	7	1	5	3	0
400		10	2	1	9	0	2	7	2	3	6	0	4	4	3	5	3	0	6	1	2	7	0	0
500	1	1	1	2	2	1	3	3	2	4	4	3	5	5	3	6	6	3	7	8	0	8	9	0
600	1	3	3	2	7	2	3	11	2	5	3	0	6	6	3	7	10	3	9	2	1	10	6	0
700	1	6	1	3	0	3	4	7	1	6	1	2	7	8	0	9	2	1	10	8	3	12	3	1
800	1	9	0	3	6	0	5	3	0	7	0	0	8	9	1	10	6	0	12	3	1	14	0	1
900	1	11	3	3	11	2	5	11	0	7	10	3	9	10	1	11	10	0	13	9	2	15	9	1
1000	2	2	1	4	4	3	6	6	3	8	9	1	10	11	2	13	1	3	15	4	0	17	6	1
ditto	*l.* .10957			*l.* .21913			*l.* .32865			*l.* .43816			*l.* .54764			*l.* .6571			*l.* .76653			*l.* .87594		

A TABLE *of* DISCOUNT *at 4 per Cent.*

Principal.	9 Days.			10 Days.				20 Days.				30 Days.				40 Days.				50 Days.				60 Days.			
l.	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			1				1			0	2				3			1	0			1	1			1	2
2			2				2			1	0			1	2			2	0			2	2			3	1
3			3				3			1	2			2	1			3	1			4	0			4	3
4		1	0			1	0			2	1			3	1			4	1			5	1			6	1
5		1	1			1	1			2	3			4	0			5	1			6	2			7	3
6		1	2			1	2			3	1			4	3			6	1			7	3			9	2
7		1	3			1	3			3	3			5	2			7	1			9	1			11	0
8		2	0			2	1			4	1			6	1			8	1			10	2		1	0	2
9		2	1			2	2			4	3			7	0			9	2			11	3		1	2	0
10		2	2			2	3			5	1			7	3			10	2		1	1	0		1	3	3
20		4	3			5	1			10	2		1	3	3		1	9	0		2	2	1		2	7	1
30		7	0			7	3		1	3	3		1	11	3		2	7	2		3	3	1		3	11	0
40		9	2			10	2		1	9	0		2	7	2		3	6	0		4	4	1		5	2	3
50	1	0	0		1	1	0		2	2	1		3	3	1		4	4	2		5	5	1		6	6	2
60	1	2	1		1	3	3		2	7	2		3	11	1		5	2	3		6	6	2		7	10	0
70	1	4	2		1	6	1		3	0	3		4	7	0		6	1	1		7	7	2		9	1	3
80	1	7	0		1	9	0		3	6	0		5	3	0		7	0	0		8	8	2		10	5	2
90	1	9	1		1	11	3		3	11	1		5	10	3		7	10	1		9	9	3		11	9	0
100	1	11	3		2	2	1		4	4	2		6	6	2		8	8	3		10	10	3		13	0	3
200	3	11	2		4	4	2		8	9	0		13	1	1		17	5	2	1	1	9	2	1	6	1	2
300	5	11	0		6	6	3		13	1	2		19	8	0	1	6	2	1	1	12	8	2	1	19	2	1
400	7	10	3		8	9	0		17	6	0	1	6	2	2	1	14	11	0	2	3	7	1	2	12	3	0
500	9	10	1		10	11	2	1	1	10	2	1	12	9	1	2	3	7	3	2	14	6	0	3	5	4	0
600	11	10	0		13	1	2	1	6	3	0	1	19	4	0	2	12	4	2	3	5	4	3	3	18	4	3
700	13	9	2		15	4	0	1	10	7	1	2	5	10	2	3	1	1	1	3	16	3	2	4	11	5	2
800	15	9	1		17	6	0	1	15	0	0	2	12	5	0	3	9	10	0	4	7	2	1	5	4	6	1
900	17	8	3		19	8	2	1	19	4	2	2	18	11	3	3	18	6	3	4	18	1	0	5	17	7	0
1000	19	8	2	1	1	10	3	2	3	8	3	3	5	6	2	4	7	3	2	5	9	0	0	6	10	7	3
ditto	*l.* .98532			*l.* 1.09469				*l.* 2.18699				*l.* 3.27689				*l.* 4.36443				*l.* 5.44959				*l.* 6.53239			

A TABLE of DISCOUNT at 4 per Cent. per Ann.

Prin-cipal.	70 Days.				80 Days.				90 Days.				100 Days.				110 Days.				120 Days.			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			1	3			2	0			2	1			2	3			3	0			3	1
2			3	3			4	1			4	3			5	1			5	3			6	1
3			4	2			6	1			7	0			7	3			8	2			9	1
4			7	1			8	1			9	2			10	2			11	2		1	0	2
5			9	1			10	2			11	3		1	1	0		1	2	1		1	3	2
6			11	0		1	0	2		1	2	0		1	3	3		1	5	1		1	6	3
7		1	0	3		1	2	2		1	4	2		1	6	1		1	8	0		1	9	3
8		1	2	2		1	4	3		1	6	3		1	8	3		1	11	0		2	1	0
9		1	4	2		1	6	3		1	9	0		1	11	2		2	1	3		2	4	0
10		1	6	1		1	8	3		1	11	2		2	2	0		2	4	3		2	7	1
20		3	0	2		3	5	3		3	11	0		4	4	1		4	9	1		5	2	1
30		4	6	3		5	2	2		5	10	1		6	6	0		7	1	3		7	9	2
40		6	1	0		6	11	2		7	9	3		8	8	0		9	6	1		10	4	3
50		7	7	1		8	9	1		9	9	1		10	10	0		11	11	0		12	11	3
60		9	1	2		10	5	1		11	8	3		13	0	0		14	3	2		15	7	0
70		10	7	3		12	2	0		13	8	0		15	0	2		16	8	0		18	2	0
80		12	2	1		13	11	0		15	7	2		17	4	1		19	0	2	1	0	9	1
90		13	8	2		15	7	3		17	7	0		19	6	1	1	1	5	2	1	3	4	2
100		15	2	3		17	4	3		19	6	2	1	1	8	1	1	3	10	0	1	5	11	2
200	1	10	5	2	1	14	9	1	1	19	0	3	2	3	4	2	2	7	7	3	2	11	11	1
300	2	5	8	0	2	12	1	3	2	18	7	1	3	5	0	2	3	11	5	3	3	17	10	3
400	3	0	11	0	3	9	6	1	3	18	1	2	4	6	8	3	4	15	3	2	5	3	10	1
500	3	16	1	2	4	6	11	0	4	17	8	1	5	8	5	0	5	19	1	1	6	9	9	3
600	4	11	4	1	5	4	3	2	5	17	2	2	6	10	1	0	7	2	11	1	7	15	9	1
700	5	6	7	0	6	1	8	0	6	1	8	3	7	11	9	1	8	6	9	0	9	1	8	3
800	6	1	9	3	6	19	0	3	7	16	3	1	8	13	5	2	9	10	7	0	10	7	8	1
900	5	17	0	1	7	16	5	1	8	15	9	2	9	15	1	2	10	14	5	0	11	13	7	3
1000	7	12	3	0	8	13	9	3	9	15	4	9	10	16	9	3	11	18	2	3	12	19	7	1
ditto	*l.* 7.61283				*l.* 8.69092				*l.* 9.76668				*l.* 10.84011				*l.* 11.9112				*l.* 12.98			

A TABLE *of* DISCOUNT *at* 4 *per Cent.* per Ann.

Principal.	130 Days.				140 Days.				150 Days.				160 Days.				170 Days.				180 Days.			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			3	2			3	3			4	0			4	1			4	2			4	3
2			6	3			7	1			7	3			8	1			8	3			9	1
3			10	1			11	0			11	3		1	0	1		1	1	1		1	2	0
4		1	1	2		1	2	2		1	2	2		1	4	2		1	5	2		1	6	2
5		1	5	0		1	6	0		1	7	2		1	8	3		1	10	0		1	11	1
6		1	8	1		1	9	3		1	11	1		2	0	3		2	2	2		2	4	0
7		1	11	3		2	1	2		2	3	1		2	5	0		2	6	3		2	8	2
8		2	3	0		2	5	0		2	7	0		2	9	0		2	11	1		3	1	0
9		2	6	1		2	8	3		2	11	0		3	1	1		3	3	2		3	5	3
10		2	9	3		3	0	1		3	2	3		3	5	2		3	7	3		3	10	2
20		5	7	2		6	0	2		6	5	2		6	10	3		7	3	3		7	8	3
30		8	5	1		9	0	3		9	8	2		10	4	1		10	11	3		11	7	1
40		11	2	3		12	1	0		12	11	1		13	9	2		14	7	2		15	5	3
50		14	0	2		15	1	1		16	2	0		17	2	3		18	3	2		19	4	1
60		16	10	1		18	1	2		19	5	0	1	0	8	0	1	1	11	2	1	3	2	2
70		19	8	0	1	1	2	0	1	2	7	3	1	4	1	2	1	5	7	1	1	7	1	0
80	1	2	5	3	1	4	2	1	1	5	10	2	1	7	6	3	1	9	3	1	1	10	11	2
90	1	5	3	2	1	7	2	2	1	9	1	1	1	11	0	1	1	12	11	1	1	14	9	3
100	1	8	1	0	1	10	2	3	1	12	4	1	1	14	5	3	1	16	7	0	1	18	8	1
200	2	16	2	1	3	0	5	2	3	4	8	1	3	8	11	1	3	13	1	3	3	17	4	2
300	4	4	3	1	4	10	8	1	4	17	0	2	5	3	4	3	5	9	8	3	5	16	0	3
400	5	12	4	2	6	0	11	0	6	9	4	3	6	17	10	1	7	6	3	3	7	14	9	0
500	7	0	5	2	7	11	1	2	8	1	8	3	8	12	4	0	9	2	10	3	9	13	5	2
600	8	8	6	3	9	1	4	1	9	14	0	3	10	6	9	2	10	19	5	3	11	12	1	2
700	9	16	7	3	10	11	7	0	11	6	5	0	12	1	3	0	12	16	0	2	13	10	10	0
800	11	4	9	0	12	1	9	3	12	18	9	1	13	15	8	2	14	12	7	2	15	9	6	0
900	12	12	10	0	13	12	0	2	14	11	1	1	15	10	2	0	16	9	2	2	17	8	2	2
1000	14	0	11	1	15	2	3	1	16	3	5	2	17	4	7	3	18	5	9	2	19	6	10	3
ditto	*l.* 14.04646				*l.* 15.11333				*l.* 16.17251				*l.* 17.23209				*l.* 18.2894				*l.* 19.34444			

A TABLE of DISCOUNT at 4 per Cent. per Ann.

Principal.	190 Days.				200 Days.				210 Days.				220 Days.				230 Days.				240 Days.			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			5	0			5	1			5	2			5	3			6	0			6	1
2			9	3			10	1			10	3			11	2		1	0	0		1	0	1
3		1	2	3		1	3	2		1	4	1		1	5	0		1	5	3		1	6	2
4		1	7	2		1	8	2		1	9	2		1	10	3		1	11	3		2	0	2
5		2	0	2		2	1	3		2	3	0		2	4	1		2	5	2		2	6	3
6		2	5	2		2	6	3		2	8	1		2	10	0		2	11	2		3	1	0
7		2	10	1		3	0	0		3	1	3		3	3	2		3	5	2		3	7	0
8		3	3	1		3	5	1		3	7	1		3	9	1		3	11	1		4	1	1
9		3	8	0		3	10	2		4	0	2		4	2	3		4	5	1		4	7	1
10		4	1	0		4	3	2		4	6	0		4	8	2		4	11	0		5	1	2
20		8	2	0		8	7	0		9	0	0		9	5	0		9	10	0		10	3	0
30		12	2	3		12	10	2		13	6	0		14	1	2		14	9	0		15	4	2
40		16	3	3		17	2	0		18	0	0		18	10	0		19	8	0	1	0	6	0
50	1	0	4	3	1	1	5	1	1	2	6	0	1	3	6	2	1	4	7	0	1	5	7	2
60	1	4	5	3	1	5	8	3	1	7	0	0	1	8	3	0	1	9	6	0	1	10	9	0
70	1	8	6	3	1	10	0	1	1	11	6	0	1	12	11	2	1	14	5	0	1	15	10	2
80	1	12	7	2	1	14	3	3	1	16	0	0	1	17	8	0	1	19	4	0	2	1	0	0
90	1	16	8	2	1	18	7	1	2	0	6	0	2	2	4	2	2	4	3	0	2	6	1	2
100	2	0	9	2	2	2	10	3	2	5	0	0	2	7	1	0	2	9	2	0	2	11	3	0
200	4	1	7	0	4	5	9	2	4	9	11	3	4	14	2	0	4	18	4	1	5	2	6	0
300	6	2	4	3	6	8	8	1	6	14	11	3	7	1	3	0	7	7	6	1	7	13	9	1
400	8	3	2	0	8	11	7	0	8	19	11	3	9	8	4	0	9	16	8	2	10	5	0	0
500	10	3	11	3	10	14	5	3	11	4	11	2	11	15	5	0	12	5	10	1	12	16	3	1
600	12	4	9	1	12	17	4	2	13	9	11	2	14	2	6	0	14	15	0	1	15	7	6	1
700	14	5	6	3	15	0	3	1	15	14	11	2	16	9	7	0	17	4	2	2	17	18	9	2
800	16	6	4	1	17	3	2	0	17	19	11	1	18	16	8	0	19	13	4	2	20	10	0	2
900	18	7	1	3	19	6	0	3	20	4	11	1	21	3	9	0	22	2	6	2	23	1	3	2
1000	20	7	11	2	21	8	11	2	22	9	11	0	23	10	10	0	24	11	8	2	25	12	6	2
ditto	*l.* 20.3972				*l.* 21.44772				*l.* 22.49598				*l.* 23.542				*l.* 24.58578				*l.* 25.62733			

A TABLE of DISCOUNT at 4 *per Cent.* per Ann.

Principal.	250 Days.				260 Days.				270 Days.				280 Days.				290 Days.				300 Days.			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			6	2			6	3			7	0			7	0			7	1			7	2
2		1	0	3		1	1	1		1	1	3		1	2	1		1	2	3		1	3	1
3		1	7	1		1	8	0		1	8	3		1	9	2		1	10	1		1	11	0
4		2	1	2		2	2	2		2	3	2		2	4	3		2	5	2		2	6	2
5		2	8	0		2	9	1		2	10	2		2	11	3		3	1	0		3	2	1
6		3	2	2		3	4	0		3	5	2		3	7	0		3	8	1		3	9	3
7		3	8	3		3	10	2		4	0	1		4	2	0		4	3	3		4	5	2
8		4	3	1		4	5	1		4	7	1		4	9	1		4	11	1		5	1	0
9		4	9	2		5	0	0		5	2	0		5	4	1		5	6	2		5	8	3
10		5	4	0		5	6	2		5	9	0		5	11	2		6	2	0		6	4	2
20		10	8	0		11	1	0		11	6	0		11	11	0		12	4	0		12	8	3
30		16	0	0		16	7	2		17	3	0		17	10	2		18	5	3		19	1	1
40	1	1	4	0	1	2	2	0	1	3	0	0	1	3	9	3	1	4	7	3	1	5	5	2
50	1	6	8	0	1	7	8	2	1	8	8	3	1	9	9	1	1	10	9	2	1	11	10	0
60	1	12	0	0	1	13	3	0	1	14	5	3	1	15	8	3	1	16	11	3	1	18	2	1
70	1	17	4	0	1	18	9	2	2	0	2	3	2	1	8	1	2	3	1	2	2	4	6	3
80	2	2	8	0	2	4	4	0	2	5	11	3	2	7	7	2	2	9	3	2	2	10	11	1
90	2	8	0	0	2	9	10	2	2	11	8	3	2	13	7	0	2	15	5	1	2	17	3	2
100	2	13	4	0	2	15	5	0	2	17	5	3	2	19	6	2	3	1	7	1	3	3	8	0
200	5	6	8	0	5	10	9	3	5	14	11	2	5	19	1	0	6	3	2	2	6	7	4	0
300	8	0	0	0	8	6	2	3	8	12	5	1	8	18	7	2	9	4	9	3	9	10	11	3
400	10	13	4	0	11	1	7	2	11	9	11	0	11	18	2	0	12	6	5	0	12	14	7	3
500	13	6	8	0	13	17	0	2	14	7	4	3	14	17	8	2	15	8	0	1	15	18	3	3
600	16	0	0	0	16	12	5	2	17	4	10	2	17	17	3	0	18	9	7	2	19	1	11	3
700	18	13	4	0	19	7	10	1	20	2	4	1	20	16	9	2	21	11	2	3	22	5	7	2
800	21	6	8	0	22	3	3	1	22	19	9	3	23	16	4	1	24	12	10	0	25	9	3	2
900	24	0	0	0	24	18	8	0	25	17	3	2	26	15	10	3	27	14	5	1	28	12	11	2
1000	26	13	4	0	27	14	1	0	28	14	9	1	29	15	5	1	30	16	0	2	31	16	7	1
ditto	*l.* 26.6 r				*l.* 27.70378				*l.* 28.73853				*l.* 29.77139				*l.* 30.8019				*l.* 31.83024			

A TABLE *of* DISCOUNT *at* 4 *per Cent.* per Ann.

Principal. l.	310 Days. l.	s.	d.	q.	320 Days. l.	s.	d.	q.	330 Days. l.	s.	d.	q.
1			7	3			8	0			8	2
2		1	3	3		1	4	2		1	4	3
3		1	11	3		2	0	1		2	1	1
4		2	7	2		2	8	2		2	9	2
5		3	3	2		3	4	3		3	6	0
6		3	11	2		4	0	3		4	2	1
7		4	7	1		4	9	0		4	10	3
8		5	3	1		5	5	1		5	7	0
9		5	11	0		6	1	1		6	3	2
10		6	6	3		6	9	1		6	11	3
20		13	1	3		13	6	3		13	11	2
30		19	8	2	1	0	4	0	1	0	11	1
40	1	6	3	2	1	7	1	1	1	7	11	1
50	1	12	10	2	1	13	10	3	1	14	11	0
60	1	19	5	1	2	0	7	3	2	1	10	3
70	2	6	0	0	2	7	5	1	2	8	10	2
80	2	12	7	0	2	14	2	2	2	15	10	1
90	2	19	1	3	3	0	11	3	3	2	10	0
100	3	5	8	3	3	7	9	1	3	9	9	3
200	6	11	5	2	6	15	6	1	6	19	7	1
300	9	17	2	0	10	3	3	2	10	9	5	0
400	13	2	11	0	13	11	0	2	13	19	2	2
500	16	8	7	2	16	18	9	3	17	9	0	1
600	19	14	4	1	20	6	6	3	20	18	10	0
700	23	0	1	0	23	14	4	0	24	8	7	2
800	26	5	9	3	27	2	1	0	27	18	5	1
900	29	11	6	1	30	9	10	1	31	8	2	3
1000	32	17	3	1	33	17	7	1	34	18	0	2
ditto	l. 32.86275				l. 33.88038				l. 34.90216			

Principal. l.	340 Days. l.	s.	d.	q.	350 Days. l.	s.	d.	q.	360 Days. l.	s.	d.	q.
1			8	3			9	0			9	1
2		1	5	1		1	5	3		1	6	1
3		2	1	2		2	2	2		2	3	1
4		2	10	2		2	11	2		3	0	2
5		3	7	0		3	8	1		3	9	2
6		4	3	2		4	5	1		4	6	3
7		5	0	1		5	2	0		5	3	3
8		5	9	0		5	11	0		6	0	3
9		6	5	2		6	7	3		6	10	0
10		7	2	1		7	4	3		7	7	0
20		14	4	2		14	9	1		15	2	1
30	1	1	6	2	1	2	2	0	1	2	9	1
40	1	8	8	2	1	9	6	2	1	10	4	2
50	1	15	11	0	1	16	11	1	1	17	11	2
60	2	3	1	1	2	4	4	0	2	5	6	2
70	2	10	3	2	2	11	8	2	2	13	1	3
80	2	17	5	3	2	19	1	1	3	0	8	3
90	3	4	8	0	3	6	6	0	3	8	4	0
100	3	11	10	1	3	13	10	2	3	15	11	0
200	7	3	8	1	7	7	9	0	7	11	9	3
300	10	15	6	1	11	1	7	2	11	7	8	3
400	14	7	4	2	14	15	6	1	15	3	7	2
500	17	19	2	2	18	9	4	3	18	19	6	2
600	21	11	0	3	22	3	3	1	22	15	5	2
700	25	2	11	0	25	17	1	3	26	11	4	2
800	28	14	9	0	29	11	0	1	30	7	3	1
900	32	6	7	0	33	4	11	0	34	3	2	1
1000	35	18	5	1	36	18	9	2	37	19	1	0
ditto	l. 35.92181				l. 36.93931				l. 37.95466			

Discount at 4 per Cent.

Part II. For Days under 10 in a Medium, to add at above 90 Days.

Principal.	365 Days. or 1 Year.				1 Day.			2 Days.			3 Days.			4 Days.		
l.	l.	s.	d.	q.	s.	d.	q.	s.	d.	q.	s.	d.	q.	s.	d.	q.
1			9	1												
2		1	6	2									1			1
3		2	3	3						1			1			1
4		3	1	0						1			1			1
5		3	10	1						1			1			2
6		4	7	1			1			1			2			2
7		5	4	3			1			1			2			3
8		6	1	3			1			2			2			3
9		6	11	1			1			2			3		1	0
10		7	8	1			1			2			3		1	0
20		15	4	3			2		1	0		1	2		2	0
30	1	3	1	0			3		1	2		2	1		3	0
40	1	10	9	1		1	0		2	0		3	0		4	0
50	1	18	5	2		1	1		2	2		3	3		5	0
60	2	6	1	3		1	2		3	0		4	2		6	0
70	2	13	10	1		1	3		3	2		5	1		7	0
80	3	1	6	2		2	0		4	0		6	0		8	0
90	3	9	2	3		2	1		4	2		6	3		9	0
100	3	16	11	1		2	2		5	0		7	2		10	0
200	7	13	10	1		5	0		10	1	1	3	0	1	8	0
300	11	10	9	1		7	2	1	3	0	1	10	3	2	6	0
400	15	7	8	1		10	0	1	8	0	2	6	0	3	4	1
500	19	4	7	1	1	0	2	2	1	0	3	1	3	4	2	1
600	23	1	6	2	1	3	0	2	6	0	3	9	1	5	0	1
700	26	18	5	2	1	5	2	2	11	1	4	4	3	5	10	1
800	30	15	4	3	1	8	0	3	4	1	5	0	1	6	8	1
900	34	12	3	3	1	10	3	3	9	1	5	7	3	7	6	2
1000	38	9	2	3	2	1	1	4	2	1	6	3	2	8	4	2
ditto	l. 38.46154				l. .10467			l. .09352			l. .31401			l. .41868		

Part II. For Days under 10 in a Medium, to add at above 90 Days.

Principal.	5 Days.			6 Days.			7 Days.			8 Days.			9 Days.		
l.	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*
1						1			1			1			1
2			1			1			1			1			2
3			1			2			2			2			3
4			2			2			3			3		1	0
5			2			3			3		1	0		1	1
6			3		1	0		1	0		1	1		1	1
7			3		1	0		1	1		1	2		1	2
8		1	0		1	1		1	2		1	2		1	3
9		1	1		1	1		1	2		1	3		2	0
10		1	1		1	2		1	3		2	0		2	1
20		2	2		3	0		3	2		4	0		4	2
30		3	3		4	2		5	1		6	0		6	3
40		5	1		6	0		7	0		8	0		9	0
50		6	1		7	2		8	3		10	0		11	2
60		7	2		9	0		10	2	1	0	0	1	1	2
70		8	3		10	2	1	0	1	1	2	0	1	3	3
80		10	0	1	0	0	1	2	0	1	4	0	1	6	0
90		11	2	1	1	2	1	3	3	1	6	0	1	8	1
100	1	0	0	1	3	0	1	5	2	1	8	0	1	10	3
200	2	1	0	2	6	0	2	11	1	3	4	1	3	9	1
300	3	1	3	3	9	1	4	4	3	5	0	1	5	7	3
400	4	2	1	5	0	1	5	10	2	6	8	1	7	6	2
500	5	2	3	6	3	2	7	4	0	8	4	2	9	5	0
600	6	3	2	7	6	2	8	9	2	10	0	2	11	3	3
700	7	4	0	8	9	2	10	3	0	11	8	3	13	2	1
800	8	4	2	10	0	2	11	8	3	13	4	3	15	0	3
900	9	5	0	11	3	3	13	2	1	15	0	3	16	11	2
1000	10	5	2	12	6	3	14	7	3	16	9	0	18	10	0
ditto	*l.* .52335			*l.* .62802			*l.* .73269			*l.* .83737			*l.* .94204		

A TABLE *of* DISCOUNT *at* 5 *per Cent.* per Ann.

Principal.	1 Day.			2 Days.			3 Days.			4 Days.			5 Days.			6 Days.			7 Days.		
l.	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*
1																		1			1
2									1			1			1			1			2
3						1			1			1			2			2			3
4						1			1			2			2			3			3
5						1			2			2			3		1	0		1	0
6			1			1			2			3		1	0		1	1		1	1
7			1			2			3			3		1	1		1	1		1	2
8			1			2			3		1	0		1	1		1	2		1	3
9			1			2			3		1	1		1	2		1	3		2	0
10			1			3		1	0		1	1		1	3		2	0		2	1
20			3		1	1		2	0		2	3		3	1		4	0		4	2
30		1	0		2	0		3	0		4	0		5	0		5	3		6	3
40		1	1		2	2		4	0		5	1		6	2		7	3		9	1
50		1	3		3	1		5	0		6	2		8	1		9	3		11	2
60		2	0		4	0		6	0		7	3		10	0		11	3	1	1	3
70		2	1		4	3		6	3		9	1		11	2	1	1	3	1	4	1
80		2	3		5	1		7	3		10	2	1	1	0	1	3	3	1	6	1
90		3	0		6	0		8	3		11	3	1	2	3	1	5	3	1	8	3
100		3	1		6	2		9	3	1	1	0	1	4	2	1	7	3	1	11	0
200		6	2	1	1	0	1	7	3	2	2	1	2	8	3	3	3	2	3	10	0
300		10	0	1	7	3	2	5	3	3	3	2	4	1	1	4	11	0	5	9	0
400	1	1	1	2	2	1	3	3	2	4	4	3	5	5	3	6	6	3	7	8	0
500	1	4	2	2	8	3	4	1	1	5	5	3	6	10	0	8	2	2	9	7	0
600	1	7	3	3	3	2	4	11	1	6	6	3	8	2	2	9	10	1	11	6	0
700	1	11	0	3	10	0	5	9	0	7	8	0	9	7	0	11	6	0	13	5	0
800	2	2	1	4	4	3	6	6	3	8	9	1	10	11	2	13	1	2	15	4	0
900	2	5	3	4	11	1	7	4	3	9	10	1	12	3	3	14	9	1	17	3	0
1000	2	8	3	5	5	3	8	2	2	10	11	2	13	8	1	16	5	1	19	2	0
ditto	*l.* .1369			*l.* .2739			*l.* .4108			*l.* .5476			*l.* .6848			*l.* .8212			*l.* .9579		

A TABLE *of* DISCOUNT *at* 5 *per Cent.* per Ann.

Principal. l.	8 Days. l.	s.	d.	q.	9 Days. l.	s.	d.	q.	10 Days. l.	s.	d.	q.
1				1				1				1
2				2				2				2
3				3				3			1	0
4			1	0			1	1			1	1
5			1	1			1	2			1	3
6			1	2			1	3			2	0
7			1	3			2	0			2	1
8			2	0			2	1			2	3
9			2	1			2	3			3	0
10			2	3			3	0			3	1
20			5	1			5	3			6	2
30			7	3			8	3			10	0
40			10	2			11	3		1	1	0
50		1	1	0		1	2	3		1	4	2
60		1	3	3		1	5	3		1	7	3
70		1	6	1		1	8	3		1	11	0
80		1	9	0		1	11	3		2	2	1
90		1	11	3		2	2	2		2	5	2
100		2	2	1		2	5	2		2	8	3
200		4	4	2		4	11	0		5	5	3
300		6	6	3		7	4	3		8	2	2
400		8	9	0		9	10	1		10	11	2
500		10	11	2		12	3	3		13	8	0
600		13	1	2		14	9	1		16	5	0
700		15	4	0		17	2	3		19	1	3
800		17	6	0		19	8	2	1	1	10	3
900		19	8	2	1	2	2	0	1	4	7	2
1000	1	1	10	3	1	4	7	2	1	7	4	1
ditto	*l.* 1.0947				*l.* 1.2313				*l.* 1.3679			

Principal. l.	20 Days. l.	s.	d.	q.	30 Days. l.	s.	d.	q.	40 Days. l.	s.	d.	q.
1				3			1	0			1	1
2			1	1			2	0			2	2
3			2	0			3	0			4	0
4			2	3			4	0			5	1
5			3	1			5	0			6	2
6			4	0			6	0			7	3
7			4	3			6	3			9	1
8			5	1			7	3			10	2
9			6	0			8	3			11	3
10			6	2			9	3		1	1	0
20		1	1	0		1	7	2		2	2	0
30		1	7	2		2	5	2		3	3	1
40		2	2	1		3	3	1		4	4	1
50		2	8	3		4	1	0		5	5	1
60		3	3	1		4	11	0		6	6	2
70		3	10	0		5	8	3		7	7	2
80		4	4	2		6	6	2		8	8	2
90		4	10	0		7	4	2		9	9	3
100		5	5	2		8	2	1		10	10	3
200		10	11	1		16	4	2	1	1	9	2
300		16	4	3	1	4	6	3	1	12	8	1
400	1	1	10	1	1	12	9	0	2	3	7	0
500	1	7	4	0	2	0	11	0	2	14	6	0
600	1	12	9	2	2	9	1	1	3	5	5	0
700	1	18	3	0	2	17	3	2	3	16	3	3
800	2	3	8	2	3	5	5	3	4	7	2	1
900	2	9	2	1	3	13	8	0	4	18	1	0
1000	2	14	7	3	4	1	10	1	5	9	0	0
ditto	*l.* 2.7322				*l.* 4.0925				*l.* 5.4496			

A TABLE *of* DISCOUNT *at* 5 *per Cent.* per Ann.

Principal.	50 Days.				60 Days.				70 Days.				80 Days.				90 Days.				100 Days.			
l.	l.	s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.
1			1	3			2	0			2	1			2	2			3	0			3	1
2			3	1			4	0			4	3			5	1			5	3			6	2
3			5	0			6	0			6	3			7	3			9	3			9	3
4			6	2			7	3			9	0			10	2			11	3		1	1	0
5			8	1			9	3			11	2		1	1	0		1	2	2		1	4	1
6			9	3			11	3		1	1	3		1	3	3		1	5	2		1	7	2
7			11	2		1	1	3		1	4	0		1	6	1		1	8	2		1	10	3
8		1	1	0		1	3	3		1	6	1		1	8	3		1	11	2		2	2	0
9		1	2	3		1	5	2		1	8	2		1	11	2		2	2	1		2	5	1
10		1	4	2		1	7	2		1	10	3		2	2	0		2	5	1		2	8	2
20		2	8	3		3	3	1		3	9	2		4	4	0		4	10	2		5	5	0
30		4	1	0		4	10	3		5	8	1		6	6	0		7	3	3		8	1	2
40		5	5	1		6	6	1		7	7	1		8	8	0		9	9	0		10	10	1
50		6	9	3		8	1	3		9	6	0		10	10	0		12	2	1		13	6	2
60		8	2	0		9	9	2		11	5	0		13	0	0		14	7	1		16	3	1
70		9	6	1		11	5	0		13	3	2		15	2	0		17	0	2		18	11	3
80		10	10	3		13	0	2		15	2	1		17	4	1		19	5	3	1	1	8	1
90		12	3	0		14	8	0		17	1	1		19	6	1	1	1	11	0	1	4	5	0
100		13	7	1		16	3	3		19	0	0	1	1	8	1	1	4	4	1	1	7	1	1
200	1	7	2	2	1	12	7	1	1	18	0	0	2	3	4	2	2	8	8	2	2	14	2	3
300	2	0	9	3	2	8	11	0	2	16	11	3	3	5	0	2	3	13	0	3	4	1	4	0
400	2	14	5	1	3	5	2	2	3	15	11	3	4	6	8	3	4	17	5	1	5	8	5	2
500	3	8	0	1	4	1	6	1	4	14	11	3	5	8	4	3	6	1	9	2	6	15	6	2
600	4	1	7	2	4	17	10	0	5	13	11	3	6	10	1	0	7	6	1	3	8	2	8	0
700	4	15	2	3	5	14	1	2	6	12	11	3	7	11	9	1	8	10	6	0	9	9	9	1
800	5	8	10	1	6	10	5	1	7	11	11	2	8	13	5	1	9	14	10	1	10	16	10	3
900	6	2	5	2	7	6	8	3	8	10	11	2	9	15	1	2	10	19	2	2	12	4	0	0
1000	6	16	0	3	8	3	0	2	9	9	11	2	10	16	9	3	12	3	7	0	13	10	1	1
ditto	*l.* 6.8027				*l.* 8.1521				*l.* 9.4976				*l.* 10.8401				*l.* 12.1786				*l.* 13.5 *r.*			

A TABLE *of* DISCOUNT *at* 5 *per Cent.* per Ann.

Principal. l.	110 Days.				120 Days.				130 Days.				140 Days.				150 Days.				160 Days.			
	l.	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			3	2			4	0			4	1			4	2			5	0			5	1
2			7	0			7	3			8	2			9	0			9	3			10	1
3			10	3			11	3		1	0	2		1	1	2		1	2	2		1	3	2
4		1	2	1		1	2	2		1	4	3		1	6	0		1	7	1		1	8	2
5		1	5	3		1	7	2		1	9	0		1	10	2		2	0	0		2	1	3
6		1	9	1		1	11	2		2	1	1		2	3	1		2	5	0		2	6	3
7		2	0	3		2	3	1		2	5	2		2	7	2		2	9	3		3	0	0
8		2	4	2		2	7	0		2	9	2		3	0	0		3	2	3		3	5	1
9		2	8	0		2	11	0		3	1	3		3	4	2		3	7	2		3	10	1
10		2	11	3		3	2	3		3	6	0		3	9	1		4	0	1		4	3	2
20		5	11	1		6	5	2		7	0	0		7	6	1		8	0	2		8	7	0
30		8	11	0		9	8	2		10	6	0		11	3	2		12	1	0		12	10	2
40		11	10	2		12	11	1		14	0	0		15	0	2		16	1	1		17	2	0
50		14	10	1		16	2	0		17	6	0		18	9	3	1	0	1	2	1	1	5	2
60		17	9	3		19	5	0	1	1	0	0	1	2	7	0	1	4	2	0	1	5	8	3
70	1	0	9	2	1	2	7	3	1	4	6	0	1	6	4	1	1	8	2	1	1	10	0	1
80	1	3	9	0	1	5	10	2	1	8	0	0	1	10	1	1	1	12	2	2	1	14	3	3
90	1	6	8	3	1	9	1	1	1	11	6	0	1	13	10	2	1	16	3	0	1	18	7	1
100	1	9	8	1	1	12	4	1	1	15	0	0	1	17	7	2	2	0	3	1	2	2	10	3
200	2	19	4	3	3	4	8	1	3	9	11	3	3	15	3	1	4	0	6	2	4	5	9	2
300	4	9	0	3	4	17	0	1	5	4	11	3	5	12	11	0	6	0	9	3	6	8	8	1
400	5	18	9	0	6	9	4	3	6	19	11	3	7	10	6	2	8	1	0	3	8	11	7	0
500	7	8	5	1	8	1	8	3	8	14	11	2	9	8	2	0	10	1	4	1	10	14	5	3
600	8	18	1	2	9	14	0	3	10	9	11	2	11	5	9	3	12	1	7	1	12	17	4	2
700	10	7	10	0	11	6	5	0	12	4	11	2	13	3	5	1	14	1	10	2	15	0	3	1
800	11	17	6	1	12	18	9	1	13	19	11	2	15	1	1	0	16	2	1	3	17	3	2	0
900	13	7	2	2	14	11	1	1	15	14	11	1	16	18	8	2	18	2	5	0	19	6	0	3
1000	14	16	10	3	16	3	5	2	17	9	11	1	18	16	4	1	20	2	8	1	21	8	11	2
ditto	*l.* 14.8448				*l.* 16.1725				*l.* 17.4966				*l.* 18.8172				*l.* 20.1342				*l.* 21.4477			

A TABLE of DISCOUNT at 5 per Cent. per Ann.

Principal. l.	170 Days. l.	s.	d.	q.	180 Days. l.	s.	d.	q.	190 Days. l.	s.	d.	q.
1			5	2			5	3			6	0
2			11	0			11	2		1	0	0
3		1	4	2		1	5	1		1	6	1
4		1	10	0		1	11	0		2	0	1
5		2	3	1		2	5	0		2	6	2
6		2	8	3		2	10	3		3	0	2
7		3	2	1		3	4	2		3	6	3
8		3	7	3		3	10	1		4	0	3
9		4	1	0		4	4	0		4	6	3
10		4	6	3		4	9	3		5	1	0
20		9	1	1		9	7	2		10	1	3
30		13	7	3		14	5	1		15	2	3
40		18	2	2		19	3	0	1	0	3	2
50	1	2	9	0	1	4	0	3	1	5	4	2
60	1	7	3	3	1	8	10	2	1	10	5	2
70	1	11	10	0	1	13	8	1	1	15	6	1
80	1	16	5	0	1	18	6	0	2	0	7	0
90	2	0	11	2	2	3	3	3	2	5	8	0
100	2	5	6	1	2	8	1	2	2	10	8	3
200	4	11	0	1	4	16	3	0	5	1	5	2
300	6	16	6	2	7	4	4	3	7	12	2	2
400	9	2	0	3	9	12	6	1	10	2	11	1
500	11	7	7	0	12	0	7	3	12	13	8	0
600	13	13	1	0	14	8	9	1	15	4	5	0
700	15	18	7	1	16	16	10	3	17	15	1	3
800	18	4	1	2	19	5	0	1	20	5	10	2
900	20	9	7	3	21	13	1	3	22	16	7	1
1000	22	15	1	3	24	1	3	2	25	7	4	1
ditto	l. 22.7577				l. 24.06420				l. 25.36710			

Principal. l.	200 Days. l.	s.	d.	q.	210 Days. l.	s.	d.	q.	220 Days. l.	s.	d.	q.
1			6	2			6	3			7	0
2		1	0	3		1	1	2		1	2	0
3		1	7	1		1	8	1		1	9	0
4		2	1	2		1	2	3		2	4	1
5		2	8	1		1	9	2		2	11	0
6		3	2	2		3	4	2		3	6	1
7		3	8	3		3	11	0		4	1	1
8		4	3	1		4	5	3		4	8	1
9		4	9	3		5	0	1		5	3	1
10		5	4	0		5	7	0		5	10	1
20		10	8	0		11	2	1		11	8	2
30		16	0	0		16	9	1		17	6	2
40	1	1	4	0	1	2	4	2	1	3	5	0
50	1	6	8	0	1	7	11	3	1	9	3	0
60	1	12	0	0	1	13	6	3	1	15	1	1
70	1	17	4	0	1	19	1	3	2	0	11	2
80	2	2	8	0	2	4	9	0	2	6	9	3
90	2	8	0	0	2	10	4	0	2	12	8	0
100	2	13	4	0	2	15	11	1	2	18	6	0
200	5	6	8	0	5	11	10	1	5	17	0	1
300	8	0	0	0	8	7	9	1	8	15	6	2
400	10	13	4	0	11	3	8	2	11	14	0	2
500	13	6	8	0	13	19	7	2	14	12	6	2
600	16	0	0	0	16	15	6	3	17	11	0	3
700	18	13	4	0	19	11	5	3	20	9	7	0
800	21	6	8	0	22	7	2	2	23	8	1	0
900	24	0	0	0	25	3	4	0	26	6	7	0
1000	26	13	4	0	27	19	3	1	29	5	1	1
ditto	l. 26.6r1				l. 27.9627				l. 29.2553			

A TABLE *of* DISCOUNT *at* 5 *per Cent. per Ann.*

Principal.	230 Days.				240 Days.				250 Days.				260 Days.				270 Days.				280 Days.			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			7	1			7	2			8	0			8	1			8	2			8	3
2		1	2	3		1	3	1		1	4	0		1	4	2		1	5	1		1	5	3
3		1	10	0		1	11	0		1	11	3		2	0	3		2	1	3		2	2	2
4		2	5	1		2	6	2		2	7	3		2	9	0		2	10	1		2	11	2
5		3	0	3		3	2	1		3	3	3		3	5	1		3	6	3		3	8	1
6		3	8	0		3	9	3		3	11	3		4	1	2		4	3	2		4	5	1
7		4	3	1		4	5	2		4	7	2		4	9	3		5	0	0		5	2	0
8		4	10	2		5	1	0		5	3	2		5	6	0		5	8	2		5	11	0
9		5	6	0		5	8	3		5	11	2		6	2	1		6	5	0		6	7	3
10		6	1	1		6	4	2		6	7	2		6	10	2		7	1	2		7	4	3
20		12	2	2		12	8	3		13	3	0		13	9	0		14	3	1		14	9	1
30		18	4	0		19	1	1		19	10	2	1	0	7	2	1	1	5	0	1	2	2	0
40	1	4	5	1	1	5	5	2	1	6	6	0	1	7	6	0	1	8	6	2	1	9	6	3
50	1	10	6	2	1	11	10	0	1	13	1	1	1	14	4	3	1	15	8	0	1	16	11	1
60	1	16	7	3	1	18	2	1	1	19	8	3	2	1	3	1	2	2	9	2	2	4	4	0
70	2	2	9	1	2	4	6	3	2	6	4	1	2	8	1	3	2	9	11	1	2	11	8	2
80	2	8	10	2	2	10	11	1	2	12	11	3	2	15	0	1	2	17	0	3	2	19	1	1
90	2	14	11	3	2	17	3	2	2	19	7	1	3	1	11	0	3	4	2	2	3	6	6	0
100	3	1	1	0	3	3	8	0	3	6	2	3	3	8	9	2	3	11	4	0	3	13	10	2
200	6	2	2	0	6	7	4	0	6	12	5	2	6	17	6	3	7	2	8	0	7	7	9	0
300	9	3	3	1	9	10	11	3	9	18	8	0	10	6	4	1	10	14	0	0	11	1	7	2
400	12	4	4	2	12	14	7	3	13	4	10	3	13	15	1	2	14	5	4	0	14	15	6	0
500	15	5	5	1	15	18	3	3	16	11	1	2	17	3	6	1	17	16	8	0	18	9	4	3
600	18	6	6	2	19	1	11	2	19	17	4	1	20	12	8	1	21	8	0	0	22	3	3	1
700	21	7	7	2	22	5	7	2	23	3	7	0	24	1	5	3	24	19	4	0	25	17	1	3
800	24	8	8	2	25	9	3	2	26	9	9	2	27	10	3	1	28	10	8	0	29	11	0	1
900	27	9	9	2	28	12	11	1	29	16	0	1	30	19	0	2	32	2	0	0	33	4	11	0
1000	30	10	10	3	31	16	7	1	33	2	3	0	34	7	10	0	35	13	4	0	36	18	9	2
ditto	*l.* 30.5444				*l.* 31.8302				*l.* 33.1125				*l.* 34.3915				*l.* 35.6671				*l.* 36.9393			

A TABLE *of* DISCOUNT *at* 5 *per Cent.* per Ann.

Principal. *l.*	290 *Days.* *l.*	*s.*	*d.*	*q.*	300 *Days.* *l.*	*s.*	*d.*	*q.*	310 *Days.* *l.*	*s.*	*d.*	*q.*	320 *Days.* *l.*	*s.*	*d.*	*q.*	330 *Days.* *l.*	*s.*	*d.*	*q.*	340 *Days.* *l.*	*s.*	*d.*	*q.*
1			9	1			9	2			9	3			10	0			10	2			10	3
2		1	6	1		1	7	0		1	7	2		1	8	0		1	8	3		1	9	1
3		2	3	2		2	4	2		2	5	1		2	6	1		2	7	1		2	8	0
4		3	0	3		3	1	3		3	3	0		3	4	2		3	5	2		3	6	3
5		3	10	0		3	11	2		4	0	3		4	2	2		4	4	0		4	5	2
6		4	7	0		4	8	3		4	10	3		5	0	2		5	2	1		5	4	1
7		5	4	1		5	6	1		5	8	2		5	10	2		6	0	3		6	2	3
8		6	1	1		6	3	3		6	6	1		6	8	2		6	11	0		7	1	2
9		6	10	2		7	1	1		7	4	0		7	6	3		7	9	2		8	0	0
10		7	7	3		7	10	3		8	1	3		8	5	0		8	7	3		8	11	0
20		15	3	2		15	9	2		16	3	2		16	9	2		17	3	3		17	9	3
30	1	2	11	0	1	3	8	1	1	4	5	1	1	5	2	1	1	5	11	2	1	6	8	2
40	1	10	6	3	1	11	7	0	1	12	7	0	1	13	7	0	1	14	7	1	1	15	7	1
50	1	18	2	2	1	19	5	3	2	0	8	3	2	2	0	0	2	3	3	0	2	4	6	0
60	2	5	10	1	2	7	4	2	2	8	10	3	2	10	4	3	2	11	10	3	2	13	4	3
70	2	13	6	0	2	15	3	1	2	17	0	1	2	18	9	2	3	0	6	2	3	2	3	3
80	3	1	1	2	3	3	2	0	3	5	2	0	3	7	2	1	3	9	2	2	3	11	2	2
90	3	8	9	1	3	11	0	2	3	13	4	0	3	15	7	0	3	17	10	1	4	0	1	1
100	3	16	5	0	3	18	11	2	4	1	5	3	4	3	11	3	4	6	6	0	4	9	0	0
200	7	12	10	0	7	17	10	3	8	2	11	2	8	7	11	3	8	13	0	0	8	18	0	0
300	11	9	3	0	11	16	10	1	12	4	5	0	12	11	11	3	12	19	6	0	13	7	0	0
400	15	5	8	0	15	15	9	2	16	5	10	3	16	15	11	2	17	6	0	0	17	16	0	1
500	19	2	1	0	19	14	8	3	20	7	4	1	20	19	11	2	21	12	6	0	22	5	0	1
600	22	18	6	0	23	13	8	1	24	8	10	0	25	3	11	1	25	19	0	0	26	14	0	1
700	26	14	11	0	27	12	7	2	28	10	3	3	29	7	11	0	30	5	6	0	31	3	0	2
800	30	11	4	0	31	11	7	0	32	11	9	1	33	11	11	0	34	12	0	0	35	12	0	2
900	34	7	9	0	35	10	6	1	36	13	3	0	37	15	11	0	38	18	6	0	40	1	0	2
1000	38	4	2	0	39	9	5	3	40	14	8	2	41	19	10	3	43	5	0	0	44	10	0	2
ditto	*l.* 38.2081				*l.* 39.4736				*l.* 40.7358				*l.* 41.9947				*l.* 43.2503				*l.* 44.5026			

Discount at 5 per Cent.

Principal.	350 Days.				360 Days.				365 Days.			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1			11	0			11	1			11	1
2		1	10	0		1	10	2		1	11	0
3		2	9	0		2	9	3		2	10	1
4		3	8	0		3	9	1		3	9	2
5		4	7	0		4	8	2		4	9	0
6		5	6	0		5	7	3		5	8	2
7		6	5	0		6	7	0		6	8	0
8		7	4	0		7	6	1		7	7	2
9		8	3	0		8	5	2		8	6	3
10		9	2	0		9	4	3		9	6	1
20		18	3	3		18	9	2		19	0	2
30	1	7	5	2	1	8	2	2	1	8	6	3
40	1	16	7	1	1	17	7	1	1	18	1	1
50	2	5	9	0	2	7	0	0	2	7	7	2
60	2	14	10	3	2	16	4	3	2	17	1	3
70	3	4	0	2	3	5	9	2	3	6	8	0
80	3	13	2	2	3	15	2	2	3	16	2	1
90	4	2	4	1	4	4	7	1	4	5	8	2
100	4	11	6	0	4	14	0	0	4	15	2	3
200	9	3	0	0	9	7	11	3	9	10	5	3
300	13	14	6	0	14	1	11	3	14	5	8	2
400	18	6	0	0	18	15	11	3	19	0	11	2
500	22	17	6	1	23	9	11	3	23	16	2	1
600	27	9	0	1	28	3	11	2	28	11	5	1
700	32	0	6	1	32	17	11	2	33	6	8	0
800	36	12	0	1	37	11	11	2	38	1	11	0
900	41	3	6	1	42	5	11	2	42	17	1	3
1000	45	15	0	1	46	19	11	1	47	12	4	3
ditto	*l.* 45.7516				*l.* 46.9973				*l.* 47.619			

Part II. For Days under 10 in a Medium, to add at above 90 Days.

Principal.	1 Day.			2 Days.			3 Days.		
l.	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*
1									
2									1
3						1			1
4						1			1
5						1			2
6			1			1			2
7			1			2			3
8			1			2			3
9			1			2			3
10			1			2		1	0
20			2		1	1		2	0
30		1	0		1	3		2	3
40		1	1		2	1		3	3
50		1	2		3	1		4	3
60		1	3		3	3		5	2
70		2	1		4	2		6	2
80		2	2		5	0		7	2
90		3	0		5	3		8	2
100		3	1		6	1		9	1
200		6	1	1	0	2	1	6	3
300		9	2	1	6	3	2	4	1
400	1	0	2	2	1	0	3	1	2
500	1	3	3	2	7	1	3	11	0
600	1	6	3	3	1	2	4	8	1
700	1	10	0	3	7	3	5	5	3
800	2	1	0	4	2	0	6	3	1
900	2	4	1	4	8	1	7	0	2
1000	2	7	1	5	2	2	7	9	3
ditto	*l.* 0.1304			*l.* 0.2608			*l.* 0.3912		

Part II. For Days under 10 in a Medium, to add at above 90 Days.

Principal.	*4 Days.*			*5 Days.*			*6 Days.*			*7 Days.*			*8 Days.*				*9 Days.*			
l.	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1						0			1			1				1				1
2			1			1			1			2				2				2
3			1			2			2			3				3				3
4			2			2			3			3			1	0			1	0
5			2			3		1	0		1	0			1	1			1	2
6			3		1	0		1	0		1	1			1	2			1	3
7			3		1	0		1	1		1	2			1	3			2	0
8		1	0		1	1		1	2		1	3			2	0			2	1
9		1	0		1	2		1	3		2	0			2	1			2	2
10		1	1		1	2		2	0		2	1			2	2			3	0
20		2	2		3	1		3	3		4	2			5	0			5	3
30		3	3		4	3		5	3		6	2			7	2			8	2
40		5	0		6	1		7	2		8	3			10	0			11	0
50		6	1		7	3		9	2		11	0		1	0	2		1	2	1
60		7	2		9	2		11	1	1	1	1		1	3	0		1	5	0
70		8	3		11	0	1	1	1	1	3	1		1	5	2		1	7	3
80		10	0	1	0	2	1	3	0	1	5	2		1	8	0		1	10	2
90		11	1	1	2	0	1	5	0	1	7	3		1	10	2		2	1	1
100	1	0	2	1	3	3	1	6	3	1	10	0		2	1	0		2	4	1
200	2	1	0	2	7	1	3	1	2	3	7	3		4	2	0		4	8	1
300	3	1	2	3	11	0	4	8	1	5	5	3		6	3	0		7	0	2
400	4	2	0	5	2	2	6	3	1	7	3	3		8	4	0		9	4	3
500	5	2	2	6	6	1	7	10	0	9	1	2		10	5	1		11	8	3
600	6	3	1	7	10	0	9	4	3	10	11	2		12	6	1		14	1	0
700	7	3	3	9	1	2	10	11	2	12	9	1		14	7	1		16	5	1
800	8	4	1	10	5	1	12	6	1	14	7	1		16	8	1		18	9	1
900	9	4	3	11	8	3	14	1	0	16	5	1		18	9	1	1	1	1	2
1000	10	5	1	13	0	0	15	7	2	18	3	0	1	0	10	1	1	3	5	3
ditto	*l.* 0.5216			*l.* 0.652			*l.* 0.7824			*l.* 0.9128			*l.* 1.0432				*l.* 1.1736			

The Use of the Tables of Discount foregoing.

Quest. 1. What is the Discount of *l.* 700 (if paid 137 days before due) at 4 *per Cent?*

In the 1st Table against *l.* 700, under 130 days is *l.* 9 : 16 : 7¼
700 under 7 days (part the 2d, at the end of the Table)—— } 00 : 10 : 3

Sum, or Answer = *l.* 10 : 6 : 10¼

Quest. 2. What is the Discount of *l.* 1500 for 57 Days at 5 *per Cent?*

By the Method in the Margin, the Answer is found *l.* 11 : 12 : 4½.

l.	*Days.*	
1000	for 50 =	*l.* 6 : 16 : 0¼
500	—— ——	3 : 8 : 0¼
1000	for 7 =	0 : 19 : 2
500	—— ——	0 : 9 : 1½
		l. 11 : 12 : 4½ = *Sum*

Note, That for 1, 2, 3, 4, 5, 6, 7, 8, and 9 Days, from and above 90, &c. to 360; I take those Tabular Numbers against 1, 2, &c. to 9, at the end of the Table of Discount.

I shall give another Example, which takes in all the Varieties of the Use of the Table.

Example 2.] What Discount at 5 *per Cent.* is to be allowed, to satisfy a Debt of *l.* 5000, 91 Days before the same is due?

l. *Days.*
Against 1000 (or ditto) under 90 stands 12.1786 } add
And under 1 (part 2.) = 0.1304 }

Sum = 12.3090
Which multiply by the Thousands —— = 5

The Answer or Product = 61.545 = *l.* 61 : 10 : 11

But had this Question been done by the Tables in Dr *Harris*'s *Lexicon*, which he says are accurate; it will be found *l.* 2 : 3 : 8½ too little. For 91 Days multiplied in *l.* 5000 gives 455000. And by those Tables you have against 400000 = 52.1853
50000 = 6.5232
5000 = 0.6523

Sum = 59.3608 = *l.* 59 : 7 : 2½.

 Which

Which taken from my Discount above, is too little by *l.* 2 : 3 : 8½ as I have fully proved before the Tables above.

III. *Of Compound Interest: The Use of Decimals.*

I have shewed above, what Compound Interest is, and now proceed to give the Reader the Use of Decimals in computing thereof.

Case 1. To find the Amount of any Sum of Money for any Number of Years, as of *l.* 650, for 5 Years at 5 *per Cent. per Ann.* Compound Interest.

Rule.] First find the Amount of *l.* 1 for 5 Years, and multiply the 5th Year by *l.* 650, the Principal given, thus:

Example.] 1st, 100. 105 : : 1. 1.05 = at the end of 1 Year.

2dly, 100. 105 : : 1.05
105

3dly, 100. 105 : : 1.1025 = of the 2d Year.
105

4thly, 100. 105 : : 1.157625 = of the 3d Year.
105

5thly, 100. 105 : : 1.21550625 = the 4th Year.
105

Answer = 1.2762815625 = 5th Year.
650 multiply.

The Amount of *l.* 650 for 5 Years.—— } =829.5830156 = Answer.

Or it will be the same thing to multiply the 3d Number by 1.05 continually; *i. e.* to multiply the Sursolid of 1.05 by 650.

In this Example you see that every Year requires a Multiplication performed as *per Compend.* 2. of Multiplication of Decimals, and the last by the 650, as *per Compend.* 6. whereby 'tis found, that if *l.* 650 be put out and forborn 5 Years, the Interest of that Principal, and of the Interest accruing, will amount to *l.* 829 : 11 : 8.

Case 2. To find the present Worth of any Sum due any Number of Years hence, as of *l.* 829 : 11 : 8 (or *l.* 829.583015625) due at the end of 5 Years. See the Operation at 5 *per Cent.* Compound Interest, or divide the Number given, *&c.* by 1.05. continually.

1.05.

1.05. 100 : : 829.5, &c.

1.05) 82958.3015625 (790.076200595 = 1 *Year hence.*

945
800
651
210
625
1000
550
25

105) 79007.620059500 (752.453524376 .... *Worth 2 Years hence.*

550
257
476
562
370
550
255
459
395
800
650

3dly, 105. 100 : : 752.453524376. 716.622423 = 3 *Years hence.*
4thly, 105. 100 : : 716.622423. 682.497164 = *present Worth* 4 (*Years hence.*
5thly, 105. 100 : : 682.497164. 650. = *the present Worth of the Sum given*, viz. *l.* 829 : 11 : 8 due at the end of 5 Years.

Here you may observe, that the 4th Proportional is always the 3d Number in the Proportion the next stating: And having for every Year multiplied by 100, and divided by 105, and the proposed Rate of Interest, (directly contrary to what you did in the last

 Case)

Case) the 4th Proportional in the 5th Operation is *l.* 650, which is the present Worth of *l.* 829 : 11 : 8, due at the end of 5 Years; which proves the Truth of the last Case.

And after the same manner, had you desired to have found the present Worth of *l.* 1 due 5 Years hence, the several Years Decrease would be as in the following Table, Column 2. from that of Years.

Years.	*The Amount of* l. 1.	*The present Worth of* l. 1.	*The Amount of* l. 1. *Annuity.*	*The present Worth of* l. 1. *Annuity.*
1	1.05	.95238095	1	.95238095
2	1.1025	.90702947	2.05	1.85941042
3	1.157625	.86383759	3.1525	2.72324801
4	1.21550625	.82270247	4.310125	3.54595048
5	1.27628156	.78352616	5.52563125	4.32947664

Case 3. To find the Amount of Annuities forborn any Number of Years, you will easily consider, that at the end of 1 Year there is 1 Year's Income due without any Interest; for the 2d Year, at the end of that there is 2 Years Income due, and the Interest of 1 Year; at the end of the 3d Year, there is due 3 Years Income; 2 Years Interest of the 1st Year's Income, 1 Year's Interest of the 2d Year's Income, and 1 Year's Interest of the Interest of the 1st Year's Income, &c.

Thus suppose the Rent 1 *l. per Ann.* the 3 Years income is *l.* 3
2 Years Interest of the 1st Years Income = 0.1
1 Years Interest of the 2d Years Income = 0.05
And 1 Year's Interest of the Interest of the 1st Year's Income. — — — — — — — — } = 0.0025

The Sum is the utmost Improvement of this Annuity for 3 Years, *viz.* — — — } = 3.1525

Hence it follows, that a Table of the Amount of 1 *l.* Annuity, as the 3d Column above, is made from the Column of the Amount of *l.* 1, by making the Tabular Number for the 1st Year 1; for the 2d in the 3d Column the Sum of the 2 first Numbers in the 1st and 3d Columns; the 3d Number in the 3d Column is the Sum of the 2d Number in the 1st and 3d Columns, &c.

Case

Case 4. To find the present Worth of an Annuity due any Number of Years to come, it must be considered that the present Worth of 1 *l.* Annuity to continue 1 Year is the same with the present Worth of *l.* 1 due a Year hence. So that the present Worth of an Annuity to continue 2 Years, is the Sum of the present Worth thereof for 1 Year, and the present Worth of the like Sum with the Annuity to continue 2 Years; and thus the 4th Column in the above Table is made: For

N°.		*Col.*		
The 1 in the	4 is	.95238095	Sum = the 2d in Column 4	
2	2	.90702947	*viz.* 1.85941042.	
2	4	1.85941042	Sum = the 3d in Column 4	
3	2	.86383759	*viz.* 2.72324801, &c.	

Case 5. To find what Annuity, to continue any Number of Years, any Sum will purchase. Divide a Unit by any of the Numbers which are the present Worth of *l.* 1 Annuity for any Years, and the Quotient shews the Annuity that *l.* 1 will purchase to continue those Years. Thus if I would know what *l.* 1 will purchase *per Ann.* to continue 5 Years.

As 4.32947664. 1 : : 1. .23097480 = Answ. = 4 *s.* 7 *d.* ½ *per Ann.*

Farther Rules concerning the five Cases above.

1. To enlarge the Numbers tabulated above, and what is under the 5th Case, to any Number of Years, two ways.
2. How to use them for any other Sum, above *l.* 1.

1. You may augment the first and second Columns by several Operations, as taught above, from 5 Years to 50, &c. and from them may make the third and fourth Columns, and another shewing what *l.* 1 will purchase to continue those Years.

Or, Because the Numbers in the 1st Column of Compound Interest are in proportion as 100 to 105, or as 1 to 1.05, therefore that Column is a Series in Geometrical Proportion, whose Ratio is 1.05, so that (as in *Progression*, *Sect.* 2. *Chap.* II.) if you have no occasion for the intermediate Numbers, but would know the Amount, present Worth of any Sum or Annuity, &c. for 41 Years; for the Amount of *l.* 1. multiply the 1.27628156 by itself, and it produceth the Amount for 10 Years; which if you multiply by itself, produceth the Amount for 20 Years; and that multiplied by itself, produceth the Amount for 40 Years. And having done the like with

with the 2d Column (of the present Worth of *l.* 1) and found the present Worth for 40 Years; you may easily by the Rules under the first and second Case find the Amount or present Worth for one Year more. And having those two for 40 and 41 Years, you may easily from thence calculate the Amount, present Worth, or Purchase of Annuities, as is shewed under the 3d, 4th and 5th Cases.

For Sums of Money or Annuities above *l.* 1, you have nothing to do but to multiply respectively by such Pounds: Thus for *l.* 500 5 Years.

l.
If you multiply 1.27628156 by 500 you have the Amount of (*l.* 500, 5 Years.
.78352616 by 500, you have the present (Worth of *l.* 500 due 5 Years hence.
5.52563125 by 500, you have the Amount (of *l.* 500 *per Ann.* for 5 Years.
4.32947664 by 500, you have the present Worth of *l.* 500 *per Ann.* to continue 5 Years.
And as in Case 5. .2309748 by 500, you have the Annuity that *l.* 500 will purchase to continue 5 Years.

And the like for any other Sums or Term of Years.

But all these things (if you are without Tables of Interest) are most easily and concisely done by the Logarithms, as appears in that Part of Arithmetic hereafter following.

Compound

COMPOUND INTEREST *at* 3 *per Cent.*

Years.	TABLE I. *The Amount of* l. 1.	TABLE II. *The present Worth of* l. 1.	TABLE III. *The Amount of* l. 1 *Annuity.*	TABLE IV. *The present Worth of* l. 1. *Annuity.*
1	1.03	.9708738	1	.9708738
2	1.0609	.9425959	2.03	1.9134697
3	1.092727	.9151417	3.0909	2.8286114
4	1.1255088	.8884870	4.183627	3.7170984
5	1.1592741	.8626088	5.3091358	4.5797072
6	1.1940523	.8374843	6.4684099	5.4171914
7	1.2298739	.8130915	7.6624622	6.2302829
8	1.2667701	.7894092	8.8923360	7.0196922
9	1.3047732	.7664167	10.1591061	7.7861089
10	1.3439164	.7440939	11.4638793	8.5302028
11	1.3842339	.7224213	12.8077957	9.2526241
12	1.4257609	.7013799	14.1920296	9.9540040
13	1.4685337	.6809513	15.6177904	10.6349553
14	1.5125897	.6611178	17.0863242	11.2960731
15	1.5579674	.6418619	18.5989139	11.9379351
16	1.6047064	.6231669	20.1568813	12.5611020
17	1.6528476	.6050164	21.7615877	13.1661185
18	1.7024331	.5873946	23.4144354	13.7535131
19	1.7535061	.5702860	25.1168684	14.3237991
20	1.8061112	.5536758	26.8703745	14.8774748
21	1.8602946	.5375493	28.6764857	15.4150241
22	1.9161034	.5218925	30.5367803	15.9369166
23	1.9735865	.5066917	32.4528837	16.4436084
24	2.0327941	.4919337	34.4264702	16.9355421
25	2.0937779	.4776056	36.4592643	17.4131477
26	2.1565913	.4636947	38.5530422	17.8768420
27	2.2212890	.4501891	40.7096335	18.3270315
28	2.2879277	.4370768	42.9309225	18.7641082
29	2.3565655	.4243464	45.2188502	19.1884546
30	2.4272625	.4119868	47.5754157	19.6004413
31	2.5000803	.3999871	50.0026782	20.0004285

COMPOUND INTEREST *at 3 per Cent.*

Years.	TABLE I. Continu'd. *The Amount of* l. 1.	TABLE II. Continu'd. *The present Worth of* l. 1.	TABLE III. Continu'd. *The Amount of* l. 1. *Annuity.*	TABLE IV. Continu'd. *The present Worth of* l. 1. *Annuity.*
32	2.5750828	.3883370	52.5027585	20.3887655
33	2.6523352	.3770262	55.0778413	20.7657918
34	2.7319053	.3660449	57.7301765	21.1318367
35	2.8138624	.3553834	60.4620818	21.4872200
36	2.8982783	.3450324	63.2759443	21.8322525
37	2.9852267	.3349829	66.1742226	22.1672354
38	3.0747835	.3252262	69.1594493	22.4924616
39	3.1670270	.3157535	72.2342327	22.8082151
40	3.2620378	.3065568	75.4012597	23.1147719
41	3.3598989	.2976280	78.6632975	23.4123999
42	3.4606959	.2889592	82.0231964	23.7013592
43	3.5645168	.2805429	85.4838923	23.9819021
44	3.6714523	.2723718	89.0484191	24.2542739
45	3.7815959	.2644386	92.7198614	24.5187125
46	3.8950437	.2567365	96.5014572	24.7754490
47	4.0118950	.2492588	100.3965009	25.0247078
48	4.1322519	.2419988	104.4083960	25.2667066
49	4.2562194	.2349503	108.5406479	25.5016569
50	4.3839060	.2281071	112.7968673	25.7297640
51	4.5154232	.2214632	117.1807733	25.9512271
52	4.6508859	.2150128	121.6961966	26.1662399
53	4.7904125	.2087503	126.3470825	26.3749902
54	4.9341249	.2026702	131.1374949	26.5776040
55	5.0821486	.1967672	136.0716198	26.7744276
56	5.2346131	.1910361	141.1537684	26.9654637
57	5.3916515	.1854719	146.3883815	27.1509356
58	5.5534010	.1800698	151.7800329	27.3310054
59	5.7200030	.1748251	157.3334339	27.5058305
60	5.8916031	.1697331	163.0534369	27.6755636
61	6.0683512	.1647894	168.9450400	27.8403530

COMPOUND INTEREST *at* 4 *per Cent.*

Years.	TABLE V. *The Amount of* l. 1.	TAB. VI. *The present Worth of* l. 1.	TABLE VII. *The Amount of* l. 1 *Annuity.*	TAB. VIII. *The present Worth of* l. 1. *Annuity.*
1.	1.04	.9615385	1.0	.9615385
2	1.0816	.9245562	2.04	1.8860947
3	1.124864	.8889964	3.1216	2.7750910
4	1.1698586	.8548042	4.246464	3.6298952
5	1.2166529	.8219271	5.4163226	4.4518223
6	1.2653190	.7903145	6.6329755	5.2421369
7	1.3159318	.7599178	7.8982945	6.0020547
8	1.3685691	.7306901	9.2142263	6.7327448
9	1.4233118	.7025866	10.5827953	7.4353314
10	1.4802443	.6755641	12.0061071	8.1108955
11	1.5394541	.6495808	13.4863514	8.7604763
12	1.6010322	.6245970	15.0258055	9.3850733
13	1.6650735	.6005740	16.6268377	9.9856473
14	1.7316764	.5774750	18.2919112	10.5631223
15	1.8009435	.5552644	20.0235876	11.1183868
16	1.8729812	.5339081	21.8245311	11.6522949
17	1.9479005	.5133732	23.6975124	12.1656680
18	2.0258165	.4936281	25.6454129	12.6592961
19	2.1068492	.4746424	27.6712294	13.1339385
20	2.1911231	.4563869	29.7780786	13.5903253
21	2.2787681	.4388335	31.9592017	14.0291589
22	2.3699188	.4219553	34.2479698	14.4511142
23	2.4647155	.4057263	36.6178886	14.8568405
24	2.5633042	.3901214	39.0826041	15.2469619
25	2.6658363	.3751168	41.6459083	15.6220787
26	2.7724698	.3606892	44.3117446	15.9827678
27	2.8833686	.3468165	47.0842144	16.3295844
28	2.9987033	.3334774	49.9675830	16.6630618
29	3.1186515	.3206514	52.9662863	16.9837132
30	3.2433975	.3083186	56.0849377	17.2920318
31	3.3731334	.2964602	59.3283352	17.5884921

COMPOUND INTEREST *at 4 per Cent.*

Years.	TABLE V. Continu'd. *The Amount of* l. 1.	TABLE VI. Continu'd. *The present Worth of* l. 1.	TABLE VII. Continu'd. *The Amount of* l. 1. *Annuity.*	TABLE VIII. Continu'd. *The present Worth of* l. 1. *Annuity.*
32	3.5080587	.2850579	62.7014687	17.8735500
33	3.6483811	.2740941	66.2095274	18.1476441
34	3.7943163	.2635521	69.8579045	18.4111962
35	3.9460890	.2534154	73.6522248	18.6646116
36	4.1039326	.2436687	77.5983138	18.9082803
37	4.2680899	.2342968	81.7022464	19.1425771
38	4.4388134	.2252854	85.9703362	19.3678625
39	4.6163660	.2166206	90.4091497	19.5844831
40	4.8010206	.2082890	95.0255157	19.7927721
41	4.9930615	.2002779	99.8265363	19.9930500
42	5.1927839	.1925749	104.8195978	20.1856250
43	5.4004953	.1851682	110.0123817	20.3707931
44	5.6165151	.1780463	115.4128769	20.5488395
45	5.8411757	.1711984	121.0293920	20.7200378
46	6.0748227	.1646138	126.8705677	20.8846517
47	6.3178156	.1582825	132.9453904	21.0429342
48	6.5705282	.1521947	139.2632060	21.1951289
49	6.8333494	.1463411	145.8337342	21.3414700
50	7.1066833	.1407126	152.6670836	21.4821826
51	7.3909507	.1353006	159.7737670	21.6174832
52	7.6865887	.1300967	167.1647176	21.7475800
53	7.9940522	.1250930	174.8513063	21.8726729
54	8.3138143	.1202817	182.8453586	21.9929546
55	8.6463669	.1156555	191.1591729	22.1086100
56	8.9922216	.1112072	199.8055398	22.2198172
57	9.3519105	.1069300	208.7977614	22.3267472
58	9.7259869	.1028173	218.1496719	22.4295645
59	10.1150263	.0988628	227.8756588	22.5284273
60	10.5196274	.0950604	237.9906851	22.6234877
61	10.9404125	.0914042	248.5103125	22.7148919

Ff 2

COMPOUND INTEREST *at 5 per Cent.*

Years.	TABLE IX. *The Amount of* l. 1.	TAB. X. *The present Worth of* l. 1.	TABLE XI. *The Amount of* l. 1. *Annuity.*	TAB. XII. *The present Worth of* l. 1. *Annuity.*
1	1.05	.9523809	1.0	.9523809
2	1.1025	.9070294	2.05	1.8594103
3	1.157625	.8638376	3.1525	2.7232480
4	1.2155063	.8227025	4.310125	3.5459505
5	1.2762816	.7835262	5.5256312	4.3294767
6	1.3400956	.7462154	6.8019128	5.0756921
7	1.4071064	.7106813	8.1420084	5.7863734
8	1.4774554	.6768394	9.5491089	6.4632128
9	1.5513282	.6446089	11.0265643	7.1078217
10	1.6288946	.6139133	12.5778925	7.7217349
11	1.7103393	.5846929	14.2067871	8.3064142
12	1.7958563	.5568374	15.9171265	8.8632516
13	1.8856491	.5303213	17.7129828	9.3935730
14	1.9799316	.5050680	19.5986320	9.8986409
15	2.0789282	.4810171	21.5785636	10.3796580
16	2.1828746	.4581115	23.6574918	10.8377695
17	2.2920183	.4362967	25.8403664	11.2740662
18	2.4066192	.4155207	28.1323847	11.6895869
19	2.5269502	.3957340	30.5390039	12.0853208
20	2.6532977	.3768895	33.0659541	12.4622103
21	2.7859626	.3589424	35.7192518	12.8211527
22	2.9252607	.3418499	38.5052144	13.1630026
23	3.0715238	.3255713	41.4304751	13.4885739
24	3.2251000	.3100679	44.5019989	13.7986418
25	3.3863549	.2953028	47.7270988	14.0939445
26	3.5556727	.2812407	51.1134537	14.3751853
27	3.7334563	.2678483	54.6691264	14.6430336
28	3.9201291	.2550936	58.4025827	14.8981272
29	4.1161356	.2429463	62.3227119	15.1410735
30	4.3219424	.2313774	66.4388474	15.3724510
31	4.5380395	.2203595	70.7607898	15.5928104

COMPOUND INTEREST *at 5 per Cent.*

Years.	TABLE IX. Continu'd. *The Amount of* l. 1.	TABLE X. Continu'd. *The present Worth of* l. 1.	TABLE XI. Continu'd. *The Amount of* l. 1. *Annuity.*	TAB. XII. Continu'd. *The present Worth of* l. 1. *Annuity.*
32	4.7649415	.2098562	75.2988293	15.8026766
33	5.0031885	.1998725	80.0637708	16.0025491
34	5.2533480	.1903548	85.0669593	16.1929039
35	5.5160154	.1812903	90.3203073	16.3741942
36	5.7918161	.1726574	95.8363226	16.5468516
37	6.0814069	.1644356	101.6281387	16.7112872
38	6.3854773	.1566054	107.7095457	16.8678926
39	6.7047511	.1491480	114.0950229	17.0170406
40	7.0399887	.1420457	120.7997741	17.1590862
41	7.3919881	.1352816	127.8397628	17.2943678
42	7.7615875	.1288396	135.2317509	17.4232074
43	8.1496669	.1227044	142.9933385	17.5459118
44	8.5571503	.1168613	151.1430054	17.6627732
45	8.9850078	.1112965	159.7001556	17.7740697
46	9.4342582	.1059967	168.6851634	17.8800663
47	9.9059711	.1009492	178.1194216	17.9810155
48	10.4012696	.0961421	188.0253927	18.0771576
49	10.9213331	.0915639	198.4266623	18.1687215
50	11.4674000	.0872037	209.3479954	18.2559253
51	12.0407698	.0830512	220.8153952	18.3389764
52	12.6428082	.0790963	232.8561649	18.4180728
53	13.2749486	.0753298	245.4989731	18.4934026
54	13.9386961	.0717427	258.7739218	18.5651453
55	14.6356309	.0683264	272.7126179	18.6334720
56	15.3674124	.0650727	287.3482488	18.6985444
57	16.1357831	.0619740	302.7156612	18.7605185
58	16.9425722	.0590229	318.8514442	18.8195414
59	17.7897008	.0562123	335.7940164	18.8757536
60	18.6791858	.0535355	353.5837172	18.9292882
61	19.6131451	.0509862	372.2629031	18.9802743

COMPOUND INTEREST *at* 6 *per Cent.*

Years.	TAB. XIII. *The Amount of* l. 1.	TAB. XIV. *The present Worth of* l. 1.	TABLE XV. *The Amount of* l. 1. *Annuity.*	TABLE XVI. *The present Worth of* l. 1. *Annuity.*
1	1.06	.9433962	1.0	.9433962
2	1.1236	.8899964	2.06	1.8333926
3	1.191016	.8396193	3.1836	2.6730119
4	1.2624769	.7920937	4.3746016	3.4651056
5	1.3382256	.7472582	5.6370930	4.2123638
6	1.4185191	.7049605	6.9753187	4.9173244
7	1.5036303	.6650571	8.3938378	5.5823815
8	1.5938481	.6274124	9.8974681	6.2097939
9	1.6894790	.5918985	11.4913162	6.8016923
10	1.7908477	.5583948	13.1807958	7.3600871
11	1.8982980	.5267875	14.9716435	7.8868747
12	2.0121965	.4969694	16.8699420	8.3838440
13	2.1329283	.4688390	18.8821385	8.8526831
14	2.2609039	.4423010	21.0150667	9.2949840
15	2.3965582	.4172651	23.2759707	9.7122491
16	2.5402517	.3936463	25.6725289	10.1058953
17	2.6927728	.3713644	28.2128806	10.4772597
18	2.8543392	.3503438	30.9056534	10.8276035
19	3.0255995	.3305130	33.7599925	11.1581165
20	3.2071355	.3118047	36.7855920	11.4699213
21	3.3995636	.2941554	39.9927275	11.7640767
22	3.6035374	.2775051	43.3922911	12.0415818
23	3.8197497	.2617973	46.9958285	12.3033790
24	4.0489346	.2469786	50.8155782	12.5503576
25	4.2918707	.2329986	54.8645128	12.7833562
26	4.5493829	.2198100	59.1563835	13.0031663
27	4.8223459	.2073680	63.7057664	13.2105342
28	5.1116866	.1956301	68.5281123	13.4061644
29	5.4183878	.1845567	73.6397990	13.5907211
30	6.7434911	.1741101	79.0581868	13.7648312
31	6.0881006	.1642548	84.8016779	13.9290861

COMPOUND INTEREST *at 6 per Cent.*

Years.	TAB. XIII. Continu'd. *The Amount of* l. 1.	TAB. XIV. Continu'd. *The present Worth of* l. 1.	TAB. XV. Continu'd. *The Amount of* l. 1. *Annuity.*	TAB. XVI. Continu'd. *The present Worth of* l. 1. *Annuity.*
32	6.4533866	.1549574	90.8897785	14.0840435
33	6.8405898	.1461862	97.3431652	14.2302297
34	7.2510252	.1379115	104.1837550	14.3681412
35	7.6860867	.1301052	111.4347802	14.4982465
36	8.1472519	.1227407	119.1208669	14.6209872
37	8.6360870	.1157932	127.2681188	14.7367804
38	9.1542523	.1092388	135.9042059	14.8460192
39	9.7035074	.1030555	145.0584581	14.9490747
40	10.2857178	.0972222	154.7619655	15.0462969
41	10.9028609	.0917190	165.0476833	15.1380160
42	11.5570326	.0865274	175.9505442	15.2245434
43	12.2504545	.0816296	187.5075769	15.3061730
44	12.9854818	.0770091	199.7580314	15.3831821
45	13.7646107	.0726500	212.7435132	15.4558321
46	14.5904873	.0685378	226.5081239	15.5243699
47	15.4659166	.0646583	241.0986112	15.5890282
48	16.3938716	.0609984	256.5645278	15.6500266
49	17.3775039	.0575457	272.9583994	15.7075723
50	18.4201541	.0542884	290.3359032	15.7618610
51	19.5253634	.0512154	308.7560573	15.8130761
52	20.6968852	.0483164	328.2814207	15.8613925
53	21.9386983	.0455816	348.9783059	15.9069741
54	23.2550202	.0430015	370.9170041	15.9499760
55	24.6503214	.0405674	394.1720243	15.9905430
56	26.1293406	.0382712	418.8223456	16.0288141
57	27.6971011	.0361049	444.9516863	16.0649190
58	29.3589272	.0340612	472.6487874	16.0989802
59	31.1204628	.0321332	502.0077145	16.1311134
60	32.9876905	.0303143	533.1281773	16.1614277
61	34.9669520	.0285984	566.1158679	16.1900261

COMPOUND INTEREST *at* 8 *per Cent.*

Years.	TAB. XVII. *The Amount of* l. 1.	TAB. XVIII. *The present Worth of* l. 1.	TABLE XIX. *The Amount of* l. 1. *Annuity.*	TABLE XX. *The present Worth of* l. 1. *Annuity.*
1	1.08	.9259259	1.0	.9259259
2	1.1664	.8573388	2.08	1.7832648
3	1.259712	.7938224	3.2464	2.5770979
4	1.3604890	.7350299	4.506112	3.3121268
5	1.4693281	.6805832	5.866010	3.9927100
6	1.5868743	.6301696	7.3359290	4.6228797
7	1.7138243	.5834904	8.9228034	5.2063701
8	1.8509302	.5402689	10.6366276	5.7466389
9	1.9990046	.5002490	12.4875579	6.2468879
10	2.1589250	.4631935	14.4865626	6.7100814
11	2.3316390	.4288829	16.6454876	7.1389643
12	2.5181701	.3971138	18.9771266	7.5360780
13	2.7196237	.3676979	21.4952967	7.9037759
14	2.9371936	.3404610	24.2149204	8.2442370
15	3.1721691	.3152417	27.1521140	8.5594790
16	3.4259426	.2918905	30.3242831	8.8513691
17	3.7000181	.2702689	33.7502258	9.1216381
18	3.9960195	.2502490	37.4502438	9.3718871
19	4.3157011	.2317121	41.4462633	9.6035992
20	4.6609571	.2145482	45.7619644	9.8181474
21	5.0338337	.1986557	50.4229215	10.0168031
22	5.4365404	.1839405	55.4567552	10.2007436
23	5.8714636	.1703153	60.8932956	10.3710589
24	6.3411807	.1576993	66.7647593	10.5287582
25	6.8484752	.1460179	73.1059400	10.6747761
26	7.3963532	.1352018	79.9544152	10.8099779
27	7.9880615	.1251868	87.3507684	10.9351647
28	8.6271064	.1159137	95.3388299	11.0510784
29	9.3172749	.1073275	103.9659363	11.1584059
30	10.0626569	.0993773	113.2832112	11.2577833
31	10.8676694	.0920160	123.3458680	11.3497993

COMPOUND INTEREST *at* 8 *per Cent.*

Years.	TAB. XVII. Continu'd. *The Amount of* l. 1.	TAB. XVIII. Continu'd. *The present Worth of* l. 1.	TAB. XIX. Continu'd. *The Amount of* l. 1. *Annuity.*	TAB. XX. Continu'd. *The present Worth of* l. 1. *Annuity.*
32	11.7370830	.0852000	134.2135375	11.4349993
33	12.6760496	.0788889	145.9506205	11.5138883
34	13.6901336	.0730453	158.6266701	11.5869336
35	14.7853443	.0676345	172.3168037	11.6545681
36	15.9681718	.0626246	187.1021480	11.7171927
37	17.2456255	.0579857	203.0703198	11.7751784
38	18.6252756	.0536905	220.3159454	11.8288689
39	20.1152976	.0497134	238.9412209	11.8785823
40	21.7245214	.0460309	259.0565186	11.9246132
41	23.4624832	.0426212	280.7810400	11.9672344
42	25.3394818	.0394641	304.2435232	12.0066985
43	27.3666404	.0365408	329.5830050	12.0432394
44	29.5559716	.0338341	356.9496454	12.0770735
45	31.9204493	.0313279	386.5056169	12.1084014
46	34.4740853	.0290073	418.4260663	12.1374087
47	37.2320121	.0268586	452.9001515	12.1642673
48	40.2105730	.0248691	490.1321636	12.1891363
49	43.4274189	.0230269	530.3427367	12.2121633
50	46.9016124	.0213212	573.7701556	12.2334845
51	50.6537414	.0197419	620.6717680	12.2532264
52	54.7060407	.0182795	671.3255094	12.2715059
53	59.0825240	.0169255	726.0315501	12.2884314
54	63.8091259	.0156717	785.1140741	12.3041031
55	68.9138560	.0145109	848.9232000	12.3186140
56	74.4269644	.0134360	917.8370559	12.3320500
57	80.3811216	.0124408	992.2640203	12.3444908
58	86.8116113	.0115192	1072.6451419	12.3560100
59	93.7565402	.0106660	1159.4567532	12.3666760
60	101.2570634	.0098759	1253.2132934	12.3765518
61	109.3576285	.0091443	1354.4703569	12.3856962

COMPOUND INTEREST *at* 10 *per Cent.*

Years.	TAB. XXI. *The Amount of* l. 1.	TAB. XXII. *The present Worth of* l. 1.	TAB. XXIII. *The Amount of* l. 1 *Annuity.*	TAB. XXIV. *The present Worth of* l. 1. *Annuity.*
1	1.1	.9090909	1.	.9090909
2	1.21	.8264463	2.1	1.7355372
3	1.331	.7513148	3.31	2.4868520
4	1.4641	.6830135	4.641	3.1698654
5	1.61051	.6209213	6.1051	3.7907868
6	1.771561	.5644739	7.71561	4.3552607
7	1.9487171	.5131581	9.487171	4.8684188
8	2.1435888	.4665074	11.4358881	5.3349262
9	2.3579477	.4240976	13.5794769	5.7590238
10	2.5937425	.3855433	15.9374246	6.1445671
11	2.8531167	.3504939	18.5311671	6.4950610
12	3.1384284	.3186308	21.3842838	6.8136918
13	3.4522712	.2896644	24.5227121	7.1033562
14	3.7974983	.2633313	27.9749834	7.3666875
15	4.1772482	.2393920	31.7724817	7.6060795
16	4.5949730	.2176291	35.9497299	7.8237086
17	5.0544703	.1978446	40.5447028	8.0215533
18	5.5599173	.1798588	45.5991731	8.2014121
19	6.1159090	.1635080	51.1590904	8.3649201
20	6.7274999	.1486436	57.2749995	8.5135637
21	7.4002499	.1351306	64.0024994	8.6486943
22	8.1402749	.1228460	71.4027494	8.7715403
23	8.9543024	.1116782	79.5430243	8.8832184
24	9.8497327	.1015256	88.4973268	8.9847440
25	10.8347059	.0922960	98.3470594	9.0770400
26	11.9181765	.0839055	109.1817654	9.1609455
27	13.1099942	.0762777	121.0999419	9.2372232
28	14.4209936	.0693433	134.2099361	9.3065665
29	15.8630930	.0630394	148.6409297	9.3696059
30	17.4494023	.0573086	164.4940227	9.4269145
31	19.1943425	.0520987	181.9434250	9.4790132

COMPOUND INTEREST at 10 per Cent.

Years..	TABLE XXI. Continu'd. The Amount of l. 1.	TAB. XXII. Continu'd. The present Worth of l. 1.	TAB. XXIII. Continu'd. The Amount of l. 1. Annuity.	TAB. XXIV. Continu'd The present Worth of l. 1. Annuity.
32	21.1137767	.0473624	201.1377674	9.5263756
33	23.2251544	.0430568	222.2515442	9.5694324
34	25.5476699	.0391425	245.4766986	9.6085749
35	28.1024368	.0355841	271.0243685	9.6441590
36	30.9126805	.0323492	299.1268053	9.6765082
37	34.0039486	.0294083	330.0394858	9.7059165
38	37.4043434	.0267349	364.0434344	9.7326514
39	41.1447778	.0243044	401.4477779	9.7569558
40	45.2592556	.0220949	442.5925557	9.7790507
41	49.7851811	.0200863	487.8518112	9.7991370
42	54.7636992	.0182603	537.6369924	9.8173973
43	60.2400692	.0166002	592.4006916	9.8339975
44	66.2640761	.0150911	652.6407608	9.8490887
45	72.8904837	.0137192	718.9048368	9.8628079
46	80.1795321	.0124720	791.7953205	9.8752799
47	88.1974853	.0113382	871.9748526	9.8866181
48	97.0172338	.0103074	960.1723378	9.8969255
49	106.7189572	.0093704	1057.1895716	9.9062959
50	117.3908529	.0085186	1163.9085288	9.9148145
51	129.1299382	.0077441	1281.2993810	9.9225586
52	142.0429320	.0070401	1410.4293198	9.9295987
53	156.2472252	.0064001	1552.4722518	9.9359989
54	171.8719477	.0058183	1708.7194769	9.9418171
55	189.0591425	.0052893	1880.5914247	9.9471065
56	207.9650567	.0048085	2069.6505671	9.9519150
57	228.7615624	.0043714	2277.6156238	9.9562864
58	251.6377186	.0039740	2506.3771862	9.9602603
59	276.8014905	.0036127	2758.0149048	9.9638730
60	304.4816395	.0032843	3034.8163953	9.9671573
61	334.9298035	.0029857	3339.2980349	9.9701430

I have designed these Rates thus: The Columns of 3, 4 or 5 *per Cent.* for Incomes of Lands, *&c.* that are certain; That Table at 6 *per Cent.* for Houses new or in good Repair, Brick; That of 8 *per Cent.* for the Purchase, *&c.* of Houses that are pretty old; That of 10 *per Cent.* for very old Houses, or not in very good Repair. And they may be used as every one's Discretion directs, according to the foregoing Rules. And as to this Table of Lives, if the Years Purchase be multiplied by the Annuity, the Product shews the Value of such Estate for such Life. Examples follow.

The Value of a Life at the several Ages thereof.

Age.	Years Purchase.	Age.	Years Purchase.	Age.	Years Purchase.
1	10.28	25	12.27	50	9.21
5	13.4	30	11.72	55	8.51
10	13.44	35	11.12	60	7.60
15	13.33	40	10.57	65	6.54
20	12.78	45	9.91	70	5.32

The Use of these Tables of Compound Interest.

Prop. 1.] To find the Amount of any Sum of Money for any Number of Years. This is done by the first Column towards the left hand; so the Amount of *l.* 50 in 50 Years Compound Interest allow'd at 5 *per Cent.* is *l.* 573 : 7 : 5; found by multiplying the Tabular Number against 50 Years by *l.* 50 thus.

11.4674091
50
l. 573.3704550

Prop. 2.] An Office is worth *l.* 500: What may be paid in present Money to enter upon it 15 Years hence? This is done by the 2d Column at 5 *per Cent.* by multiplying the present Worth of *l.* 1. due 15 Years hence by 500, as in the Margin, where the Answer is *l.* 240 : 10 : 2.

.4810171
500
l. 240.5085500

Prop. 3.] Annuity of 70 *l. per Ann.* is forborn 33 Years, what is the Improvement, Compound Interest being

being allow'd the Owner at 5 *per Cent?* By multiplying the Amount of *l.* 1 *per Ann.* in 33 Years by the Annuity *l.* 70, you have the Answer *l.* 5604 : 9 : 4 ½.

	Years.
l. 1. *per Ann.*	33 = 80.0638328
	70
	l. 5604.4682960

Prop. 4.] There are 13 Years to come in the Lease of an House that is pretty old, but in good Repair, and is *l.* 50 *per Ann.* clear; what is a farther Lease for 31 Years worth in present Money, at the Rate of 8 *per Cent?*

I add the 31 Years Reversion to the 13 *in esse* makes 44.

The present Worth of *l.* 1 due 44 Years hence, is 12.0770735; from which deduct the present Worth for 13 Years, and the Rest is multiplied by 50 for Answer: *All by the Table of* 8 *per Cent.*

	Years.	
l. 1 *per Ann.*	44 =	*l.* 12.0770735
l. 1 — —	13 =	*l.* 7.9037759
	Rests = *l.*	4.1732976
		50
	Answ. *l.*	208.66488

And if you multiply any of the Numbers in the 4th Column (according as the Estate is in Land or Houses, as abovesaid) by the Annuity you would know the present Worth of, the Product shews the present Value of such Annuity for any Years required under 62.

To find the Present Worth of Estates for Lives.

Prop. 5.] To find the Value of an Estate of *l.* 35 *per Ann.* in Land for a Life of 45 Years. By the little Table above, as computed by the Learned Dr *Halley*, that Life is Years Purchase — 9.91
Which being multiplied by the Annuity 35, the 35
Product is *l.* 346 : 17 : 00 = the Answer.
Prod. *l.* 346.85

Prop. 6.] It many times happens, that a Life and so many Years certain to come, are proposed in the same Question; therefore it becomes necessary, as in Computations for two Lives also, *to reduce the Years Purchase of any Life into Years certain to come, or Years in a Lease:* which is done thus.

I find a Life of 50 worth 9.21 Years Purchase. Now if you turn to any of the four Columns in the Tables above, which shew the present Worth of Annuities according to the several Rates, you will find at 5 *per Cent.* the next to the said 9.21 is 9.393573, against which in the Column under Years is 13 Years to come. At 6 *per Cent.* the next to 9.21 is 9.294984, against which under Years are 14 Years. At 8 *per Cent.* are 9.1216381, against which under Years are

are 17 Years Leaſe. And at 10 *per Cent.* ſtands 9.2372232, (as next to the ſaid 9.21) and againſt that are 27 Years Leaſe.

Example. What is the Value of an Houſe of *l.* 96 *per Ann.* clear Rent for 23 Years after the Death of a Perſon 60 Years old?

The Age being found = 12 Years to come; therefore by the Table of 8 *per Cent.* Column 4. the Anſwer, as under *Prop.* 4. is found to be as *per* Margin, *l.* 395.37504 = *l.* 395 : 7 : 6.

Years.	*Years Purch.*	*Col.* 4. 8 *per C.*	*Years to come.*
60 —	7.60 ——	or 7.536078	or = 12

The pref. Worth of l. 1 per Ann.— } *Years.* for 35 = *l.* 11.654568
for 12 = 7.536078

of *l.* 1 *Reverſion* = 4.11849
96

Anſw. = 395.37504

Prop. 7.] *In Computations of the Value of Eſtates for two Lives*; as ſuppoſe 50 and 55. Firſt, find by the little Table of the Value of Lives the Years Purchaſe that the elder Life is worth; then find in the fourth Column at 5 *per Cent.* (ſuppoſing the Purchaſe to be a clear certain Income) the 8.51 found in the ſaid little Table, or the next to it, which is = 8.3064142; right againſt which, in the Column of Years, is 11 Years: to which add the Difference in the Ages (5) and the Sum is 16 Years; againſt which, in the 4th Column from the Years, is 10.8377695, the Value of *l.* 1 *per Ann.* for the ſaid two Lives: which multiplied by the Annuity, gives the Anſwer or Value thereof for the ſaid two Lives, near enough the Truth. Farther Directions might be given on this head; but I have not room to enlarge on things ſo uncertain, and which at beſt depend on many caſual Circumſtances.

A TABLE

A TABLE *shewing by Inspection the Fines payable for any Number of Years lapsed or expired in a Church or College Lease of their Lands, to make up such Lease* 21 *Years.*

Annual Rent of Lands.	*1 Year lapsed* Fines Payable.				*2 Years laps.* Fines Payable.				*3 Years laps.* Fines Payable.				*4 Years laps.* Fines Payable.				*5 Years laps.* Fines Payable.				*6 Years laps.* Fines Payable.			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1		2	0	0		4	3	0		6	9	0		9	6	2		12	7	3		16	1	2
2		4	0	1		8	6	0		13	6	0		19	1	0	1	5	3	3	1	12	3	0
3		6	0	1		12	9	0	1	0	3	1	1	8	7	2	1	17	11	2	2	8	4	2
4		8	0	2		17	0	0	1	7	0	1	1	18	2	0	2	10	7	2	3	4	6	0
5		10	0	2	1	1	3	1	1	13	9	1	2	7	8	3	3	3	3	1	4	0	7	2
6		12	0	3	1	5	6	1	2	0	6	1	2	17	3	1	3	15	11	1	4	16	9	1
7		14	1	0	1	9	9	1	2	7	3	1	3	6	9	3	4	8	7	0	5	12	10	3
8		16	1	0	1	14	0	1	2	14	0	1	3	16	4	1	5	1	3	0	6	9	0	1
9		18	1	1	1	18	3	1	3	0	9	2	4	5	10	3	5	13	10	3	7	5	1	3
10	1	0	1	1	2	2	6	1	3	7	6	2	4	15	1	1	6	6	6	3	8	1	3	1
20	2	0	2	2	4	5	0	3	6	15	1	0	9	10	10	2	12	13	1	2	16	2	6	3
30	3	0	3	3	6	7	7	0	10	2	7	2	14	6	4	0	18	19	8	1	24	3	10	0
40	4	0	5	0	8	10	1	1	13	10	2	0	19	1	9	1	25	6	3	0	32	5	1	1
50	5	0	6	1	10	12	7	3	16	17	8	2	23	17	2	2	31	12	9	3	40	6	4	3
60	6	0	7	2	12	15	2	0	20	5	3	0	28	12	7	3	37	19	4	2	48	7	8	0
70	7	0	8	3	14	17	8	1	23	12	9	1	33	8	1	1	44	5	11	1	56	8	11	1
80	8	0	10	0	17	0	2	3	27	0	3	3	38	3	6	2	50	12	6	0	64	10	2	3
90	9	0	11	1	19	2	9	0	30	7	10	1	42	18	11	3	56	19	0	3	72	11	6	0
100	10	1	0	2	21	5	3	2	33	15	4	3	47	14	5	1	63	5	7	2	80	12	9	1
200	20	2	0	1	42	10	7	0	67	10	9	3	95	8	10	1	126	11	3	0	161	5	7	0
300	30	3	1	2	63	15	10	2	101	6	2	2	143	3	3	1	189	16	10	2	241	18	4	1
400	40	4	2	0	85	1	1	2	135	1	7	2	190	17	8	2	253	2	6	0	322	11	1	2
500	50	5	2	2	106	6	5	0	168	17	0	0	238	12	1	1	316	8	1	1	403	3	11	0

A TABLE *shewing by Inspection the Fines payable for any Number of Years lapsed or expired in a Church or College Lease, &c.*

Annual Rent of Lands.	*7 Years lapſ. Fines Payable.*				*8 Years lapſ. Fines Payable.*				*9 Years lapſ. Fines Payable.*				*10 Years lapſ. Fines Payable.*				*11 Years lapſ. Fines Payable.*			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1	1				1	4	4	0	1	9	1	3	1	14	6	1	2	0	6	0
2	2				2	8	7	3	2	18	3	1	3	9	1	0	4	1	0	1
3	3				3	12	11	2	4	7	5	0	5	3	6	2	6	1	6	2
4	4				4	17	3	2	5	16	6	2	6	18	0	3	8	2	0	2
5	5				6	1	7	1	7	5	8	1	8	12	6	3	10	2	6	3
6	6				7	5	11	0	8	14	10	0	10	7	1	0	12	3	0	3
7	7				8	10	3	0	10	3	11	2	12	1	7	1	14	3	7	0
8	8				9	14	7	0	11	13	1	1	13	16	1	2	16	4	1	1
9	9				10	18	10	2	13	2	2	3	15	10	7	2	18	4	7	1
10	10				12	3	2	2	14	11	4	2	17	5	1	3	20	5	1	2
20	20				24	6	5	0	29	2	9	0	34	10	3	2	40	10	2	3
30	30				36	9	7	1	43	14	1	2	51	15	5	1	60	15	4	1
40	40				48	12	9	3	58	5	6	0	69	0	7	0	81	0	5	3
50	50				60	16	0	0	72	16	10	2	86	5	8	3	101	5	7	0
60	60				72	19	2	2	87	8	3	0	103	10	10	2	121	10	8	2
70	70				85	2	5	0	101	19	7	2	120	16	0	1	141	15	10	0
80	80				97	5	7	1	116	11	0	0	138	1	2	0	162	0	11	1
90	90				109	8	9	3	131	2	4	3	155	6	3	3	182	6	0	3
100	100				121	12	0	0	145	13	9	0	172	11	5	2	202	11	2	1
200	200				243	4	0	0	291	7	6	2	345	2	11	1	405	2	4	2
300	300				364	16	0	0	437	1	3	2	517	14	4	3	607	13	6	3
400	400				486	8	0	0	582	15	0	2	690	5	10	1	810	4	9	0
500	500				608	0	0	0	728	8	10	0	862	17	3	3	1012	15	11	1

A TABLE *shewing by Inspection the Fines payable for any Number of Years lapsed or expired in a Church or College Lease, &c.*

Annual Rent of Lands.	*12 Years lapsed.*				*13 Years lapsed.*				*14 Years lapsed.*				*15 Years lapsed.*				*16 Years lapsed.*			
	Fines Payable.				*Fines Payable.*				*Fines Payable.*				*Fines Payable.*				*Fines Payable.*			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1	2	7	2	2	2	14	8	0	3	3	0	0	3	12	3	2	4	2	7	3
2	4	14	4	3	5	9	4	0	6	5	11	3	7	4	6	3	8	5	3	2
3	7	1	7	1	8	4	0	0	9	8	11	3	10	16	10	1	12	7	11	1
4	9	8	9	3	10	18	8	0	12	11	11	3	14	9	1	3	16	10	7	1
5	11	16	0	0	13	13	4	0	15	14	11	2	18	1	5	0	20	13	3	0
6	14	3	2	2	16	8	0	0	18	17	11	2	21	13	8	2	24	15	10	3
7	16	10	5	0	19	2	8	0	22	0	11	2	25	5	11	3	28	18	6	2
8	18	17	7	1	21	17	4	0	25	3	11	1	28	18	0	3	33	1	2	1
9	21	4	9	3	24	12	0	0	28	6	11	1	32	10	4	1	37	3	10	0
10	23	12	0	1	27	6	8	0	31	9	11	1	36	2	10	0	41	6	5	3
20	47	4	0	2	54	13	4	0	62	19	10	1	72	5	8	0	82	12	11	3
30	70	16	0	3	82	0	0	0	94	9	9	2	108	8	6	1	123	19	5	3
40	94	8	1	0	109	6	8	0	125	19	8	3	144	11	4	1	165	5	11	2
50	118	0	1	1	136	13	4	0	157	9	8	0	180	14	2	1	206	12	5	2
60	141	12	1	2	164	0	0	0	188	19	7	1	216	17	0	1	247	18	11	1
70	165	4	1	3	191	6	8	0	220	9	6	2	252	19	10	1	289	5	5	1
80	188	16	2	0	218	13	4	0	251	19	5	3	289	0	8	2	330	11	11	0
90	212	8	2	1	246	0	0	0	283	9	4	3	325	3	6	2	371	18	5	0
100	236	0	2	3	273	6	8	0	314	19	4	1	361	8	4	2	413	4	10	3
200	472	0	5	2	546	13	3	0	629	18	8	1	722	16	9	1	826	9	10	0
300	708	0	8	1	819	19	10	2	944	18	0	1	1084	5	1	3	1239	14	8	2
400	944	0	11	0	1093	6	6	0	1259	17	4	2	1445	13	6	2	1652	19	7	1
500	1180	1	1	1	1366	13	1	1	1574	16	8	2	1807	1	11	0	2066	4	6	1

A TABLE *shewing by Inspection the Fines payable for any Number of Years lapsed or expired in a Church or College Lease, &c.*

Annual Rent of Lands.	17 *Years lapsed.* *Fines Payable.*				18 *Years lapsed.* *Fines Payable.*				19 *Years lapsed.* *Fines Payable.*				20 *Years lapsed.* *Fines Payable.*				21 *Years lapsed.* *Fines Payable.*			
l.	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1	4	14	2	2	5	7	1	2	6	1	6	1	6	17	6	0	7	15	6	0
2	9	8	5	0	10	14	2	3	12	3	0	1	13	14	11	3	15	11	0	0
3	14	2	7	3	16	1	4	1	18	4	6	2	20	12	5	3	23	6	6	0
4	18	16	10	1	21	8	5	2	24	6	0	3	27	9	11	3	31	2	0	0
5	23	11	0	3	26	15	7	0	30	7	6	3	34	7	5	3	38	17	6	0
6	28	5	3	2	32	2	8	2	36	9	1	0	41	4	11	2	46	13	0	0
7	32	19	6	0	37	9	9	3	42	10	7	1	48	2	5	2	54	8	6	0
8	37	13	8	2	42	16	11	1	48	12	1	1	54	19	11	2	62	4	0	0
9	42	7	11	0	48	4	0	2	54	13	7	2	61	17	5	2	69	19	6	0
10	47	2	1	3	53	11	2	0	60	15	1	3	68	14	11	1	77	15	0	0
20	94	4	3	1	107	2	4	0	121	10	3	1	137	9	10	3	155	10	0	1
30	141	6	5	0	160	13	6	0	182	5	5	0	206	4	10	0	233	5	0	2
40	188	8	6	3	214	4	8	1	243	0	6	3	274	19	9	2	311	0	0	3
50	235	10	8	1	267	15	10	1	303	15	8	1	343	14	8	3	388	15	0	3
60	282	12	10	0	321	7	0	1	364	10	10	0	412	9	8	0	466	10	1	0
70	329	14	11	2	374	18	2	1	425	5	11	2	481	4	7	2	544	5	1	1
80	376	17	1	1	428	9	4	1	486	1	1	1	549	19	6	3	622	0	1	1
90	423	19	3	0	482	0	6	1	546	16	3	0	618	14	6	1	699	15	1	2
100	471	1	4	2	535	11	8	2	607	11	4	2	687	9	5	2	777	10	1	3
200	942	2	9	1	1071	3	5	0	1215	2	9	1	1374	18	11	1	1555	0	3	2
300	1413	4	1	3	1606	15	1	1	1822	14	1	3	2062	8	4	3	2332	10	5	1
400	1884	5	6	2	2142	6	10	0	2430	5	6	2	2749	17	10	1	3110	0	7	0
500	2355	6	11	0	2677	18	6	1	3037	16	11	0	3437	7	3	3	3887	10	8	3

The Use of this Table.

Example 1.] What Fine is to be paid, to make up a Leafe 21 Years, when 9 Years of 21 are lapfed (or let flip) fuppofing the Rent *l.* 147 *per Annnm?*

	per Ann.		*Fine.*
In the Table under 9 Years lapfed againft	*l.* 100	is	*l.* 145 : 13 : 9
againft	40	—	58 : 5 : 6
	7	—	10 : 3 : 11½
Sums,	*Rent* 147		*Fine* 214 : 3 : 2½

There are two Books that afford us a Table of this kind: The firft was written by the Reverend *John Newton*, D. D. one of the King's Chaplains, *Anno* 1668. which contains only Fines, or Value of 1 *l. per Ann.* Rent, all in Decimals. The other is faid to be approved of by a very great Author, (who perhaps never faw it) which is very tedious in its Ufe; as I fhall fhew by the Example above, which is performed by my Table: To find the Fine payable to make up 9 Years lapfed 21 Years; Rent 147 *l. per Ann.* his Anfwer is 1 : 1 : 2 : 5 Purchafe (*Year. Qr. Mon. Tenths.*), taken out of one Table; which he values by another thus:

per Ann.			*l.*	*s.*	*d.*
l. 147	for 1 Year	=	147	00	00
100	for 1 Quarter	=	25	00	00
	2 Months	=	16	13	4
	5 Tenths of a Mon.		4	3	4
l. 40	for 1 Quarter	— —	10	00	00
	2 Months	— —	6	13	4
	5 Tenths of a Mon.		1	13	4
l. 7	for 1 Quarter	— —	1	15	00
	2 Months	— —	1	3	4
	5 Tenths of a Mon.		00	5	10

Sums.

l. per. Ann.	*Fine.*
147.	*l.* 214 : 7 : 6

But 'tis plain the Ufe of my Table (as above) is much more brief, and will be allowed more accurate.

Example 2.] To make up a Leafe of 14 Years any Number of Years lefs than 21: As fuppofe I would add 5 Years to 14 in a Leafe of *l.* 40 *per Ann.* take 5 from 7 there refts 2; then take the Fine payable for 2 Years lapfed = *l.* 8 : 10 : 1 : 1 from the Annual Rent, (here 40) and the Remainer is = *l.* 31 : 9 : 10 : 3 = the Sum to be paid to add 5 Years.

Example 3.] To find the Value of a Church or College full Leaſe of Lands of 21 Years: as ſuppoſe *l.* 80 *per Ann.* The laſt Column in the Table againſt *l.* 80 gives you *l.* 622 : 00 : 1 : 1, the Anſwer.

I need not tell the Reader, That in caſe of Improvement by the Seller, he may advance the Rent to the Buyers in proportion.

SECT. IX. *To Extract the Square and Cube Roots of Decimals and Mixt Numbers.*

1. *For the Square Root.* What is the Square Root of .000976541?

To what is ſaid under the Extraction of the Square Root for Intire Numbers, it is only neceſſary to add for Decimals, that you begin to point over Seconds place of the Decimal given, and ſo proceed over every other towards the right hand: and if the Decimal Places given are not an even Number, you muſt make them ſo, by adding a Cypher towards the right hand.

```
.0009765410 (.03124 = Root.
   6) 76
  62) 1554
 624)  31010
         6034 reſts.
```

2. That for every Decimal Place you would have in the Root, you muſt (by adding Cyphers towards the right hand, if need be) make twice ſo many Decimal Places in that given to have its Root extracted.

There are ſeveral Ways of extracting the Roots, but I chuſe that natural one, according to the Algebraic Canon, as in whole Numbers foregoing.

Secondly, For the Cube Root.

Example 1.] *Of a Mixt Number.* What is the Cube Root of 32.934168093464? Anſwer 3.2054.

In theſe Caſes you muſt point over the Third's place of the Decimal, &c. over every Third toward the right hand adding Cyphers, if the Point fall not over the place next the right hand.

Example 2.] What is the Cube Root of 91? See the Work.

In Examples of this kind, where the whole Number is not a compleat Cube, you muſt add ſo many times 3 Cyphers towards the right hand, as you intend to have Decimal Places in the Root:

as

as here are 6 Cyphers for the 2 Decimal Places in the Root. And if you desire more, it is but putting 3 Cyphers towards the right hand of the Remainer 481151, and proceed to make that Root as exact as you please.

3. If you have the Square or Cube Roots of a Vulgar Fraction to extract, you may reduce it to a Decimal, and then proceed as in the Example above.

4. And if the Decimals have Cyphers before next the Point, keep to the Rule under the last Example of the Cube Root above for the Points over: as .014672, point thus .014672; .0014672, point thus, .001467200; and .00014672 point thus, .000146720, &c. and extract as in Whole Numbers: For in this Case of the Cube of Decimals, their Places must always be, or be made, 3, 6, or 9, &c. by putting Cyphers toward the right hand of the Decimal whose Root is required.

```
91.000000 (4.49
---------
27000
-----
   12
   48
-----
  492
-----
   64
  192
 192
-----
21184
-------
5816000
-------
    132
   5808
-------
  58212
-------
    729
  10692
 52272
-------
5334849
-------
 481151 rests.
```

Having already shewn how to extract the Square, Cube, &c. Roots of Intire Numbers, and Fractions Vulgar and Decimal; I shall in the Use of Logarithms give a much more easy and short Way of Extraction, and also shew the Use of Logarithms in resolving Questions of Interest: on both which accounts, Logarithms (among many other of their Uses) are extremely to be valued.

Here ends DECIMAL ARITHMETIC.

CHAP.

CHAP. IV.

DUODECIMAL ARITHMETIC.

I. NUMERATION.] I have shew'd under *Duodecimals*, in the Definition of Numbers, *Chap.* I. what these Fractions are; as in 1 : 1 : 1 : 1 : 1. (*Foot.* ′ ″ ‴ ⁗) This is read 1 Foot, 1 Prime (or 12th of a Foot) 1 Second (or 12th of a Prime) 1 Third (or 12th of a Second) 1 Fourth (or 12 of a Third) *&c.* which is sufficient to know how to read or write Duodecimals. I shall proceed to

II. *Addition.*] This is no more of consequence to what is above, but to divide the Sum of each Series by 12, putting down the Remainer, and carrying the Quotes to the next. But that next the left hand is done as all other things that have but one Denomination. The Example makes it clear.

	Feet.	′	″	‴	⁗	′′′′′
	321	11	10	9	8	7
	210	10	9	8	7	6
	109	9	8	7	6	5
	98	8	7	6	5	4
	87	7	6	5	4	3
	76	6	5	4	3	2
	65	5	4	3	2	1
Sum	971	0	4	9	1	4

III. *Substraction of Duodecimals.*] A Joiner having lined several Rooms with Cedar, as above; finds the Deductions for the Apertures, as Chimnies, Windows, Beauvets, Arches, Vistoes, *&c.* to be — — —

89	9	8	7	6	5

How many Feet must he be paid for?

Substracting the lesser from the greater, adding 12 where the upper is too little, or 10 in that to the left the Remainer is the Answer — —

881	2	8	1	6	11

IV. *Multiplication.*] This is the principal Rule to be insisted on in this kind of Arithmetic, as being much the oftenest in use, and consequently the most to be observed.

Example

Example 1.] Multiply 9 *F.* 10′ by 8 : 8′ Feet. See the Work in the Margin: Where I say 8 times 10 Primes is 80 or 6′ to carry, and 8″ to be put down; then 8 times 9 is 72, and 6 is 78 Primes, which divided mentally by 12 is 6 *F.* 6′ which I put down as you see. Then I multiply 9 : 10′ by the 8 *F*, saying 8 times 10 is 80 Primes, which is 8′ and 6 Foot to carry; then 8 times 9 is 72, and 6 carried is 78 : the Sum of which 2 Lines or Products is 85 : 2′ : 8″ = the Answer.

	Feet.	′	″
	9 :	10	
	8 :	8	
	6 :	6 :	8
	78 :	8	
		′	″
Foot	85 :	2 :	8 = Answ.

By which Example you may perceive the Necessity of having in your mind the Product of any 2 Numbers under 13, or not exceeding 12, by 12, as in the Multiplication Table.

Example 2.] Multiply 16 : 10 : 6 by 9 : 3. See the Operation. Here 'tis plain I multiply, 1st, the Seconds, then the Primes, and then the Integers by the 3, carrying the Quote of each Product divided by 12 to that towards the left hand; placing the Remainer in each Division under the Degrees of the Factors, as the Example indicateth.

	F.	′	″	
	16 :	10 :	6	
	9 :	3 :	0	
	4 :	2 :	7 :	6
	151 :	10 :	6	
Prod.	156 :	1 :	1 :	6

Example 3.] Multiply 372 : 11′ : 4″ by 25 : 6′ : 3″. I have inserted the whole Work. As, 1*st*, I multiply 4″ by 3 produceth 12, so I carry 1; 11′ by 3 is 33, and 1 is 34, I put down 10 and carry 2: then I multiply 372 by the 3, adding the 2 makes 1118″, all which I put down as you see. Then I multiply 372 : 11 : 4 by 6′, and it produceth 2237′ : 8″ : 0‴, as you see.

		′	″	‴	⁗
	372	11	4		
	25	6	3		
1*st Product*			1118	10	
2*d Product*		2237	8		
3*d Product*		283	4		
Sum		2520	1130	r 10	
Quote		94	r 2		
Sum		2614			
Quote	217	r 10			
Prod. 372 by 25 =	9300				
Add.					
Prod. or Answ. =	9517	10	2	10	

And this I take to be the most easy and natural Way of working, when the Integers are large Numbers.

3*dly*,

3*dly*, I multiply 372 : 11 : 4 by 25 : as 4 times 25 is 100, or $8' : 4''$; put 4 down, and carry 8; 11 times 5 is 55, and 8 is $63'$, put 3 down, and carry 6; 11 times 2 is $22'$, and 6 is 28, or $283' : 4''$.

4*thly*, I sum up those 3 Lines makes $2520' : 1130'' : 10'''$.

5*thly*, I divide the 1130 Seconds mentally by 12, produceth $94'$, which makes $2614' : 2''$.

6*thly*, I divide, as before, $2614'$ by 12, produceth 217 Integers, and $10'$ remains.

7*thly*, I multiply the 372 by the 25 at once, as taught in Decimals, and the Product is 9300: So the Sum or Answer is 9517 Integers $10' : 2'' : 10''' : 0''''$.

Notes to be observed as appears by the abovesaid Examples.

If you multiply 2ds by 2ds, the Product is 4ths; Primes by 2ds place gives 3ds, Primes by Primes gives Seconds, and Integers by Primes gives Primes, and by 2ds gives 2ds, &c. And accordingly it is proper to place them Degrees towards the right hand, that so the Degrees of each part of the Operation may fall under the like in the given Duodecimals.

So also 2 Places given in order, as Integers and Primes, to be multiplied by 2 Places or Denominations, produce 3 Places; 3 Places by 2 produce 4 Places; and 3 Places by 3 produce 5 Denominations, or Degrees, in the Product; *i.e.* 1 less than the Sum of the Exponents or Places, or Names in both the Factors given.

V. *Division of Duodecimals.*] Divide 156 *F.* $1' : 1'' : 6''' : 9''''$ by 9. See the Operation: Where the rest is always reduced into the next Denomination or Degree, and the respective Figures of each Degree added.

$$
\begin{array}{l}
\quad\ \ F.\quad\quad\ \ ' \quad '' \quad ''' \quad '''' \\
9\,(156 : 1 : 1 : 6 : 9\,(17 : 4 : 1 : 6 : 1 = \text{Quote.} \\
\quad\ \ 66 \\
\quad\quad 37 \\
\quad\quad\ \ 13 \\
\quad\quad\quad 54 \\
\quad\quad\quad\ \ 9
\end{array}
$$

There is rarely occasion ('tis to be supposed) to divide by a Number of more than 1 Degree or Name: yet in case there should,

should, it may be done as in the Margin, where $85 : 2' : 8''$ is divided by $8 : 8'$. Where I say, the Eights in 85 are 9 times, which put in the Quote, and say 9 times 8′ is 72, (which is 6 to carry, and put down 0) 9 times 8 Integers is 72, and 6 is 78,

```
 F.  '      '  ''     '
8 : 8) 85 : 2 : 8 (9 : 10 = the Quote.
       78 : 0
       ------
        7 : 2
          86 : 8
          86 : 8
          ------
               0 rests.
```

which put down, and deducted, the Remainer is $7 : 2'$ or $86'$; to which bring down the $8''$, and say 8 in 86 is 10 times; 10 times 8′ is 80, or $6' : 8$, put the 8 down, and carry the 6, saying 10 times 8 is 80, and 6 is 86, which deducted, 0 remains: which proves the Truth of the first Example in Multiplication.

And by the same Rules the Example in the Margin, or any other, is performed. But it may be observed, that in this Division more than one Place of Figures may, and often must, be put in the Quote at once.

```
  '         '  ''  '''     '   ''
9 : 3) 156 : 1 : 1 : 6 (16 : 10 : 6
       148 : 0              = Quote.
       -------
         8 : 1
        97 : 1
        92 : 6
       -------
         4 : 7
           55 : 6
           55 : 6
           ------
                0 rests.
```

CHAP. V.

SEXAGESIMAL ARITHMETIC.

THESE Fractions, or 60th Parts, are by some called Astronomicals, because used in the Mensuration of Time and Motion in Astronomy: as $1\,h : 1' : 1'' : 1''' : 1''''$, that is, 1 Hour, 1 Minute (or 60 part of an Hour) 1 Second (or 60th part of a Minute) 1 Third (or 60th part of a Second) 1 Fourth (or 60th of a Third) &c. of Time. And in Motion or Measure in the Heavens, Earth, or Sea, $1^{\circ} : 1' : 1''$ is 1 Degree (or 60, but according to some 'tis 69 *English* Miles upon the Terraqueous Globe) $1'$ is 1 Minute (or 60th part of a Degree) $1''$ is 1 Second (or 60th of a Minute) $1'''$ is the 60th of a Second, &c. See *Sexagesimal Numbers*, at the beginning of *Chap.* I.

Addition is only to sum up each Column, and divide them by 60, carrying the Quotes, &c. And *Substraction* is so easy, that neither of them need an Example. But

In Multiplication

I think the best way is to work as *per* Margin; as being a methodical and easy Way, which does not charge the Memory: and I am sure 'tis more brief than any Way I have seen. The Rule is plain; for having multiplied the Denominations one in another, and placed the whole Products as you see, I add them; and find the Sum as under the second black Line.

	h.	'	''		
	57 :	29 :	52		Multiply.
	6 :	12 :	10		
			570 :	290 :	520
		684 :	348 :	624	
	342 :	174 :	312		
	342 :	858 :	1230 :	914 :	520
	14 :	20 :	15 :	8	
Answer =	356 :	38 :	45 :	22 :	40

Then I begin at the right hand to divide by 60, and find $520''''$ to be $8''' : 40''''$, placed as you see; and so I proceed with the rest, not making a Digit more than what is down.

Example

Example 2.] 75 h : 33′ : 52″ : 27‴ by 32 : 35′ : 24″ : 42‴. See the Operation just in the same Method with the last Example above.

	h.	′	″	‴	⁗	′′′′′	′′′′′′
Multiply.	75	33	52	27			
	32	35	24	42			
Multiplication of Sexagesimals.				3150	1386	2184	1134
			1800	792	1248	648	
		2625	1155	1820	945		
	2400	1056	1664	864			
Sums Totals	2400	3681	4619	6626	3579	2832	1134
Quotes	= 62	78	111	60	47	18	
Product =	2462	39	50	26	26	30	54 *Answer.*

Here it may be noted, that I multiply 27‴, 52″, &c. by 42‴, without making 2 Lines (as is taught in Multiplication of Decimals).

2*dly*, I begin at 1134′′′′′′ to divide the Line of Totals by 60, (or all except Units place by 6) so the Quote is 18, and Remainer 54; then I divide $\frac{2832}{18}$ by 60, and 47 is the Quotient, and 30 the Remainer, &c.

I shall give the Reader the Proof of this by Division, for a Conclusion, *viz.*

′ ″ ‴ ′ ″ ‴ ⁗ ′′′′′ ′′′′′′ ′ ″ ‴

32 : 35 : 24 : 42) 2462 : 39 : 50 : 26 : 26 : 30 : 54 (75 : 33 : 52 : 27 = Quote.
2444 : 15 : 52 : 30 = the Divisor multiplied in the 75 in the Quote, (and divide by 60.

Division of Sexagesimals.

18 : 23 : 57 : 56 the Difference between the 4 first places in the (two last Lines.
1103 : 57 : 56 : 26 ditto with the Integ. reduc'd to Primes, &c.
1075 : 28 : 35 : 6 the Divisor multiplied in 33′.

28 : 29 : 21 : 20 the Difference between the two last.
1709 : 21 : 20 : 30 ditto with the 28′ reduced to ″, &c.
1694 : 41 : 24 : 24 the Divisor multip. in 52″ in the Quote.

14 : 39 : 56 : 6 the Difference between the two last.
879 : 56 : 6 : 54 ditto with the 14″ reduced to ‴.
879 : 56 : 6 : 54 the Divisor multip. in 27 in Quote.

0 Remains.

CHAP. VI.

POLITICAL ARITHMETIC.

THIS Specie of Arithmetic has nothing new in it, as to the Nature of the Numbers themselves, nor as to the Manner of Operation; but only in the Application or Subject about which the Numbers are employ'd, which is Political (or relating to Polity or Government): As to discover the Riches and Strength of a Nation, by estimating its Income from Land, Trade, Commerce, Art and Labour, Exports, Imports, its Number of People, Males, Females, Fighting-Men, *&c.*

And there have been several Pieces written on this Subject, as Mr *Grant*'s Observations on the Bills of Mortality of *London* in 1676, done at the request of the Royal Society; Sir *William Petty*'s Political Arithmetic in 1687, and his Book of the Use of Duplicate Proportion; the Learned Dr *Halley*'s Observations on the Bills of Mortality of *Breslaw* in 1692, whereby he computes the Value of Annuities for Lives at different Ages thereof, *&c.* as mentioned a little farther; and Dr *Davenant* and *Gregory King* Esq; on Revenues and Trade, *&c.* I shall give some Examples of this Way of Computing: As

Prop. 1. To find the Number of People in *England*, and how many are Males, Females, and Fighting-Men.

First, I find the Number of Houses within the Weekly Bills of *London*, by considering the Number of those Families who pay to the Poor in each Parish, and of the Poor themselves; or by the Poll-Books, King's-Tax-Books, *&c.* Which suppose I find 108000 (as they are thereabout) these at 5 in each Family, one with another, makes 540000 People in the said Compass. Now admit that by a Poll-Tax, *&c.* it has been found that the Heads in *London* are to those in all the rest of *England* as 1 to 9; there must at this rate be 4860000 Souls in *England*. And it having been found by many Years

Years Obſervation upon the Weekly Bills, that there are 14 Males to 13 Females; ſo that of the ſaid 4860000 Perſons, there muſt be 2520000 Males, and 2340000 Females: for

27. 14 :: 4860000. 2520000. Or 27. 13 :: 4860000. 2340000

And hence to find the Fighting-Men, it has been computed, That 37.4 of 100 Souls are above 16, and under 60; therefore

100. 37.4 :: 2520000. 942480 = the Fighting-Men.

Prop. 2. To find the Number of People by the Coals imported: I find, ſuppoſe, by the Coal-Meeters Books 404200 Chaldron imported in one Year, *comm. Ann.* It may eaſily, by conſidering the Medium of the ſeveral Claſſes of Houſekeepers, be found that each Houſe, one with another, burneth about 3¾ Chaldron *per Ann.* So that by Diviſion I find 107786 Houſes, which is but 214 ſhort of the Number of Houſes in the Weekly Bills, found as in the laſt Proportion.

Prop. 3. To find the Number of Houſes within the Bills, by the Yards Square of the Ground. I find (ſuppoſe, as *per* Mr *Grant*) 54 Houſes in every hundred Yards Square, and 2000 ſuch Squares built upon within the Bills of Mortality; which multiplied together gives 108000 Houſes, as *per* the firſt Method.

From theſe and ſuch-like Reaſonings by Political Arithmetic, the Vitality *per Cent.* of ſeveral Ages of Perſons is found:

As that 64 of 100	born are alive	at 6	Years old.
40 of 100	———	———	at 16
25 of 100	———	———	at 26
16 of 100	———	———	at 36
10 of 100	———	———	at 46
6 of 100	———	———	at 56
3 of 100	———	———	at 60
1 of 100	———	———	at 76

And that the Income of *England per An.* is, from Trade *l.* 9000000
Land ——— 14000000
Art and Labour 20000000

So that the whole Annual Income is 43000000 *l. Sterl. per Ann.*

The Income of *France l.* 81000000

Of *Holland l.* 18000000

Alſo that the Exports of *England* in *Engliſh* Commodities are about *l.* 5600000 *per Ann.*

The Current Coin of *England* is about *l.* 7000000.

The

The Expence for the abovesaid Number of People at 8 *l. per* Head *per Ann.* is *l.* 38,880000 *per Ann.* So that *England* increaseth in Riches at that rate, *l.* 4120000 *per Ann.*

The Number of Acres of the Land of *England* has been by the very Judicious esteemed (of the several sorts of Land) as follows: *viz.*

	N° of Acres.
Of Arable Land	9,000000
Pasture and Meadow	12,000000
Wood and Coppice	3,000000
Forrests, Parks, and Commons	3,000000
Houses, Orchards, and Church-yards	1,000000

	N° of Acres.
In the last Column	28,000000
Heaths and Barren Lands	10,000000
Rivers, Lakes and Ponds	500000
Roads and Waste-Land	500000
Sum = the Acres in *England*	39,000000
In *Scotland*	24,000000
Ireland	29,000000

So that by the above said Estimate there is 8$\frac{12}{486}$ Acres for each Head in *England*, and about *l.* 1 : 8 : 10 each in Cash.

	N° of Acres.
There is computed to be in *France*	117,000000
The 17 Provinces	13,000000
Spain and *Portugal*	78,000000
Italy, *Venice*, and Isles thereabouts	66,000000
Germany	120,000000
Sweden, *Norway*, and *Denmark*	156,000000
Poland about	116,000000
Muscovy	468,000000
Turkey in *Europe*	192,000000
Turkey in *Asia*	234,000000
Arabia	585,000000
Persia	507,000000
Acres in *Great Britain* and *Ireland*	92,000000
In last Column	2652,000000
Mogul's Empire	741,000000
China and *Tonquin*	702,000000
Tartary	2730,000000
Georgia	117,000000
Madagascar	78,000000
Sumatra, and Isles thereabout	80,000000
Japan	117,000000
All *Africa*	4680,000000
America	3510,000000
Isles *Wight*, *Man*, *Guernsey*, *Jersey*, and in the *West-Indies*	50,000000

So that the Acres on the whole Earth are computed	15549,000000
Area of the Earth and Sea, as the Superficies of a Sphere in Acres	127675,955000

So

So that the Area of the Sea in Acres is ——— 112126,000000

And the People of the whole World are computed at ——— ——— ——— } 300,000000

But the People in the World, in proportion as the Acres of *England* is to the People of *England*, would be 1937,600000. For

39,000000. 4,860000 : : 15549,000000. 1937,600000

This Difference above 300,000000 (which is about $\frac{1}{6}$ of the whole 1937, *&c.*) is caused by the vast Quantities of Barren Ground, as $\frac{2}{13}$ of *Africa*, $\frac{3}{4}$ *America*, $\frac{5}{7}$ *Tartary*, $\frac{2}{3}$ *Russia*, $\frac{7}{8}$ *Arabia*, &c. which are probably desert and unpeopled.

The very Ingenious and Accurate Dr *Halley*, in his said judicious Remarks upon the *Breslaw* Bills of Mortality, (wherein both the Ages and Sexes of all that died were monthly delivered and compared with the Number of Births, for the Years 1687, 1688, 1689, 1690, and 1691) hath calculated a Table, (published in the *Philosophical Transactions* in 1692, and in *Miscellanea Curiosa*, Vol. I.) and hath shew'd these Uses thereof; as, 1*st*, In finding the Proportion of Men able to bear Arms in any Multitude, from 18 to 56 Years of Age. 2*dly*, The different Degrees of Vitality in all Ages; as at what Number of Years it is an even Lay, that a Person of any Age shall die: for instance, that a Man of 30 Years of Age liveth between 27 and 28 Years; That 'tis 80 to 1, that a Person of 25 Years does not die in a Year; That 'tis $5\frac{1}{2}$ to 1, that one of 40 lives 7 Years. 3*dly*, He computes the Value of Annuities for Lives, (as in the curious little Table after the Tables of Compound Interest foregoing) and the Price of Insurances; and that one half of those who are born, do not live above 17 Years, *&c.*

Mr *Grant* (besides what he computeth, as above, of the Vitality *per Cent.*) saith, That the People of *London* are about one 14th of the People of *England*. And

Sir *William Petty* saith, That there are more People living between the Ages of 16 and 26 than any other Ages; and thence infers, That the Square Root of every Person's Age under 16 (whose Square Root is 4) sheweth the Proportion of Probability of such Person's living to 70 Years: *i. e.* It is 4 times more likely that one of 16 Years of Age lives to be 70, than a Child of 1 Year; 'Tis thrice as probable that one of 9 Years lives to 70, than that a new-born Child does; That the odds is 5 to 4, that one of 25 dieth before 1 of 16 Years old; That 'tis 6 to 5, that one of 36 dieth before 1 of 25 Years: These Proportions being the Roots of 25, 16,

16, and 36 = the Ages, as is above said. He also says, That the Shipping of *Europe* is about 2000000 Tons, of which *England* hath 500000, the *Dutch* 900000, the *French* 100000, *Spain*, *Portugal*, and *Italy* 250000, *&c.*

And besides what is said above of the Number of Acres of each kind of Land in *England*, Messieurs *King* and *Davenant* say farther, That the Increase of the People of *England* is 9000 *per Annum*, Allowances being made for War, Plague, Shipping, and the Plantations. They reckon the Souls of *London* 530000, in the Cities and Market-Towns in *England* 870000, the Villages and Hamlets 4100000. The Annual Produce by Cattel in Butter, Cheese, and Milk, about *l.* 2500000; the Value of the Wool yearly shorn, about *l.* 2000000: of Horses bred yearly *l.* 250000; Value of Flesh yearly spent as Food *l.* 3350000; of the Tallow and Hides about *l.* 600000; Hay yearly consumed by Horses *l.* 1300000; by other Cattel *l.* 1000000, *&c.*

CHAP. VII.

LOGARITHMICAL ARITHMETIC.

IN this kind of Arithmetic the Work of Multiplication, Division, and Extraction of Roots, are performed not by the Numbers themselves, but by Artificial Numbers adapted to those given; so that their Addition performs Multiplication; Substraction does the business of Division; and Dividing by 2, 3, 4, *&c.* of these Artificial Numbers, gives the Square, Cube, Biquadrate, *&c.* Roots of the respective Natural Numbers.

2. There are to every Natural or Common Number, an Artificial one proportioned; which being formed into a Table, where each Natural Number has its own Artificial standing right against it, 'tis called a Table of Artificial Numbers or Logarithms.

3. Of these Tables some have the Logarithms of all Natural Numbers from 1 to 1000, some to 10000, and some to 100000. And

And again, theſe Logarithms conſiſt of Places from 5 to 14; but one and the ſame Table has always but one Number of Places: and the more extenſive any Table is, the more it is univerſally uſeful: and therefore I ſhall by and by give Rules for enlarging a Table.

4. There may be ſeveral kinds of Logarithms contrived; for any Series of Numbers in Arithmetical Progreſſion are the Logarithms of thoſe right againſt them in Geometrical Proportion: As

1,	2,	4,	8,	16,	32,	64,	128,	256,	512,	1024	Natural Numbers in Geom. Progr.
0,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.	Their Logarith. in Arith. Progr.

And the aforeſaid Uſes of Logarithms will appear even in this ſmall Example of a Table: As,

1. *In Multiplication.* As 32 by 16. Againſt theſe in the upper Series, you have 5 and 4 in the lower (or that of Logarithms); therefore add 5 to 4 maketh 9, which ſee in the Logarithms, and over it is 512 = the Product of 32 by 16, done by only adding their reſpective Logarithms 5 and 4. So 32 by 32 is 1024; 8 by 16 = 128, &c.

2. *In Diviſion.* To divide 1024 by 64: Under 1024 ſtands the Logarithm 10; and under 64 ſtands the Log. = 6, therefore take 6 from 10, and the Remainer is the Log. = 4, over which ſtands the Quotient 16. And ſo 512 divided by 16, quotes 32, and proves the firſt Example in Multiplication, &c.

3. *In the Extraction of the Square Root.* Admit you would have the Square Root of 1024: Under that you have the Logar. 10. the half of which is 5, over which Log. is 32 = the Square Root of 1024, by only taking half its Logarithm; ſo the Root of 256 is 16, &c.

4. *In Extracting the Cube Root:* If you would have the Cube Root of 512, under that is the Log. 9. a 3d of which is 3; over which Log. ſtands the Anſwer 8 = the Cube-Root ſought, &c.

5. By this very ſmall Specimen it may be obſerved how neceſſary (becauſe exceeding uſeful) a Table of Artificial Numbers muſt needs be, when they are made ſuch a univerſal Series, as to be fitted for all Numbers from 1 to 100000; and this was the happy Contrivance or Thought of the Famous Lord *Neper*, a *Scotch* Baron, *Anno* 1614. whoſe Memory will always be precious, eſpecially to all Trigonometrical Calculators. But the firſt Table, as aboveſaid, was made

by that Learned Mathematician of our own Country, Mr *Henry Briggs*, ſometime *Savilian* Profeſſor of Geometry in the Univerſity of *Oxford*, for Natural Numbers to 100000, and the Log. to 14 places; which new Invention was extremely approved of and promoted by *Kepler*, and other Foreigners.

6. Theſe Logarithms being a Series of Numbers in Arithmetical Proportion to anſwer 1, 2, 3, 4, &c. to 10 or 100 Thouſand, muſt needs be Decimals, except the Index or firſt Figure towards the left hand, ſeparated from the Logarithm by a Point: for theſe ſhew how many Places the Natural Number conſiſts of, if it be not a Decimal; but if a Decimal, the Index is marked with a Note of Negation, as —, put commonly under the Index or Characteriſtic: which Index, in caſe of Logarithms of Decimals, does ſhew how many Places from the Point the firſt ſignificant Figure ſtands.

The Index of the Log. of all Intire Numbers. from	*In Decimals the Index of*
1, to 10 excl. is 0	.1, .2, .3, &c. is $\underline{1}$
10, to 100 — is 1	.01, .02, .03, &c. $\underline{2}$
100, to 1000 .. is 2	.001, .002, &c. $\underline{3}$
1000, to 10000 is 3	.0001, .0002, &c. $\underline{4}$
10000, to 100000 = 4	.00001, &c. —— $\underline{5}$
&c.	&c.

And 'tis plain, that the Indexes of Logarithms of intire Numbers are always 1. leſs in Value, than the Number of Places in thoſe of whole Numbers.

Note, That for taking the Logarithm out of my Table, &c. you may ſee *Sect.* 2, and 4. of this Chapter.

7. Indefatigable muſt the Trouble and Pains be ſuppoſed, which Mr *Briggs* had in his firſt Table; as muſt be granted, when the Method is underſtood of making but one commonly uſed, conſiſting of half the Number of Places. Which is this: They firſt found ſo many Mean Proportionals between 10 and 1, by the continual adding Cyphers, about 28, and half ſo many afterwards towards the right hand of the 10, and then extracting the Roots, till at laſt the Root was ſo ſmall, as to have as many Cyphers next the Point in the Decimal, as they intended the Log. ſhould conſiſt of Places; which could not be done under 27 very long Extractions: theſe ſe-

veral

veral Means they put down in order, from the extreme 10 downward.

2*dly*, They found Logarithms to these Extreme and Means, by putting down against the Extreme 10.000, &c. the Log. 1.000, &c. (as is beforesaid) and proceeding to take half that Log. continually. half of that half, &c. and placing them orderly against their respective Mean Proportionals found as before.

3*dly*, Then when they had a mind to make the Log. of any Number, they, by extracting the Square Root, found so many continual Means between the Number given and 1, that the last Mean was the least Root that had so many Cyphers next the Point, as the Logarithm was to have Places.

4*thly*, Then to find Logarithms to these Means; the Proportion is
As the significant Figures of the Mean Proportional found between 10 and 1 (of the like place with the last continual Mean in these you are upon)
Is to its respective Logarithm (omitting the Cyphers)
So is the last Mean in the Series you are now a working,
To its Logarithm.

5*thly*, This Logarithm of the last continual Mean between the Number you are investigating the Log. of, and 1, being found, is to be doubled, and that, &c. Products doubled continually; putting the several Products against the respective Means last found, till at last the Product falling against the Number whose Log. is sought, gives you that Logarithm, by which several others are easily found: for the Log. of 4 multiplied by 2, gives the Log of 16; $\frac{1}{2}$ Log. of 4 is that of 2: so have you the Logarithms of all the Powers of 2; and the Log. of 10 (or 1.0) less the Log. of 2. leaveth the Log. of 5. So you have the Log. of all the Powers of 5, and Product by 2 and 5.

I did intend to have inserted here the Whole of finding the Log. of any Number between 1 and 10; but there being lately so many briefer Ways of finding the Logarithms, published by Mr *Mercator*, Mr *Gregory*, Mr *Speidal*, and especially by Dr *Halley*, in *Philosophical Transactions*, N° 216, that I thought it not worth while to give an Example of the old tedious Way above-mentioned.

It could hardly be expected, that in one Treatise handling so many different Parts of Arithmetic, I should insert a Table of Logarithms; which alone, with its Use, makes a considerable Volume: And especially considering there are so many Tables already publish'd; for besides Mr *Briggs*'s Original, there are Transcripts by

Norwood, *Gunter*, *Wingate*, *Moor*, *Newhouse*, *Leyburn*, *Sturmy*, *Taylor*, *Harris*, *Ward*, &c. *Englishmen*; besides those of *Kepler*, *Vlacq*, *Ursin*, &c. Foreigners. But I have however (the better to make this Book answer to the Title) inserted a Table to 10000, and Rules to make it to 100000000.

SECT. II. *To Add*, *Substract*, *Multiply*, *and Divide Logarithms*; *or to perform the Work of Multiplication*, *Division*, *to Square and Cube*, *and Extraction of Roots of Intire Numbers by the Logarithms.*

BUT it must first be observed, That

I. *To take any Logarithm in my Table belonging to any Natural Number*:

If the Natural Number consist of three Places only, you have them in the Column under *Natural Numbers*; and in the next Column toward the right hand are their respective Logarithms under (0).

But if the Number given consist of four Places, look for all but Units place under *Natural Numbers* as before, right against which, and under the Digit in Units place, is your Logarithm required. And in like manner for any Logarithm under 100, see for that in Tens place under *Natural Numbers*; and for that in Units at the head of the Column, and in the common Angle of meeting stands the Logarithm sought.

II. *To find the Natural Number to a Logarithm.*

This you will easily do by the help of your Index, and especially by considering the Nature of the Numbers which increase from the left hand towards the right; as the Digits at the head of the Columns, and those in the left hand Columns respectively increase where four Places is in the Natural Number. But downwards the Logarithms are to such Natural Numbers as increase by Ten: Thus the Logarithm 8123785 I find under 2, and against 649; so that it is the Logarithm of 6492: but if it were the Logarithm 8122447, I find that under (0) and against 649; so that 6490 is the Natural Number sought. And the same Logarithm is also for 649: As all are Logarithms for the Natural Number expressed in the Table, or for that Number multiplied by 1 or more Tens; but you are to be guided by your Index what places your Number is.

So

So much for the Logarithm of Integers: And for those of Fractions and Mixt Numbers, see *Sect.* 3. following, and more of the above, *Sect.* 4.

I. *Multiplication of Intire Numbers by the Logarithms.*

Example 1.] Multiply 91 by 93.

Rule. To the Logarithm of 91, which is = 1.9590414
Add the Logarithm of 93 —— —— = 1.9684829

Product = 8463; the Nat. Numb. to the Log. 3.9275243 Sum.

Example 2.] Multiply 4 by 177.

Rule. To the Log. of 177, which is = 2.2479733 } Add.
Add the Log. of 4, which is = 0.6020600 }

Product 708; its Logarithm = 2.8500333 Sum.

These two Examples are sufficient to shew how to multiply any intire Number by another; by the Logarithms, where I need not give many words to shew that against the Natural Numbers given stand their Logarithms; and against the Sum of those Logarithms stand the respective Natural Numbers 8463 and 708: *i. e.* In my Table you have the Log. 9275243 under 3, and against 846 (the Units place, where there are 4) being always over the Logarithm. See *Sect.* 4. and the Rules above for finding the Logarithms, and their Natural Numbers.

II. *Division by Logarithms.*

Example 1.] Divide 8463 by 93.

Rule. From the Logarithm of 8463, which is = 3.9275243
Deduct the Log. of 93, which is = 1.9684829

Rests the Log. of the Quote 91, which is = 1.9590414

Example 2.] Divide 708 by 177.

From the Logarithm of 708, which is = 2.8500333
Deduct the Log. of 177, which is = 2.2479733

Rests the Log. of the Quote 4; which is = 0.6020600

III. *To Square any Intire Number, and Extract the Square Root by the Logarithms.*

1. To square (or involve in itself) any whole Number.

Example.]

Example.] What is the Square of 99?

Rule. Multiply the Logarithm of 99, which is = 1.9956352
by ——— ——— 2

9801, the Square sought; being the Natural Number to the Product ——— ——— 3.9912704

2. To evolve, or extract the Square Root of any *Intire Number.*

Example.] What is the Square Root of 9801?

Rule. Take half of the Logarithm of 9801. Log. is 3.9912704
Half that Log. of 9801 is = 1.9956352

Which half is the Log. of 99 = the Root sought. And this proves the last Question.

IV. *To Cube, or Extract the Cube Root of any Intire Number, by the Logarithms.*

1. To Cube any *Whole Number.*

Example.] What is the Cube of 21?

Rule. Multiply the Log. of 21, which is ——— = 1.3222193
by ——— ——— 3

And the Cube required is 9261 = the Natural Number of the Log. ——— ——— 3.9666579

2. To extract the Cube Root of any *Intire Number.*

Example.] What is the Root of 9261?

The Log. of 9261 is as before = 3.9666579
$\frac{1}{3}$ of that Logarithm is = 1.3222193

Whose Natural Number is 21 = the Cube Root required.

3. If you desire the Biquadrate of any Number, multiply its Log. by 4, gives the Log. of the Answer: And if you would have the Biquadrate or Sursolid Roots, &c. of a Number, divide its Logarithm by 4 or 5, &c. Examples are needless; but note, that if the Index is less than the Divisor, take so often as the Divisor can be had in 2 places.

SECT. III. *To perform the Work of Multiplication, Division, Square or Cube, &c. or Extract the Roots of Fractions and mixt Numbers, by the Logarithms.*

BEFORE I give Examples of Logarithms, it will be necessary to give a Rule for *Adding or Substracting Negative Indices or Characteristics.*

Note,

Note, That when the Index is mark'd under thus (—) it is so much less than nothing as those beforegoing, or $\underline{2}$, $\underline{3}$, $\underline{1}$, *&c.*

1. The Rules for adding and substracting the Indices of Logarithms are numerous, and not easy to retain in mind: I have therefore supposed (which taking the Reason of the thing along with it, easiy solves all, without charging the Memory) when the Index is negative, that it is a Person owing so many *l.*100 more than he is worth, as the Index-Digit expresseth.

Addition of Indices.		*Substraction.*	
Exam. 1. Add the Index	$\underline{3}$	From ———	$\underline{2}$
to ——— ———	1	Take ———	1
Sum =	$\underline{2}$	Rests	$\underline{3}$
Exam. 2. Add ——— ———	3	2*dly*, From ———	2
to ——— ———	$\underline{1}$	Take ———	$\underline{1}$
Sum =	2	Rests	3
Exam. 3. Add ——— ———	$\underline{2}$	3*dly*, From ———	$\underline{5}$
to ——— ———	$\underline{3}$	Take ———	$\underline{2}$
Sum =	$\underline{5}$	Rests	$\underline{3}$
		These prove the Truth of those in Addition.	

This being done, the adding to or taking from the upper Index acording to the Nature of each, makes the Sum or Remainer plainly appear.

1. So in the first Example of Addition above, I consider that if I add *l.* 100 to a Person who is 3 worse than nothing, the Sum must be 2; that is, I then make him worth but 2 less than nothing.

2. In the second Example, if I add 1 less than (0) to; that is, if I take 1 from a Person who hath 3, the Sum is but 2; for that is all that is left. And if I add 3 less than nothing to him who before was 2 worse than (0) I leave him 5 less than (0), as in Example 3.

So in Substraction of Indices:

3. If (as in the first Example) I take 1 from him who already is 2 worse, or less than (0), I leave him 3 less than nothing; and so $\underline{3}$ is the Remainer.

4. If

4. If (as in the 2d Example) I take $\bar{1}$ (or 1 less than nothing) from him that hath 2 in possession already, I then leave him worth 3, because I take 1 less than nothing from him: therefore I add 1 to him, that is, I make 2 into 3, which is therefore the Remainer. (See the words *Negative Arithmetic* in Algebra.) And all this consider'd right, is but one single, short, and easy Rule.

II. *To find the Logarithm of a mixt Number, or of a Decimal, by the Table.*

Prob. 1. *To find the Logarithm of a mixt Number, as of* 9.177.

Rule. Take the Log. of 9177, except the Index; and to that Logarithm prefix an Index according to the Number of Places in the integral Part of the Number given, as by the little Table under the 6th Head of the first Sect. of this Chapter, and you have the Answer.

Examp. Thus the Log. of 9177 (without the Index) is 9627007; and the proper Index for 1 Place being 0, therefore the Answer is 0.9627007.

So also the Log. of 91.77 is = 1.9627007, and of 917.7 'tis = 2.9627007.

Prob. 2. *To find the Logarithm of a Decimal, as of* .5

Rule. Take the Log. of 5, except the Index, and prefix to that Log. an Index agreeable to the Decimal, as *per* the said little Table under the 6th Head of the 1st Sect. of this Chapter.

Examples. The Log. of 5 is = 6989700, without its Index: to which Log. if you put its proper Index $\bar{1}$, (there being no Cyphers between the Point and the 5) the Answer is $\bar{1}$.6989700. And by the same Rule the Log. of .05 is $\bar{2}$.6989700; of .005 = $\bar{3}$.6989700; and of .000579 the Log. is $\bar{4}$.7626785; being the Log. of 759, with the Index of .000579 put before it.

III. *To find Mixt Numbers or Decimals to Logarithms by the Table.*

Prob. 1. *To find a Mixt Number to a Logarithm.*

Rule. Regard not the Index; but look for the Logarithm among those which would have the greatest Indexes, which having found, take the Natural Number standing against it, and point off for Integers according to the Index given.

Example. What is the Natural Number to the Log. 1.9627007?

My Table extending to 10000, the greatest Index is 3; among which I find my Logarithm given, and right against it I find

9177.

9177. Now the Index given being 1, requires that 2 of the 4 Places be Integers, so that the Answer is 91.77: And also the Natural Number to 2.9627007 must be 917.7, *&c.* as *per* the Rule for Indexes above.

Prop. 2. *To find the Decimal to a Logarithm.*

Rule. Look for your Logarithm in that part of the Table where 'tis to be found, as before in the last, and put down the Natural Number that stands right against it: then according to the Greatness of the Index given, make the Point of the Decimal.

Example. What is the Natural Number to $\bar{4}$.7626785?

Against this Logarithm I find 5790. Now because my Index given is $\bar{4}$, I place 5 in the 4th place from the Point, and say therefore that .000579 is the true Decimal sought.

IV. *To Multiply Mixt Numbers and Decimals, by adding their Logarithms.*

I have so fully prepared the Reader for this, that I only need to give Examples.

Example 1. A Mixt Number by a Mixt, as 9.336 by 12.09

To the Logarithm of 9.336, which is 0.9701608
Add the Log. of 12.09, which is —— 1.0824263

The Sum is the Log. of 112.9 the Product, *viz.* 2.0525871

Example 2. A Decimal by a Decimal, as .09 by .123

To the Log. of .09, which is —— $\bar{2}$.9542425
Add the Log. of 123, which is —— $\bar{1}$.0899051

The Sum is the Log. of the Product .01107, which is $\bar{2}$.0441476

Example 3. A Whole Number by a Decimal, as 765 by .00345

To the Log. of 765, which is 2.8836614
Add the Log. of .00345, which is $\bar{3}$.5378191

The Sum is the Log. of the Product 2.639, *viz.* 0.4214805

Example

Example 4. A Mixt Number by a Decimal, as 1.23 by .00009

To the Log. of 1.23, which is —— 0.0899051

Add the Log. of .00009, which is $\underline{5}$.9542425

The Sum is the Log. of the Product .0001107 = $\underline{4}$.0441476

Note, That *Whensoever* 1 *is carried to the Index*, *that* 1 *is affirmative*, and must be added to the lower Index, and that Sum to the upper, as *per* the Rules and Examples of adding Indices at the beginning of this Section; and more particularly in the three last Examples.

V. *To divide Mixt Numbers, Decimals,* &c. *by Substracting their Logarithms.*

In the following Scheme you have all the Cases that can happen under this Head: which will be easily performed, only observing the first and second Examples of substracting Indices, and the Rule at the beginning of this Section.

The

The 9 Cases of Division, done by Logarithms.		
Cases.	*Numbers given, or Dividends and Divisors, &c.*	*Their Logarithms; Substract that of the Divisor.*
1.	Divide 237.3 by 25.32 Quote = 9.376	—— 2.3752977 —— 1.4034637 Rests 0.9718340
2.	Divide 11.5 by 932 Quote .01234	—— 1.0606978 —— 2.9694159 Rests $\bar{2}$.0912819
3.	Divide .1234 by .321 Quote .3846	—— $\bar{1}$.0913151 —— $\bar{1}$.5065050 Rests $\bar{1}$.5848101
4.	Divide .463 by 3214 Quote = .000144	—— $\bar{1}$.6655810 —— 3.5070459 Rests $\bar{4}$.1585351
5.	Divide 2 by 3.214 Quote .6222	—— 0.3010300 —— 0.5070459 Rests $\bar{1}$.7939841
6.	Divide 5 by 365 Quote = .0137	—— 0.6989700 —— 2.5622929 Rests $\bar{2}$.1366771
7.	Divide 1.26 by .002 Quote = 630	—— 0.1003705 —— $\bar{3}$.3010300 Rests 2.7993405
8.	Divide .5 by 7.5 Quote = .06 *r* 1	—— $\bar{1}$.6989700 —— 0.8750612 Rests $\bar{2}$.8239088
9.	Divide 999 by .00013 Quo. 7684615.384	—— 2.9995655 —— $\bar{4}$.1139433 Rests 6.8856222

VI. *To Square any Mixt Number or Decimal, or to Extract the Square Root thereof.*

1. *To square such Numbers.*

Rule. Multiply the *Logarithm* of the Root given by 2; but if the Index be negative, out of the Product of that by the 2 deduct what Tens are carried from the place next toward the right hand, and put down the Remainer for the Index of the Product (or *Log.* of the Square).

Examples.

I. For Squaring Decimals, *&c.* by *Logarithms.*

The Square Roots given.	*The Logarithms of those Roots.*	*Double the last Col. or the Log. of the Squares.*	*The Squares sought.*
987.6.	2.9945811	5.9891622	975353.76
98.76.	1.9945811	3.9891622	9753.5376
9.876.	0.9945811	1.9891622	97.5353
.9876.	$\underline{1}$.9945811	$\underline{1}$.9891622	.9753
.09876.	$\underline{2}$.9945811	$\underline{3}$.9891622	.009753
.009876.	$\underline{3}$.9945811	$\underline{5}$.9891622	.00009753
.0009876.	$\underline{4}$.9945811	$\underline{7}$.9891622	.0000009753
.00009876.	$\underline{5}$.9945811	$\underline{9}$.9891622	.000000009753

Or, II. For the Square Roots; these Titles observe.

Squ. Roots sought (or the Natural N° to the half Logarithms.)	*½ the Logarithms of the Squares; or those of the Roots.*	*Logarithms of the Squares given.*	*Squares given, whose Roots are required.*

2. *Observe in Extracting the Square Root*, that is to say, in taking half of the *Log.* of the Square; if the Index be an odd Number, add 1 to it in your mind, and so proceed: As in taking half $\underline{3}$.989, *&c.* I say half of 4 is 2; of 19, = 9: of 18, = 9, *&c.*

VII. To

VII. *To Cube, or Extract the Cube Root of Decimals, &c.*

Examples.

I. For the Cubing of Decimals by *Logarithms.*

The Cube Roots given.	*Logarithms of those Roots.*	*Three times the Logarithms.*	*The Cube Numbers sought.*
3.1	0.4913617	1.4740851	29.791
.31	$\underline{1}$.4913617	$\underline{2}$.4740851	.029791
.031	$\underline{2}$.4913617	$\underline{5}$.4740851	.000029791
.0031	$\underline{3}$.4913617	$\underline{8}$.4740851	.000000029791
.00031	$\underline{4}$.4913617	$\underline{11}$.4740851	.000000000029791
.000031	$\underline{5}$.4913617	$\underline{14}$.4740851	.000000000000029791

Or, II. For the Cube Root observe these Titles of the Columns.

Cube Roots sought.	*⅓ of those Logarith. being those of the Roots.*	*Logarithms of those Cube Numbers.*	*Cube Numbers given; whose Roots are required.*

1. You see by the Examples, that to Cube any Number (as those next the left hand) is to multiply their *Logarithms* (as those in the 2d Column) by 3; and the Products are those in the 3d Column, whose Natural Numbers required are those in the 4th Column. And here note, That what Tens you carry, you must abate out of 3 times the Index; as taught for the Square Root.

2. For the Examples of Extracting the Cube Root: The Number in the Column next the right hand are Examples of Cube Numbers given, the next Column are their *Logarithms*, and the third Column are third parts of those *Logarithms*. Where note, That if your Index cannot be divided by 3 without a Remainer, you must add so many Units to it, as is necessary for that end: And so many Units as you borrow'd there, you must pay Tens to the

the next Figure, and then take a 3d of the whole Log. The Examples make all plain.

Or *for finding the Biquadrate, Surſolid, or Squared Cube Powers.*

Rule 1. Multiply the Log. of the Root given by 4, 5, or 6; obſerving the Rules given before, abating what you carry, out of the Product of the Index reſpectively.

Or *for finding the Biquadrate, Surſolid, or Squared Cube Roots.*

Rule 2. Divide the Log. of the Power by 4, 5, or 6; obſerving the Rule of adding ſo many to the Index, that you can divide it without a Remainer, and paying ſo many Tens as you added or borrow'd Units, to the next Figure.

Examples to Rule 1.	*Logarithm of the Powers.*	*Whoſe Nat. N° are the Anſ. or Powers requir'd*
Thus *the Biquadrate* of .3; its Log. being = $\bar{1}.4771212$; 4 times that —	= $\bar{3}.9084848$	= .0081
The *Surſolid Power* of .3; its Log. being $\bar{1}.4771212$; 5 times that — —	= $\bar{3}.3856060$	= .00243
The *Squared Cube* of .3; its Log. being $\bar{1}.4771212$; 6 times that — —	= $\bar{4}.8627272$	= .000729

And according to the laſt Rule.	*Log. of the Roots.*		*Roots ſought.*
The *Biquadrate Root* of .0081, its Log. being $\bar{3}.9084848$; $\frac{1}{4}$ of that is — —	= $\bar{1}.4771212$	=	.3.
The *Surſolid Root* of .00243, its Log. being $\bar{3}.3856060$; $\frac{1}{5}$ of that is — —	= $\bar{1}.4771212$	=	.3.
The *Squared Cube Root* of .000729, its Log. being $\bar{4}.8627272$; $\frac{1}{6}$ of that is	= $\bar{1}.4771212$	=	.3.

Which later Examples prove the Truth of the former.

VIII. *To find the Logarithms of Vulgar Fractions, and of the Operations thereby.*

Prop. To find the Log. of $\frac{3456}{4567}$.

Rule. From the Log. of the Numerator deduct the Log. of the Denominator, and the Remainer is the Log. for the Anſwer.

Example.

Example. From the Log. of 3456, which is $=3.5385737$
Take the Log. of 4567, which is $=3.6596310$

Remains the Log. of $\frac{3456}{4567}$, which is $=\bar{1}.8789427$

Prop. 2. To find the *Log.* of a compound Fraction.

Rule. Reduce the Compound to a Simple, and work as above.

Example. To find the *Log.* of $\frac{6}{4567}$ of $\frac{576}{1}$.

The Compound in a Simple being $\frac{3456}{4567}$, the *Log.* is as before; Or *Log.* of $\frac{6}{4567}$ of $\frac{576}{1}=\bar{1}.8789427$.

Prop. 3. To find the *Logarithm* of a Mixt Number.

Rule. Reduce it to an Improper Fraction, and find the *Logarithm* as *per* the first Proposition. This needs no Example.

Prop. 4. To reduce a Vulgar Fraction to a Decimal.

Rule. Find the *Log.* of the Vulgar Fraction as *per* the first: then against that *Log.* in the Table you have the Decimal.

Example. To Reduce $\frac{3456}{4567}$ to a Decimal. The *Log.* of this Vulgar Fraction, as before, is $=\bar{1}.8789427$; and the Natural Number to that, *per* the Table, is $.75\overline{6}7=$ the Answer.

And for reducing such Fractions as this given, whose Terms have so many places, I take this to be the most easy and speedy Way of reducing them to Decimals: and if your Table of *Logarithms* were to 100000, or where the Numerator is a place less than the Denominator, the Rule would in most Cases be exact enough.

Prop. 5. What is the Product of one Vulgar Fraction by another, giving the Answer in a Decimal Fraction?

Rule. Add the *Log.* of the one Fraction to the *Log.* of the other found as *per* the 1st Prop. and look for the total *Log.* in the Table, against which stands the Decimal required.

Example. What is the Product of $\frac{3456}{4567}$ by $\frac{765}{9876}$?

$\frac{3456}{4567}$ its *Log.* as before $=\bar{1}.8789427$
$\frac{765}{9876}$ its *Log.* — — $=\bar{2}.8890803$

The Product for Answer is .05862, whose *Log.* is $\bar{2}.7680230$

And thus 'tis plain, that knowing by the Rules delivered how to find the *Logarithms* of Vulgar and Decimal Fractions, they may be multiplied or divided simply or promiscuously. For,

6. Suppose it were required to divide .05862 by $\frac{765}{9876}$:

If

If from the *Log.* of .05862, which is = $\bar{2}.7680230$
You take the *Log.* $\frac{765}{9876}$, which is = $\bar{2}.8890803$

The Remainer is the *Log.* of .7567, or of $\frac{3456}{4567}$, *viz.* — — — — } = $\bar{1}.8789427$

Which proves the Operation above.

7. By what is abovesaid, it will follow that the Powers or Roots of Vulgar Fractions are easily found by multiplying the *Log.* of the Vulgar Fraction assigned as a Root, by 2, 3, 4, 5, 6, *&c.* to obtain the Power, or by dividing the *Log.* of the Vulgar Fraction assigned as a Power, by 2, 3, *&c.* to obtain the respective Roots. And this I take also to be the easiest and briefest Way of finding the Powers or Roots of Vulgar Fractions; especially where the Terms consist of 3, 4, or 5 places, and your Table of *Logarithms* is large enough.

SECT. IV. *The Explanation of my Table of Logarithms, whose Method somewhat differs from* Briggs, Norwood, *and some others.* 2dly, *Of the Arithmetical Complements of Logarithms.* 3dly, *Of Parts Proportional and Differences: And,* 4thly, *How to enlarge the Tables of Logarithms several Ways.*

1. THE easiest Tables for a Learner to use, are those that have the Indices put down, and *Logarithms* to the Natural Numbers which are intirely placed successively. But I would here explain to him another kind of Table placed without the Indices, and which have not the Units place of the Natural Number with the rest, but over the respective *Logarithm* (this is the best Method, and which I have therefore observed in my following Table of *Logarithms*) thus

N°.	0	1	2	3	4	*&c.* to 9
522	7176705	7177537	7178369	7179200	7180032	
523	7185017	7185847	7186677	7187507	7188337	
524	7193313	7194142	7194970	7195799	7196627	
525	7201593	7202420	7203247	7204074	7204901	

In

In Case such a Table as this is to be used; you must, for example to find the Logarithm of 5244, put down, *1st*, 3 for your Index, as *per* the Rules in the sixth Paragraph of the first Sect. of this Chapter.

2dly, Having found 524, (the three first Figures of your Natural Number given toward the left hand) cast your eye in the line toward the right hand, till you come under the Figure in Units place of the Number given, (here 4) where against 524, and under 4, you have the Log. required: for the Log. of 5244 is 3.7196627.

So the Log. of 5223 is 3.7179200; and that of 523 = 2.7185017: the Log. of the last Natural Number having all its places found together under *Number*; the Log. against 523, and under 0, is that sought, &c.

These Logarithms in Mr *Brigg*'s first and large Table, also in *Norwood* and some others, stand thus; (tho' they take up much room.)

Nº	Log.	Nº	Log.	Nº	Log.
522	2.7176705	5221	3.7177537	5231	3.7185847
523	2.7185017	5222	3.7178369	5232	3.7186677
524	2.7193313	5223	3.7179200	5233	3.7187507
525	2.7201593	5224	3.7180032	5234	3.7188337, &c.

II. *The Arithmetical Complement* of a Logarithm, is that which will make it 10.0000000; so that to find the Complement, the Logarithm given must be deducted from the 10.0, &c.

Now the best way to substract this (not according to the common way, but) to take every Digit of the Subtrahend from 9, except that in Units place, which take from 10: And to do this, you may begin at either end to substract.

	10.0000000
Log. given =	3.7204901
Its Compl. Arith. =	6.2795099

The Complement Arith. is often used in Analogies wrought by Artificial Numbers, to prevent Trouble and Confusion: For instead of substracting a Logarithm from the Sum of two others, it is usual to add the Complement Arithmetical, and from the Sum of the three Logarithms to abate Radius or 10.0 from the Index of that Sum; which gives the same Answer as tho' you had deducted the Logarithm of the first Number.

To instance in a common Question in the Rule of Proportion: If I give *l.* 25 : 10 : — for 48 Yards of Cloth; what will 15 cost at that rate?

Yards 48, its *Logarithm* = 1.6812412

l. 25.5, its *Logarithm* = 1.4065402 } Add.
Yards 15, its *Logarithm* = 1.1760912 }

Sum 2.5826314

From which deducting the first *Log.* the Remainer is that of *l.* 7.968, *viz.* — } = 0.9013902

This by the Complement Arithmetical is done thus:

48. Compl. Arith. = 8.3187588
25.5 : : its *Log.* — = 1.4065402
15, its *Log.* — = 1.1760912

Sum abating 10 in the Index, the *Log.* of 7.968 the Answer as before — } = 0.9013902

These kinds of Operations are much used in Trigonometry, with the *Logarithms* of Natural Sines, Tangents, &c.

III. *Of Differences and Proportional Parts.*

1. By the *Differences* is meant the Differences of the *Logarithms* of 2 Numbers immediately and numerically succeeding one another, or the Differences of the Products of such Numbers multiplied by 10.

2. By *Proportional Parts* you are to understand, one or more tenth Part of those Differences. And these are apply'd in Practice thus:

IV. *When you would find the Logarithm of any Number that is greater than what is contained in the subsequent Table of Logarithms,* and you would do this *by the help of the Table of Differences and Parts Proportional.*

Suppose you were to find the *Logarithm* of 51716: see the Operation.

I find in the Table the *Log.* of 5171 or 51710, which is — — — } = 7135745
And of the next Number 5172 or 51720 = 7136585

The Difference of these *Logarithms* is = 840 *Diff.*
Which found in the Table of Differences (following that of the *Logarithms*) right against that 840 and under 6 (the Digit in Units place — 504 *Add.*
7136249 *Sum.*

of

of the Number given) you find the Proportional Part 504; which added to the Logarithm of 5171, the Sum is that of 51716, which with its proper Index is 4.7136249. You may prove the Truth of this by dividing 51716 by 7, the Quote is = 7388; to the Log. of which if you add that of 7, the Sum is the Log. of 51716, as above.

2. *To prove the Truth of any Logarithms in the Table, without the Table of Proportional Parts:* as those of 7000 to 7010 exclusive.

The Difference between the Logarithms of 700 and 701, or 7000 and 7010, is 6200.

Tenths of the Difference	*Are these Proportional Parts.*	*Log. of 7000 repeated.*	*Sums of the last Logarithms and Proport. Parts respectively.*	*The Natural Numbers to the last Logarithms.*
1	620	8450981	8451601	7001
2	1240	8450981	8452221	7002
3	1860	8450981	8452841	7003
4	2480	8450981	8453461	7004
5	3100	8450981	8454081	7005
6	3720	8450981	8454701	7006
7	4340	8450981	8455321	7007
8	4960	8450981	8455941	7008
9	5580	8450981	8456561	7009

By this it appears, that the Difference between the Logarithms of 7000 and 7010 being divided into 10 parts, 1 Tenth is 620, the Part to add to the Logarithm of 7000, to give the Logarithm of 7001; 2 Tenths added to the Logarithm of 7000, gives the Logarithm of 7002; 3 Tenths of the Difference 6200, which is 1860, added to 8450981 (the Log. of 7000) giveth 8452841 = the Logarithm of 7003, &c.

And if you would have the Proportional Part immediately to make the Log. of 7000 that of 7008, it is only multiplying a Tenth of the Difference (which here is 620) by 8, and the Product is the Proportional Part 4960; which added to 8450981 = the Log. of 7000, the Sum is 8455941 = the Log. of 7008 required. And thus the 9 Logarithms between 7000 and 7010 exclusive, are examined and proved: And so may any other without a Table of Differences and Proportional Parts, or by even calculating or examining those at the same time.

 3. *To*

3. *To augment the Table of Logarithms*, so as to make that which extends only to shew the Log. of any Natural Number under 10000, subservient to give that of any Number under 100000, &c. without the Table of Parts Proportional. For Example;

1. To find the Log. of 64736.

The Log. of 6473 or 64730 is = 8111056
Of 6474 or 64740 is = 8111727

The Difference = 671

A Tenth of that Difference (being the Proportional Part for 1, that is to make the Log. of 64731)— } 67.1
6

Which multiply by 6 (the Place of Units in the given Number)-produceth —— ——— } 402.6

Or because the 6 is above one half, you may make the Proportional Part for 6 (in Units place of the Number given) ——— ——— ——— } 403

Which added to the Log. above of 64730, *viz.* —— 8111056

The Sum is the Log. of 64736 or Answer = 4.8111459

For as 10. 671 : : 6. 403 *ferè*.

i. e. As the Number of Cyphers added to the Natural Number in your Table with a 1 before such (0) towards the left hand;
Is to the Proportional Part, or Difference of the two Logarithms:
So is the Figures towards the right hand exceeding the Places for which your Table is calculated,
To the Proportional Part to be added to the Log. to answer those Figures.

2. To find (by a Table which has only the Logarithms to Numbers under 10000) what the Logarithms are for all Numbers under 1000000: For instance what is the Log. of 647365?

Log. of 647300 is = 8111056
of 647400 is = 8111727

Difference = 671

$\frac{1}{100}$ thereof, there being 100 difference between the Nat. N° of which these are Logarithms ——— } 6.71

This multiplied by 65 (the 2 places toward the right hand exceeding the Compass of the Table) —— } 65

Produceth = 436.15

Which added to Log. 647300 } 8111056

Sum = 8111492 = the Log. of 647365

For 100. 671 : : 65. 436. 15 the .15 may be omitted.

3. To

3. To find (by a Table which has only the *Logarithms* to Natural Numbers not exceeding 10000) what the *Logarithms* are for all Numbers not exceeding 10000000000. For example,

I would know the *Logarithm* of 2966820514.

	Nat. N°.	*Logarithms.*
As before, the *Log.* of the most places towards the left hand of the Number given, contained in my Table is of 2966, which is also of — —	2966000000 =	4721711
The *Log.* of the next Number is of 2967 or of — — — — — —	2967000000 =	4723175
The Difference of these *Logarithms* is =		1464
Which Difference divided by 1000000 that each Unit may have its Proportional Part of that Difference, that for 1 is — — — —	=	.001464
But because the Numb. given requires 820514 of those Parts for its Part Proportional, therefore I multiply the Difference (as *per* the former Rules) by —	=	820514
And the Product is the Part Proport. for 820514 =		1201.232496
Which (omitting the Decimal as useless) added to the *Log.* of 2966000000 as above, which is —		4721711
The Sum is the *Logarithm* sought, *viz.*		4722912

Which having a proper Index put before it, according to the Number given, as *per* the 5th Rule of the first Section of this Chapter:

The Log. of 2966820514 is found = 9.4722912

I have in this last Example made this matter so very plain that I need add nothing for that end; only to observe,

1st, That the Log. of any Number is the same, tho' that Number have never so many Cyphers added to its right hand. And therefore in

Example 1. Of this Head, I call the Log. of 6473, which is 8111056, the Log. of 64730.

In *Example* 2. I call the same Logarithm that of 647300. And

In *Example* 3. I call the Log. of 2966 (which is 4721711) the Log. of 2966000000, making up the rest of the Places toward the right hand (for which I have no Logarithms in my Table) with Cyphers.

2dly, (As in the last Example) whatever Number of Cyphers those are, I cut off so many Places of the Difference of the Logarithms, &c.

And

And thus I have endeavoured to make this Part (which has been omitted by most others) very intelligible, by shewing how to prove the Truth of *Logarithms*, or find those to Numbers that are large by the Difference and Parts Proportional, as they are found in Tables for that purpose: And have also most easily and plainly shew'd how *Logarithms* of large Numbers are found without such Tables of Differences and Proportional Parts. It remains next to shew,

1. *How to find Natural Numbers to large Logarithms by the Tables of Differences and Parts Proportional.*
2. *How to do the same without such Tables of Differences and Parts Proportional.*

4. *To find the Natural Numbers to Logarithms beyond the Compass of your Table, by the Difference and Parts Proportional.*

I would know the Natural Number to the *Logarithm* 4.8111459: See the Operation.

The *Logarithm* given	= 8111459
The next *Log.* in the Table less than that given is the *Log.* of 6473, (which are the four first Places towards the left hand of the Number required) *viz.*	= 8111056
The next *Log.* in the Table greater than the last (being that of 6474) is — —	= 8111727
The Difference of the 2 first *Logarithms*	= 403
Difference of the 2 last *Logarithms*	= 671

Now to find the 5th Figure of the Number sought, or that in Units place, (the Index of the *Log.* given shewing that it hath but 5 places) look for the 2d Difference under [*Differences*] in the Table of Proportional Parts, and you'll find the next it (greater) to be 672. Then look in the same Line toward the right hand, and you'll find the 1st Difference 403; and at the head of that Column stands (6) which put in Units place, gives 64736 for the Number sought. This proves the Truth of the 1st Example, last abovesaid.

5. *To find the Natural Numbers (tho' very large) to any Log. given, without the Tables of Parts Proportional.*

What is the Natural Number to the *Logarithm* 9,4722912?

I find the *Logarithms*, and take the Differences as above thus:

The

The *Log.* given (without the Index) is 4722912 = 1st Differ. 1201

The next less in the Table (which is the *Log.* of 2966) is — — 4721711

The next *Log.* (greater than the last) is — — — — 4723175 = 2d Differ. 1464

Now whereas the Index shews the Natural Number required to have 10 places, and there being only the first 4 found, there wants 6 places, which with a Unit towards the left hand is = 1000000: therefore I say, As the 2d Difference is to 1000000 ::

So is the 1st Difference to the rest of the Nat. N° required, *i. e.* 1464. 1000000 :: 1201. 820514.

Which 820514 placed towards the right hand of the 2966 found in the Table as above, is together 2966820514 = the Answer.

But the *Logarithms* of 7 places being too few to find so large a Natural Number by, makes the Differences of too few places; and consequently the 4th Proportional in the last Analogy will fall somewhat short, if done by a Table having only 7 places. However, the Example shews the true Method of proceeding in this Case, which is all here intended.

This last Example was designed to prove the Truth of the third; where .232496 being omitted, if that Decimal be annexed to the 1201, the third in the last Analogy, it will then be divided by the 1464, and produce the true Quotient 820514 exactly.

SECT. V. *The Use of Logarithms in solving Questions of Compound Interest.*

Quest. 1. WHAT *is the Amount of* l. 10 in 130 Years, at 5 *per Cent?*

Rule.] Multiply the *Log.* of 1.05 (which is 1 *l.* and its Interest for a Year, which in the subsequent Questions I call the Ratio) by 130, (the N°. of Years) and the Product is the *Log.* of the Amount of *l.* 1 in 130 Years.

To this *Log.* add that of *l.* 10 = the Principal; and the Sum is the *Log.* of the Answer, *viz.* of *l.* 5684.409 = the Amount required.

Example.

1.05, its *Log.* is =	0.0211893
Multiply by Years	130
Amount of 1 *l.* in 130 Years. = *l.* 568.4, its Log.	2.7546090
To which add the *Log.* of the Princip. 10 *l.*	1.0
Sum or *Log.* of 5684.409—	3.7546090

Quest.

Quest. 2. *What is the present Worth* of *l.* 5684 : 8 : 2 ¼ (or of *l.* 5684.409) due at the End of 130 Years?

Rule.] Find the *Logarithm* of the Amount of *l.* 1 for 130 Years, as before.

Substract the *Log.* of that Amount from the *Log.* of the Sum whose present Worth is required, and the Remainer is the *Logarithm* of the present Worth, as *per* Margin.

So I find *l.* 5684.409 to be received 130 Years hence, is worth *l.* 10. in present Money; which proves the 1st.

Example.

Ratio = 1.05, its *Log.* =	0.0211893
Multiply by — —	130
Product, or Amount of *l.* 1 in 130 Years = 568.4; its *Log.* —	2.7546090
which last, deduct from the *Log.* of *l.* 5684.409	3.7546090
And the remaining *Log.* is that of *l.* 10, the Answer. — —	1.0000000

Quest. 3. *What is the Amount of the Lease or Annuity* of *l.* 25 *per Ann.* in 21 Years, according to its utmost Improvement at 5 *per Cent?*

Rule.] Find the Amount of *l.* 1 for 21 Years, as above.

2. Then from the *Log.* of the Amount less 1, deduct the *Log.* of the Ratio less 1; and the Remainer is the Amount of *l.* 1 Annuity in 21 Years.

3. Add to the last *Log.* that of 25 the Annuity, and the Sum is the Answer, or *l.* 893, the Amount of 25 *per Ann.* in 21 Years.

Example.

Ratio 1.05, its Log. =	0.0211893
Multiply by Years — —	21
Product, or Amount of *l.* 1 in 21 Years 2.786; its Log. — —	0.4449753
1.786 = the Amount less 1 its *Log.* — —	0.2518815
1.05 = Ratio, less 1 = .05; its *Log.* —	$\overline{2}$.6989700
Which last deducted, there rests the *Log.* of *l.* 35.72, the Amount of *l.* 1 *per An.* in 21 Years.	1.5529115
To which adding the *Log.* of 25 the Annu.	1.3979400
The Sum is the *Log.* of *l.* 893 the Amount of the Annuity for Answ.	2.9508515

Quest.

Quest. 4. *What is the present Worth of a Lease, or Annuity* of *l.* 25 *per Annum*, to continue 21 Years at 5 *per Cent.* Interest?

Rule.] Find the Amount of *l.* 1 in 21 Years, as in the last Question.

2*dly*, To the *Log.* of that Amount add the *Log.* of the Ratio, less 1.

3*dly*, Substract that Sum from the *Log.* of the Amount, less 1; and the Remainer is the *Log.* of the present Worth of *l.* 1 Annuity to continue 21 Years.

4*thly*, To the last *Log.* add that of the Annuity given, and the Sum is the *Log.* of the present Worth required.

Example.

2.786 = the Amount of *l.* 1 in 1 Years (as above) its *Log.* — —	0.4449753
.05 = Ratio less 1, its *Log.* — — — —	$\bar{2}$.6989700
Sum = —— ——	$\bar{1}$.1439453
1.786 = Amount, less 1, its *Log.* — — —	0.2518815
Rests the *Log.* of 12.821 *l.* which is the present Worth of *l.* 1 *per Ann.* to continue 21 Years	1.1079362
To which add the *Log.* of *l.* 25 the Annuity	1.3979400
And the Sum is the *Log.* of the Answer *l.* 320.527, which is	2.5058762

Quest. 5. *What Annuity or Lease will the Sum* of *l.* 320.527, or *l.* 320 : 10 : 6½ purchase, to continue 21 Years, allowing the Purchaser 5 *per Cent.*

Rule.] Find the Amount of *l.* 1 in 21 Years, as in the 1st, &c. Questions.

2*dly*, To the *Log* of that Amount add the *Log.* of the Ratio less 1, as in the last.

3*dly*, From that Sum deduct the *Logarithm* of the Amount less 1, and the Remainer is the *Log.* of the Annuity that *l.* 1 will purchase for 21 Years

4*thly*, To the *Log.* of the Annuity of *l.* 1 add the *Log.* of the Purchase-Money, and the Sum is the *Log.* of the Annuity sought here *l.* 25 *per Ann.* as *per* the Work in the Margin.

Example.

2.786 the Amount of *l.* 1 in 21 Years, its *Log.*	0.4449753
.05 = Ratio, less 1, its *Log.* —— — —	$\bar{2}$.6989700
Sum — — =	$\bar{1}$.1439453
1.786 = Amount, less 1, deduct its *Log.* ——	0.2518815
Rests the *Log.* of the Annuity that *l.* 1 will purchase for 21 Years, *viz.* of .078, which *Log.* is —— —— ——	2.8920638
320.527 the Purchase-Money, its *Log.* add =	2.5058762
l. 25. *per Ann.* the Annuity required, whose *Log.* is the Sum of the 2 last *Log.*	1.3979400

 Quest.

Quest. 6. What is an Estate worth in Fee, which contains Arable Land *l.* 60 *per Ann.* Under-Wood which once in 15 Years is worth *l.* 200, and there is also Timber which 'tis computed after 30 Years Growth will be worth *l.* 1000?

Rule.] The Arable Land at 20 Years Purchase is worth — — — } *l.* 1200 : 00 : 00

2*dly*, *l.* 200 (the first for the Under-Wood) due 15 Years hence at 6 *per Cent.* by the Logarithms (as per the 2d Question) is found thus: 1.06 its Log. = 0.0253058

Multiply by Years — — 15

2.397 = Amount of *l.* 1 in 15 Years, its Log — — } 0.3795870

Which taken from the Log. of *l.* 200 — — } 2.3010300

The Remainer is that of the present Worth — — } 1.9214430 = Log of .83 : 9 : 00

3*dly*, By the same Rule the present Value of *l.* 200 due 30 Years hence is — — — } 34 : 16 : 00

4*thly*, By ditto Rule the Value of the 3d 200 due 45 Years hence, is — — — — — — } 14 : 10 : 00

5*thly*, The Value of the 4th *l.* 200 due 60 Years hence, is *per* ditto Rule — — — — — } 6 : 1 : 00

6*thly*, The Value of *l.* 1000 for the Timber due 30 Years hence, is in present — — — — } 174 : 2 : 00

The Sum of which six Articles is the Answer near enough (for 'tis needless in this Purchase to take notice of the 5th 200 *l.*) — — — — } 1512 : 18 : 00

Quest.

Queſt. 7. *What Annuity payable Quarterly, will l.* 320.527 *purchaſe, to continue* 21 *Years at* 5 *per Cent.?*

Rule.] Firſt take a fourth of the Log. of the Ratio.

2*dly*, Multiply that 4th by the Quarters in the Years given, and that gives the Log. of the Amount of *l.* 1 for thoſe Quarters.

3*dly*, To the Log. of that Amount add that of the Rate *per* Quarter leſs 1.

4*thly*, From that Sum deduct the Log. of the Amount for the Quarters, leſs 1; and the Remainer is the Log. of the Annuity that *l.* 1 will purchaſe for 21 Years payable quarterly.

5*thly*, To the laſt Log. add that of the Purchaſe-Money, and the Sum is the Log. of the Anſwer.

Example.

1.05 the Ratio, its *Log.*	0.0211893
A 4th of that *Log.* w[ch] is the *Log.* of 1.012. the Rate *per* Quarter.	0.0052974
Which *Log.* multip. by the Quarters in 21 Y[rs]	84
And that produceth the *Log.* of 2.786 = the Amount of *l.* 1 for 84 Quarters, *viz.*	0.4449816
To which *Log.* add that of .012 = Rate *per* Quarter, leſs 1	$\bar{2}$.0791812
Sum	$\bar{2}$.5241628
1.786 = the Amount, leſs 1, its *Log.* deduct	0.2518815
Reſts the *Log.* of .01872 *l.* = the Annuity that *l.* 1 will purchaſe, payable quarterly, *viz.*	$\bar{2}$.2722813
To which *Log.* add that of *l.* 320.527 the Purchaſe Money, which is	2.5058762
And the Sum is the *Log.* of *l.* 6, the Quarterly payment required.	0.7781575

By this Example it is obvious, that the *l.* 320.527 will purchaſe but 6 *l. per* Quarter, at the Rate of 24 *l. per Ann.* Whereas by the 5th Queſtion it appears, that Sum will purchaſe *l.* 25 *per Ann.* if payable yearly. So that there is *l.* 1 *per Ann.* difference in that Purchaſe between receiving the Rent quarterly and yearly, the Purchaſer having the Uſe of the Quarters Rents.

Queſt. 8. *What is the preſent Worth of* l. 24 per Ann. *payable by equal Quarterly Payments*, to continue 21 Years?

Rule.] Firſt find the Amount of *l.* 1 for 84 Quarters, as above.

2*dly*, To that *Log.* add the *Log.* of the Rate *per* Quarter (found as above) leſs 1.

3*dly*, Deduct that Sum from the *Log.* of the Amount leſs 1, and the Remainer is the *Log.* of the preſent Worth of *l.* 1 *per* Quarter, to continue 84 Quarters.

4*thly*, To that *Log.* add that of *l.* 6 *per* Quarter, (as the 4th of 24 *per Ann.*) and the Sum is the *Log.* of the preſent Worth of the Annuity given, payable quarterly.

This proves the truth of the laſt Queſtion.

Example.

Rate *per* Quarter found as above, is ———	1.012
2.786 = the Amount of *l.* 1 for 84 Qrs. (found as above) its *Log.* ——	0.4449816
.012 = the Rate *per* Quarter leſs 1, its *Log.* Add ———	2.0791812
Sum of thoſe 2 *Logs.*	2.5241628
1.786 = the Amount above, leſs 1, deduct its *Logarithm* ———	0.2518815
Reſts the *Log.* of *l.* 53.42 = preſent Worth of *l.* 1 *per* Quarter, to continue 21 Years——	1.7277187
6 *l. per* Quarter (as *per* the Annuity given) its *Log.* Add———	0.7781575
And the Sum is the *Log.* of 320.527 the Anſw.	2.5058762

Queſt. 9. In an Account of Conſequence 'tis found that *l.* 5684.409 is the Amount of 10 *l.* Compound Intereſt at 5 *per Cent.* what Time was required for that Increaſe?

Rule.] From the *Log.* of the Amount deduct that of the Principal put out.

2*dly*, Divide the remaining *Log.* by the *Log.* of the Ratio, and the Quotient is the Years required. See the Operation.

Example.

l. 5684.409 its Log. — —	3.7546090
10 *l.* the Principal, its Log. Deduct — — ——	1.
	Quo.
1.05, its Log. .0211893 the Divisor — — —	) 2.7546090 (130
So the Anſwer is 130 Years.	635670

Queſt.

Queſt. 10. In a certain Account *l.* 5684.409 is charged as the Amout of 10 *l.* in 130 Years Compound Intereſt: But upon a Trial the Simple Intereſt being only allowed according to the Rate agreed on, whence the ſaid Amount is computed; now it not appearing by any Bond, or Entry, what Rate or Intereſt was reckoned, the Queſtion is how that may be found out?

Rule.] From the Log. of the Amount deduct that of the Principal put forth.

2*dly*, Divide that remaining Log. by the Number of Years, and the Quote is the Log. of *l.* 1 and its Intereſt for a Year. Therefore ſay,

3*dly*, As *l.* 1 is to *l.* 1 and its Intereſt found :: So is 100 to 100 and its Intereſt. Whence taking 100, the reſt muſt be the Rate of Intereſt required.

Example.

5684.409 the Amount, its Log. ———	3.7546090
10 = the Principal, deduct its Log. —	1.
Divide by 130) reſts —	2.7546090
Quoteth the Log. of *l.* 1.05, which is —	0.0211893

Therefore

As 1. 1.05 : : 100. 105.

And 105 leſs 100 = 5 the Intereſt required *per Cent.*

Queſt. 11. A Gentleman bought a Leaſe of *l.* 25 *per Ann.* to continue 21 Years for *l.* 320.527: At what Rate was Intereſt allowed him *per Cent?*

To do this by the Logarithms, take the Log. of *l.* 320.527, which is ——— ——— ———	2.5058762
Whence deduct the Log. of the Annuity given 25, which is ——— ——— ——— ———	1.3979400
And the Remainer is the Log. of 12.821 (or the preſent Worth of *l.* 1 *per Ann.* to continue 21 Years; *which note.* ——— ——— ——— ———	1.1079362

Now to find what Intereſt this is reckon'd at, I make trial (as *per Queſt.* 4.) and find *at* 6 *per Cent.* that the preſent Worth of *l.* 1 *per Ann.* to continue 21 Years is 11.764: which being leſs than the 12.821 (the preſent Worth of *l.* 1 *per Ann.* as above) I know that 6 *per Cent.* is too much, (for the more the preſent Worth is, the leſs is the Rate of Intereſt, and the contrary.)

Therefore I make trial at 4 *per Cent.* and (by the Rules under the 4th Queſtion) I find the preſent Worth of *l.* 1.

per

per Ann. at 4 *per Cent.* is (to continue 21 Years) 14.031: which being more than the above 12.821, I know that 4 *per Cent.* is too little. So that it neither being at 6 nor 4, but finding the Rate to lie between them, I suppose it 5 *per Cent.* And trying by the Rules under the said 4th Question I find the Log. of the present Worth of *l.* 1 *per Ann.* to continue 21 Years is that of 12.821, the same as above, *viz.* ——— ——— ——— ——— ——— 1.1079362

And consequently I know that 5 *per Cent.* is the Rate sought.

Or thus to find the Rate:

Having found the present Worth at 4 and 6 *per Cent.* as above, proceed thus.

1. Take the Difference between the Years Purchase given (or found by the Logarithms) which Years Purchase as above are ——— ——— ——— ——— } 12.821

And the Years Purchase at 4 *per Cent.* found —— 14.031

Which Difference is ——— ——— = 1.210

2. The Years Purchase at 4 *per Cent.* are found = 14.031

Ditto at 6 *per Cent.* as above are = 11.764

2d Difference = 2.267

3. Say 2.267. 1.210 : : 2. 1

i. e. As the 2d Difference is to the first: so is 2 the Difference between the Rates *per Cent.* (supposed) as before, to 1 = the Proportion to be added to the lesser Rate 4 to give the Rate sought for; in which case the 1st place in the 4th Proportional is sufficient, where (0) is next the Point in the Decimal, if the Division were carry'd on farther.

Quest. 12. ***What is the present Value of the Reversion of an Annuity or Lease* of *l.* 50 *per Ann.* to continue 21 Years, to commence after the Expiration of 7 Years, at 5 *per Cent. per Ann.* allowed to the Purchaser.**

Rule.] **First add the Years (the 7 to the 21) make 28.**

2*dly*, **Find the present Worth of the Annuity to continue 28 Years by the 4th Question, which you'll find ——— ——— ——— ———** } *l.* 745 : 00 : 00

3*dly*, Find the present Worth of the *l.* 50 *per Ann.* (by the Rules under the said 4th Quest.) to continue 7 Years, which is ——— ——— ——— } *l.* 283 : 6 : 00

4thly,

4thly, Deduct the later from the former, and the Remainer is the present Worth of the Reversion required ——— ——— ——— ——— ——— } *l.* 461 : 14 : 0

Example the 2d shall be to find the Values of the Reversions of Church or College Leases of Houses; or the Fines payable for any Number of Years lapsed: As suppose 17 Years lapsed of 40, what Fine must be paid to make up the Lease again 40 Years: supposing the clear annual Rent 40 Pounds.

Rule.] Since this is the same thing as to find the Value of the Reversion of 17 Years after the Expiration of 23 in *esse*; and the Rate of Interest in these Cases is 1.116:

1st, Therefore find the present Worth of *l.* 40 *per Ann.* to continue 40 Years by the Rules above and as follows.

2dly, From that deduct the present Worth of *l.* 40 *per Ann.* to continue 23 Years, and the Remainer is the Fine or Value of the Reversion: See the whole Operation as follows.

1.116 the Rate, its Logarithm =	0.0476642
multiply by Years ——— ——— ———	40
The Amount of *l.* 1 in 40 Years or 80.64; its Log.	1.9065680
.116 the Rate less 1, add its Log. =	$\bar{1}$.0644580
Sum of the 2 last	$\bar{0}$.9710260
Which deduct from the Log. of the Amount less 1 = 79.64, its Log. ——— ——— ——— } =	1.9011313
And the Remainer is the Log. of the present Worth of *l.* 1 *per Ann.* to continue 40 Years }	0.9301053
To which add the Log. of the Annuity *l.* 40 =	1.6020600
The Sum is the Log. of 340.5 the present Worth of *l.* 40 *per Ann.* to continue 40 Years ——— }	2.5321653

2dly for the present Worth of *l.* 40 to continue 23 Years.

1.116 the Rate or *Ratio*, its Log. =	0.0476642
multiply by Years ——— ——— ——— ———	23
The Amount of *l.* 1 in 23 Years 12.48 its Log. =	1.0962766
.116 the *Ratio* less 1 add its Log. =	$\bar{1}$.0644586
Sum of the 2 last ——— ———	$\bar{0}$.1607346
Which deduct from the Amount less 1, *viz.* 11.48, its Log. ——— ——— ——— } =	1.0599419
The Remainer is the Log. of the present Worth of *l.* 1 to continue 23 Years — } =	0.8992073
To which add the Log. of the Annuity 40 =	1.6020600

And

And the Sum is the Log. of the present Worth of *l.* 40 *per Ann.* to continue 23 Years, ——— } 2.5012673

Viz. of 317.1, which deducted from the above *l.* 340.5, the Remainer is *l.* 23 : 8 : 0 the Fine required.

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13. *When it is required to purchase an Estate either in Fee or for Years mentioned, with Money due at any time to come:* as suppose I would purchase an Annuity to continue 14 Years, with *l.* 1000 due to me at the end of 5 Years; the Annuity to commence presently at 5 *per Cent.*

Rule.] First, by the 2d Question find the present Worth of *l.* 1000 due at the End of 5 Years, which is } *l.* 783 : 10 : 0

2*dly*, By the 5th Question find what Annuity to continue 14 Years *l.* 783 : 10 will purchase; which for Answer you'll find at 5 *per Cent.* ——— ——— } *l.* 79 : 2 *per An.* 14 Years.

I need not tell the Reader, that in case the *Purchase had been required in Fee, to divide the present Worth of* l. 1000 = l. 783.5 *by the Number of Years Purchase agreed on*; as at 20 it will be ——— ——— *l.* 39 : 3 : 6 *per Ann.* for ever.

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14. *When you would Fine off Rent:* As suppose a Lease is *l.* 90 *per Ann.* and the Tenant is willing to Fine off *l.* 40 *per Ann.* for his Lease of 21 Years at 5 *per Cent.* in this Case there is nothing to be done but by the 4th Quest. find the present Worth of *l.* 40 *per Ann.* to continue 21 Years; which present Worth *l.* 512 : 16 : 11 must be given the Landlord as a Fine, and *l.* 50 *per Ann.* must be paid Annual Rent. On the contrary:

15. *When you would Rent off a Fine:* As suppose a Lease for 21 Years is to be let at *l.* 50 *per Ann.* with a Fine of *l.* 512.846; but the Leasee not having so much ready Money to spare, he is willing to pay an Equivalent Rent, and no Fine. To answer this, you need only to consider by the Rules under the 5th Question, what Annuity to continue 21 Years at 5 *per Cent.* the said Fine = *l.* 512.846 will purchase; which you'll find *l.* 40 *per Ann.* which added to the *l.* 50, shews that he must pay *l.* 90 *per Ann.* if he pays no Fine.

These 15 Questions and Proportions contain the Fundamental Rules for solving all Questions of Compound Interest whatever; and whosoever doth thoroughly understand them, may be able to give a true Answer to any Question relating to Selling or Purchasing Estates, &c. without the help of Tables for that purpose; provided he has well learnt the Logarithms.

The End of Logarithmical Arithmetic, and its Use.

A TABLE

A

TABLE

OF THE

LOGARITHMS

TO ALL

NUMBERS,

Not exceeding 10000, or 4 Places, whether they be Intire, Broken, or Mixt Numbers.

And the Differences of Logarithms, and the Parts proportional, whereby the Logarithm of any Number is produced to 100000.

Particularly useful in Extracting the Square, Cube, *&c.* Roots, and solving Questions in Compound Interest, *&c.*

Natural Numbers.	0	1	2	3	4
1	0000000	0413927	0791812	1139433	1461280
2	3010300	3222193	3424227	3617278	3802112
3	4771212	4913617	5051500	5185139	5314789
4	6020600	6127838	6232493	6334684	6434527
5	6989700	7075702	7160033	7242759	7323937
6	7781512	7853298	7923917	7993405	8061800
7	8450980	8512583	8573325	8633229	8692317
8	9030900	9084850	9138138	9190781	9242793
9	9542425	9590414	9637878	9684829	9731278
100	0000000	0004341	0008677	0013009	0017337
101	0043214	0047511	0051805	0056094	0060379
102	0086002	0090257	0094509	0098756	0102999
103	0128372	0132587	0136794	0141003	0145205
104	0170333	0174507	0178677	0182843	0187005
105	0211893	0216027	0220157	0224284	0228406
106	0253059	0257154	0261245	0265333	0269416
107	0293838	0297895	0301948	0305997	0310043
108	0334237	0338257	0342273	0346284	0350293
109	0374265	0378247	0382226	0386202	0390173
110	0413927	0417873	0421816	0425755	0429691
111	0453230	0457140	0461048	0464952	0468852
112	0492180	0496056	0499928	0503797	0507663
113	0530784	0534626	0538464	0542299	0546130
114	0569048	0572856	0576661	0580462	0584260
115	0606978	0610753	0614525	0618293	0622058
116	0644580	0648322	0552061	0655797	0659530
117	0681859	0685569	0689276	0692980	0696681
118	0718820	0722499	0726175	0729847	0733517
119	0755470	0759118	0762762	0766404	0770043
120	0791812	0795430	0799045	0802656	0806265
121	0827854	0831441	0835026	0838608	0842187
122	0863598	0867157	0870712	0874264	0877814
123	0899051	0902580	0906107	0909631	0913151
124	0934217	0937718	0941216	0944711	0948204
125	0969100	0972573	0976043	0979511	0982975
126	1003705	1007151	1010593	1014033	1017471
127	1038037	1041455	1044871	1048284	1051694
128	1072100	1075491	1078880	1082266	1085650
129	1105897	1109262	1112625	1115985	1119343
130	1139433	1142773	1146110	1149444	1152776
131	1172713	1176027	1179338	1182647	1185954
132	1205739	1209028	1212314	1215598	1218880
133	1238516	1241780	1245042	1248301	1251558
134	1271048	1274288	1277525	1280760	1283993

Natural Numbers.	5	6	7	8	9
1	1760912	2041200	2304489	2552725	2787536
2	3979400	4149733	4313637	4471580	4623980
3	5440680	5563025	5682017	5797836	5910646
4	6532125	6627578	6720978	6812412	6901961
5	7403627	7481880	7558748	7634280	7708520
6	8129133	8195439	8260748	8325089	8388491
7	8750613	8808136	8864907	8920946	8976271
8	9294189	9344984	9395192	9444827	9493900
9	9777236	9822712	9867717	9912261	9956352
100	0021661	0025980	0030295	0034605	0038912
101	0064660	0068937	0073209	0077478	0081742
102	0107239	0111473	0115704	0119931	0124154
103	0149403	0153597	0157787	0161973	0166155
104	0191163	0195317	0199467	0203613	0207755
105	0232524	0236639	0240750	0244857	0248960
106	0273496	0277572	0281644	0285712	0289777
107	0314085	0318123	0322157	0326188	0330214
108	0354297	0358298	0362295	0366289	0370279
109	0394141	0398105	0402066	0406023	0409977
110	0433623	0437551	0441476	0445398	0449315
111	0472749	0476642	0480532	0484418	0488301
112	0511525	0515384	0519239	0523091	0526939
113	0549958	0553783	0557605	0561423	0565237
114	0588055	0591846	0595634	0599419	0603200
115	0625820	0629578	0633334	0637085	0640834
116	0663259	0666985	0670708	0674428	0678145
117	0700379	0704073	0707765	0711453	0715138
118	0737183	0740847	0744507	0748164	0751818
119	0773679	0777312	0780941	0784568	0788192
120	0809870	0813473	0817073	0820669	0824263
121	0845763	0849336	0852906	0856473	0860037
122	0881361	0884905	0888446	0891984	0895519
123	0916669	0920185	0923696	0927206	0930712
124	0951693	0955180	0958664	0962146	0965624
125	0986437	0989896	0993353	0996806	1000257
126	1020905	1024337	1027766	1031192	1034616
127	1055102	1058506	1061909	1065308	1068705
128	1089031	1092410	1095785	1099159	1102529
129	1122698	1126050	1129400	1132746	1136091
130	1156105	1159432	1162756	1166077	1169396
131	1189257	1192559	1195858	1199154	1202448
132	1222159	1225435	1228709	1231981	1235250
133	1254813	1258064	1261314	1264561	1267806
134	1287223	1290450	1293676	1296890	1300110

Natural Numbers.	0	1	2	3	4
135	1303338	1306553	1309767	1312978	1316187
136	1335389	1338581	1341771	1344958	1348144
137	1367206	1370374	1373541	1376705	1379867
138	1398791	1401937	1405080	1408222	1411361
139	1430148	1433271	1436392	1439511	1442628
140	1461280	1464381	1467480	1470577	1473671
141	1492191	1495270	1498347	1501422	1504494
142	1522883	1525941	1528996	1532049	1535100
143	1553360	1556396	1559430	1562462	1565491
144	1583625	1586640	1589653	1592663	1595672
145	1613680	1616674	1619666	1622656	1625644
146	1643528	1646502	1649474	1652443	1655411
147	1673173	1676127	1679078	1682027	1684975
148	1702617	1705550	1708482	1711411	1714339
149	1731863	1734776	1737688	1740598	1743506
150	1760913	1763807	1766699	1769590	1772478
151	1789769	1792645	1795518	1798389	1801259
152	1818436	1821292	1824146	1826999	1829850
153	1846941	1849752	1852588	1855421	1858253
154	1875207	1878026	1880844	1883659	1886473
155	1903317	1906118	1908917	1911714	1914510
156	1931246	1934029	1936810	1939590	1942367
157	1958996	1961762	1964525	1967287	1970047
158	1986571	1989319	1992065	1994809	1997552
159	2013971	2016702	2019431	2022158	2024883
160	2041200	2043913	2046625	2049335	2052044
161	2068259	2070955	2073650	2076344	2079035
162	2095150	2097830	2100508	2103185	2105860
163	2121876	2124540	2127201	2129862	2132521
164	2148438	2151086	2153732	2156376	2159018
165	2174839	2177471	2180100	2182727	2185355
166	2201081	2203696	2206310	2208922	2211533
167	2227165	2229764	2232363	2234959	2237554
168	2253093	2259677	2258260	2260841	2263421
169	2278867	2281436	2284003	2286570	2289134
170	2304489	2307043	2309596	2312146	2314696
171	2329961	2332500	2335038	2337574	2340108
172	2355284	2357809	2360331	2362853	2365373
173	2380461	2382971	2385479	2387986	2390491
174	2405492	2407988	2410481	2412974	2415465
175	2430380	2432861	2435341	2437819	2440296
176	2455127	2457593	2460059	2462523	2464986
177	2479733	2482186	2484637	2487085	2489536
178	2504200	2506639	2509077	2511513	2513948

Natural Numbers.	5	6	7	8	9
135	1319393	1322597	1325798	1328998	1332194
136	1351326	1354507	1357685	1360861	1364034
137	1383027	1386184	1389339	1392492	1395643
138	1414498	1417632	1420765	1423895	1427022
139	1445742	1448854	1451964	1455072	1458177
140	1476763	1479853	1482941	1486026	1489110
141	1507564	1510632	1513698	1516762	1519824
142	1538149	1541195	1544240	1547282	1550322
143	1568519	1571544	1574568	1577589	1580608
144	1598678	1601683	1604685	1607686	1610684
145	1628630	1631614	1634595	1637575	1640553
146	1658376	1661340	1664301	1667260	1670218
147	1687920	1690863	1693805	1696744	1699682
148	1717264	1720188	1723110	1726029	1728947
149	1746412	1749316	1752218	1755118	1758016
150	1775365	1778250	1781132	1784013	1786892
151	1804126	1806992	1809856	1812718	1815578
152	1832698	1835545	1838390	1841233	1844075
153	1861084	1863912	1866739	1869563	1872386
154	1889285	1892095	1894903	1897709	1900514
155	1917304	1920096	1922886	1925674	1928461
156	1945143	1947917	1950690	1953460	1956229
157	1972806	1975562	1978317	1981070	1983821
158	2000293	2003032	2005769	2008505	2011239
159	2027607	2030329	2033049	2035768	2038485
160	2054750	2057455	2060159	2062869	2065560
161	2081725	2084413	2087100	2089785	2092468
162	2108534	2111203	2113876	2116544	2119211
163	2135178	2137833	2140487	2143139	2145789
164	2161659	2164298	2166936	2169572	2172206
165	2187980	2190603	2193225	2195845	2198464
166	2214142	2216750	2219356	2221960	2224563
167	2240148	2242740	2245331	2247920	2250507
168	2265999	2268576	2271151	2273724	2276296
169	2291697	2294258	2296818	2299377	2301934
170	2317244	2319790	2322335	2324879	2327421
171	2342641	2345173	2347703	2350232	2352759
172	2367891	2370408	2372923	2375437	2377950
173	2392995	2395497	2397998	2400498	2402996
174	2417954	2420442	2422929	2425414	2427898
175	2442771	2445245	2447718	2450189	2452658
176	2467447	2469907	2472365	2474823	2477278
177	2491984	2494430	2496874	2499317	2501759
178	2516382	2518814	2521246	2523675	2526103

Natural Numbers.	0	1	2	3	4
179	2528530	2530956	2533380	2535803	2538224
180	2552725	2555137	2557548	2559957	2562365
181	2576786	2579184	2581582	2583978	2586373
182	2600714	2603099	2605484	2607867	2610248
183	2624511	2626883	2629255	2631625	2633993
184	2648178	2650538	2652896	2655253	2657609
185	2671717	2674064	2676410	2678752	2681097
186	2695129	2697464	2699797	2702128	2704459
187	2718416	2720738	2723058	2725378	2727696
188	2741578	2743888	2746196	2748503	2750809
189	2764618	2766915	2769211	2771506	2773800
190	2787536	2789821	2792105	2794388	2796669
191	2810334	2812607	2814879	2817150	2819419
192	2833012	2835274	2837534	2839793	2842051
193	2855573	2857823	2860071	2862318	2864565
194	2878017	2880255	2882492	2884728	2886963
195	2900346	2902573	2904798	2907022	2909246
196	2922561	2924776	2926990	2929203	2931415
197	2944662	2946866	2949069	2951271	2953471
198	2966652	2968845	2971036	2973227	2975417
199	2988531	2990713	2992893	2995073	2997251
200	3010300	3012471	3014641	3016809	3018977
201	3031961	3034121	3036280	3038438	3040595
202	3053514	3055663	3057811	3059959	3062105
203	3074960	3077099	3079237	3081374	3083509
204	3096302	3098430	3100557	3102684	3104809
205	3117539	3119657	3121774	3123889	3126004
206	3138672	3140780	3142887	3144992	3147097
207	3159703	3161801	3163897	3165993	3168087
208	3180633	3182721	3184807	3186893	3188977
209	3201463	3203540	3205617	3207692	3209767
210	3222193	3224260	3226327	3228393	3230457
211	3242825	3244882	3246939	3248995	3251050
212	3263359	3265407	3267454	3269500	3271545
213	3283796	3285834	3287872	3289909	3291944
214	3304138	3306167	3308195	3310222	3312248
215	3324385	3326404	3328423	3330440	3332457
216	3344537	3346548	3348557	3350565	3352572
217	3364597	3366598	3368598	3370597	3372595
218	3384565	3386557	3388547	3390537	3392526
219	3404441	3406424	3408405	3410386	3412366
220	3424227	3426200	3428173	3430145	3432116
221	3443923	3445887	3447851	3449814	3451776
222	3463530	3465486	3467441	3469395	3471348

Natural Numbers.	5	6	7	8	9
179	2540645	2543063	2545481	2547897	2550312
180	2564772	2567177	2569581	2571984	2574386
181	2588766	2591158	2593549	2595939	2598327
182	2612629	2615008	2617385	2619762	2622137
183	2636361	2638727	2641092	2643455	2645817
184	2659964	2662317	2664669	2667020	2669369
185	2683439	2685780	2688119	2690457	2692794
186	2706788	2709116	2711443	2713769	2716093
187	2730013	2732328	2734643	2736956	2739268
188	2753113	2755417	2757719	2760020	2762320
189	2776092	2778383	2780673	2782952	2785250
190	2798950	2801229	2803507	2805784	2808059
191	2821688	2823955	2826221	2828486	2830750
192	2844307	2846563	2848817	2851070	2853322
193	2866810	2869054	2871296	2873538	2875778
194	2889196	2891428	2893659	2895889	2898118
195	2911468	2913688	2915908	2918127	2920344
196	2933626	2935835	2938044	2940251	2942457
197	2955671	2957869	2960067	2962263	2964458
198	2977605	2979792	2981979	2984164	2986348
199	2999420	3001605	3003781	3005955	3008128
200	3021144	3023309	3025474	3027637	3029799
201	3042751	3044905	3047059	3049212	3051363
202	3064250	3066394	3068537	3070679	3072820
203	3085644	3087778	3089910	3092042	3094172
204	3106933	3109056	3111178	3113299	3115420
205	3128118	3130231	3132343	3134454	3136563
206	3149200	3151303	3153405	3155505	3157605
207	3170181	3172273	3174365	3176455	3178545
208	3191061	3193143	3195224	3197305	3199384
209	3211840	3213913	3215984	3218055	3220124
210	3232521	3234584	3236645	3238706	3240766
211	3253104	3255157	3257209	3259260	3261310
212	3273589	3275633	3277675	3279716	3281757
213	3293979	3296012	3298045	3300077	3302108
214	3314273	3316297	3318320	3320343	3322364
215	3334473	3336488	3338501	3340514	3342526
216	3354579	3356585	3358589	3360593	3362596
217	3374593	3376589	3378584	3380579	3382572
218	3394514	3396501	3398488	3400473	3402458
219	3414345	3416323	3418301	3420277	3422252
220	3434086	3436055	3438023	3439991	3441957
221	3453737	3455698	3457657	3459615	3461573
222	3473300	3475252	3477202	3479152	3481101

Natural Numbers.	0	1	2	3	4
223	3483049	3484996	3486942	3488887	3490832
224	3502480	3504419	3506356	3508293	3510228
225	3521825	3523755	3525684	3527612	3529539
226	3541084	3543006	3544926	3546845	3548764
227	3560259	3562171	3564083	3565994	3567905
228	3579348	3581253	3583156	3585059	3586961
229	3598355	3600251	3602146	3604040	3605934
230	3617278	3619166	3621053	3622939	3624825
231	3636120	3638000	3639878	3641756	3643633
232	3654880	3656751	3658622	3660492	3662361
233	3673559	3675423	3677285	3679147	3681008
234	3692159	3694014	3695869	3697723	3699576
235	3710679	3712526	3714373	3716219	3718065
236	3729120	3730960	3732799	3734637	3736475
237	3747483	3749316	3751147	3752977	3754807
238	3765769	3767594	3769418	3771240	3773062
239	3783979	3785796	3787612	3789427	3791241
240	3802112	3803922	3805730	3807538	3809345
241	3820170	3821972	3823773	3825573	3827373
242	3838154	3839948	3841741	3843534	3845326
243	3856063	3857850	3859636	3861421	3863206
244	3873898	3875678	3877457	3879235	3881012
245	3891661	3893433	3895205	3896975	3898746
246	3909351	3911116	3912880	3914644	3916407
247	3926969	3928727	3930485	3932241	3933997
248	3944517	3946268	3948018	3949767	3951516
249	3961993	3963734	3965480	3967223	3968964
250	3979400	3981137	3982873	3984608	3986343
251	3996737	3998467	4000196	4001925	4003653
252	4014005	4015728	4017451	4019172	4020893
253	4031205	4032921	4034637	4036352	4038066
254	4048337	4050047	4051755	4053464	4055171
255	4065402	4067105	4068807	4070508	4072209
256	4082400	4084096	4085791	4087486	4089180
257	4099331	4101021	4102710	4104398	4106085
258	4116197	4117880	4119562	4121244	4122925
259	4132998	4134674	4136350	4138025	4139700
260	4149733	4151404	4153073	4154742	4156410
261	4166405	4168069	4169732	4171394	4173056
262	4183013	4184670	4186327	4187983	4189638
263	4199557	4201208	4202859	4204509	4206158
264	4216039	4217684	4219328	4220972	4222614
265	4232459	4234097	4235735	4237372	4239009
266	4248816	4250449	4252080	4253712	4255342

Natural Numbers.	5	6	7	8	9
223	3492775	3494718	3496660	3498601	3500541
224	3512163	3514098	3516031	3517963	3519895
225	3531465	3533391	3535316	3537239	3539162
226	3550682	3552599	3554515	3556430	3558345
227	3569813	3571723	3573630	3575537	3577443
228	3588862	3590762	3592662	3594560	3596458
229	3607827	3609719	3611610	3613500	3615390
230	3626709	3628593	3630476	3632358	3634239
231	3645510	3647386	3649260	3651134	3653007
232	3664230	3666097	3667964	3669830	3671695
233	3682869	3684728	3686587	3688445	3690302
234	3701428	3703280	3705131	3706981	3708830
235	3719909	3721753	3723596	3725438	3727279
236	3738311	3740147	3741983	3743817	3745651
237	3756636	3758464	3760292	3762118	3763944
238	3774884	3776704	3778524	3780343	3782161
239	3793055	3794868	3796680	3798492	3800302
240	3811151	3812956	3814761	3816565	3818368
241	3829171	3830969	3832766	3834563	3836359
242	3847117	3848908	3850698	3852487	3854275
243	3864990	3866773	3868555	3870337	3872118
244	3882789	3884565	3886340	3888114	3889888
245	3900515	3902284	3904052	3905819	3907585
246	3918169	3919931	3921691	3923452	3925211
247	3935752	3937506	3939260	3941013	3942765
248	3953264	3955011	3956758	3958504	3960249
249	3970705	3972446	3974185	3975924	3977662
250	3988077	3989811	3991543	3993275	3995007
251	4005380	4007106	4008832	4010557	4012282
252	4022614	4024333	4026052	4027771	4029488
253	4039780	4041492	4043205	4044916	4046627
254	4056878	4058584	4060289	4061994	4063698
255	4073909	4075608	4077307	4079005	4080703
256	4090874	4092567	4094259	4095950	4097641
257	4107772	4109459	4111144	4112829	4114513
258	4124605	4126285	4127964	4129643	4131320
259	4141374	4143047	4144719	4146391	4148063
260	4158077	4196744	4161410	4163076	4164741
261	4174717	4176377	4178037	4179696	4181355
262	4191293	4192947	4194601	4196254	4197906
263	4207806	4209454	4211101	4212748	4214394
264	4224257	4225898	4227539	4229180	4230820
265	4240645	4242281	4243915	4245550	4247183
266	4256972	4258601	4260230	4261858	4263486

Natural Numbers.	0	1	2	3	4
267	4265113	4266739	4268365	4269990	4271614
268	4281348	4282968	4284588	4286207	4287825
269	4297523	4299137	4300751	4302364	4303976
270	4313638	4315246	4316853	4318460	4320067
271	4329693	4331295	4332897	4334498	4336098
272	4345689	4347285	4348881	4350476	4352071
273	4361626	4363217	4364807	4366396	4367985
274	4377506	4379090	4380674	4382258	4383841
275	4393327	4394906	4396484	4398062	4399639
276	4409091	4410664	4412237	4413809	4415380
277	4424798	4426365	4427932	4429499	4431065
278	4440448	4442010	4443571	4445132	4446692
279	4456042	4457598	4459154	4460709	4462264
280	4471580	4473131	4474681	4476231	4477780
281	4487063	4488608	4490153	4491697	4493241
282	4502491	4504031	4505570	4507109	4508647
283	4517864	4519399	4520932	4522466	4523998
284	4533183	4534712	4536241	4537769	4539296
285	4548449	4549972	4551495	4553018	4554540
286	4563660	4565179	4566696	4568213	4569731
287	4578819	4580332	4581844	4583356	4584868
288	4593925	4595433	4596940	4598446	4599953
289	4608978	4610481	4611983	4613484	4614985
290	4623980	4625477	4626974	4628470	4629966
291	4638930	4640422	4641914	4643405	4644895
292	4653828	4655316	4656802	4658288	4659774
293	4668676	4670158	4671640	4673120	4674601
294	4683473	4684950	4686427	4687903	4689378
295	4698220	4699692	4701163	4702634	4704105
296	4712917	4714384	4715850	4717317	4718782
297	4727564	4729027	4730488	4731949	4733410
298	4742163	4743620	4745076	4746533	4747988
299	4756712	4758164	4759616	4761067	4762518
300	4771212	4772660	4774107	4775553	4776999
301	4785665	4787108	4788550	4789991	4791432
302	4800069	4801507	4802945	4804381	4805818
303	4814426	4815859	4817292	4818724	4820156
304	4828736	4830164	4831592	4833019	4834446
305	4842998	4844422	4845845	4847268	4848690
306	4857214	4858633	4860052	4861470	4862888
307	4871384	4872798	4874212	4875626	4877039
308	4885507	4886917	4888326	4889735	4891144
309	4899585	4900990	4902395	4903799	4905203
310	4913617	4915018	4916418	4917818	4919217

Natural Numbers.	5	6	7	8	9
267	4273238	4274861	4276484	4278106	4279727
268	4289443	4291060	4292677	4294293	4295908
269	4305588	4307199	4308809	4310419	4312029
270	4321673	4323278	4324883	4326487	4328090
271	4337698	4339298	4340896	4342494	4344092
272	4353665	4355258	4356851	4358444	4360035
273	4369573	4371161	4372748	4374334	4375920
274	4385423	4387005	4388587	4390167	4391747
275	4401216	4402792	4404368	4405943	4407517
276	4416951	4418522	4420092	4421661	4423229
277	4432630	4434195	4435759	4437322	4438885
278	4448252	4449811	4451370	4452928	4454485
279	4463818	4465372	4466925	4468477	4470029
280	4479329	4480877	4482424	4483971	4485517
281	4494784	4496326	4497868	4499410	4500951
282	4510184	4511721	4513258	4514794	4516329
283	4525531	4527062	4528593	4530124	4531654
284	4540823	4542349	4543875	4545400	4546924
285	4556061	4557582	4559102	4560622	4562142
286	4571246	4572762	4574277	4575791	4577305
287	4586378	4587889	4589399	4590908	4592417
288	4601458	4602963	4604468	4605972	4607475
289	4616486	4617986	4619485	4620984	4622482
290	4631461	4632956	4634450	4635944	4637437
291	4646386	4647875	4649364	4650853	4652341
292	4661259	4662743	4664227	4665711	4667194
293	4676081	4677560	4679039	4680518	4681996
294	4690853	4692327	4693801	4695275	4696748
295	4705575	4707044	4708513	4709982	4711450
296	4720247	4721711	4723175	4724639	4726102
297	4734870	4736329	4737788	4739247	4740705
298	4749443	4750898	4752352	4753806	4755259
299	4763968	4765418	4766867	4768316	4769765
300	4778445	4779890	4781334	4782778	4784222
301	4792873	4794313	4795754	4797192	4798631
302	4807254	4808689	4810124	4811559	4812993
303	4821587	4823018	4824448	4825878	4827307
304	4835873	4837299	4838725	4840150	4841574
305	4850112	4851533	4852954	4854375	4855795
306	4864305	4865721	4867138	4868554	4869969
307	4878451	4879863	4881275	4882686	4884097
308	4892552	4893959	4895366	4896773	4898179
309	4906607	4908009	4909412	4910814	4912216
310	4920616	4922014	4923413	4924810	4926207

Natural Numbers.	0	1	2	3	4
311	4927604	4929000	4930396	4931791	4933186
312	4941546	4942938	4944329	4945720	4947110
313	4955443	4956831	4958218	4959604	4960990
314	4969296	4970679	4972062	4973444	4974825
315	4983106	4984484	4985862	4987240	4988617
316	4996871	4998245	4999619	5000992	5002365
317	5010593	5011962	5013332	5014701	5016069
318	5024271	5025637	5027002	5028366	5029731
319	5037907	5039268	5040629	5041989	5043349
320	5051500	5052857	5054213	5055569	5056925
321	5065050	5066403	5067755	5069107	5070459
322	5078559	5079907	5081255	5082603	5083950
323	5092025	5093370	5094713	5096057	5097400
324	5105450	5106790	5108130	5109469	5110808
325	5118834	5120170	5121505	5122841	5124175
326	5132176	5133508	5134840	5136171	5137501
327	5145478	5146805	5148133	5149460	5150787
328	5158738	5160062	5161386	5162709	5164031
329	5171959	5173279	5174598	5175917	5177236
330	5185139	5186455	5187771	5189086	5190400
331	5198280	5199592	5200903	5202214	5203525
332	5211381	5212689	5213996	5215303	5216610
333	5224442	5225746	5227050	5228353	5229656
334	5237465	5238765	5240064	5241364	5242663
335	5250448	5251744	5253040	5254335	5255631
336	5263393	5264685	5265977	5267269	5268560
337	5276299	5277588	5278876	5280163	5281451
338	5289167	5290452	5291736	5293020	5294303
339	5301997	5303278	5304558	5305839	5307118
340	5314789	5316066	5317343	5318619	5319895
341	5327544	5328817	5330090	5331363	5332635
342	5340261	5341531	5342800	5344069	5345338
343	5352941	5354207	5355473	5356738	5358003
344	5365584	5366847	5368109	5369370	5370631
345	5378191	5379450	5380708	5381966	5383223
346	5390761	5392016	5393271	5394525	5395779
347	5403295	5404546	5405797	5407048	5408298
348	5415792	5417040	5418288	5419535	5420781
349	5428254	5429498	5430742	5431986	5433229
350	5440680	5441921	5443161	5444401	5445641
351	5453071	5454308	5455545	5456781	5458017
352	5465427	5466660	5467894	5469126	5470359
353	5477747	5478977	5480207	5481436	5482665
354	5490033	5491259	5492486	5493712	5494937

Natural Numbers.	5	6	7	8	9
311	4934580	4935974	4937368	4938761	4940154
312	4948500	4949890	4951279	4952667	4954056
313	4962375	4963761	4965145	4966529	4967913
314	4976206	4977587	4978967	4980347	4981727
315	4989994	4991370	4992745	4994121	4995496
316	5003737	5005109	5006481	5007852	5009222
317	5017437	5018805	5020172	5021539	5022905
318	5031094	5032458	5033821	5035183	5036545
319	5044709	5046068	5047426	5048785	5050142
320	5058280	5059635	5060990	5062344	5063697
321	5071810	5073160	5074511	5075860	5077210
322	5085297	5086644	5087990	5089335	5090680
323	5098743	5100085	5101427	5102768	5104109
324	5112147	5113485	5114823	5116160	5117497
325	5125510	5126844	5128178	5129511	5130844
326	5138832	5140162	5141491	5142820	5144149
327	5152113	5153439	5154764	5156089	5157414
328	5165354	5166676	5167997	5169318	5170639
329	5178554	5179872	5181189	5182506	5183823
330	5191715	5193028	5194342	5195655	5196968
331	5204835	5206145	5207455	5208764	5210073
332	5217916	5219222	5220528	5221833	5223138
333	5230958	5232260	5233562	5234863	5236164
334	5243961	5245259	5246557	5247854	5249151
335	5256925	5258219	5259513	5260807	5262100
336	5269851	5271141	5272431	5273721	5275010
337	5282738	5284024	5285311	5286596	5287882
338	5295587	5296869	5298152	5299434	5300716
339	5308398	5309677	5310955	5312234	5313512
340	5321171	5322446	5323721	5324996	5326270
341	5333907	5335179	5336450	5337721	5338991
342	5346606	5347874	5349141	5350408	5351675
343	5359267	5360532	5361795	5363059	5364322
344	5371892	5373153	5374413	5375672	5376932
345	5384481	5385737	5386994	5388250	5389506
346	5397032	5398286	5399538	5400791	5402043
347	5409548	5410798	5412047	5413296	5414544
348	5422028	5423274	5424519	5425765	5427010
349	5434472	5435714	5436956	5438198	5439439
350	5446880	5448119	5449358	5450596	5451834
351	5459253	5460489	5461724	5462958	5464193
352	5471591	5472823	5474055	5475286	5476517
353	5483894	5485123	5486351	5487578	5488806
354	5406162	5497387	5498612	5499836	5501060

Natural Numbers.	0	1	2	3	4
355	5502283	5503507	5504730	5505952	5507174
356	5514500	5515720	5516939	5518158	5519377
357	5526682	5527898	5529114	5530330	5531545
358	5538830	5540043	5541256	5542468	5543680
359	5550944	5552154	5553362	5554572	5555781
360	5563025	5564231	5565437	5566643	5567848
361	5575072	5576275	5577477	5578680	5579881
362	5587086	5588285	5589484	5590683	5591882
363	5599066	5600262	5601458	5602654	5603849
364	5611014	5612207	5613399	5614592	5615784
365	5622929	5624118	5625308	5626497	5627685
366	5634811	5635997	5637183	5638369	5639555
367	5646661	5647844	5649027	5650209	5651392
368	5658478	5659658	5660838	5662017	5663196
369	5670264	5671440	5672617	5673793	5674969
370	5682017	5683191	5684364	5685537	5686710
371	5693739	5694910	5696080	5697249	5698419
372	5705429	5706597	5707764	5708930	5710097
373	5717088	5718252	5719416	5720580	5721743
374	5728716	5729877	5731038	5732198	5733358
375	5740313	5741471	5742628	5743786	5744943
376	5751878	5753033	5754188	5755342	5756496
377	5763413	5764565	5765717	5766868	5768019
378	5774917	5776067	5777215	5778363	5779511
379	5786392	5787538	5788683	5789828	5790973
380	5797836	5798979	5800121	5801263	5802405
381	5809250	5810389	5811529	5812668	5813807
382	5820634	5821770	5822907	5824043	5825179
383	5831988	5833122	5834255	5835388	5836521
384	5843312	5844443	5845574	5846704	5847834
385	5854617	5855735	5856863	5857990	5859117
386	5865873	5866998	5868123	5869247	5870371
387	5877110	5878232	5879353	5880475	5881596
388	5888317	5889436	5890555	5891674	5892792
389	5899496	5900612	5901728	5902844	5903959
390	5910646	5911759	5912873	5913985	5915098
391	5921768	5922878	5923988	5925098	5926208
392	5932861	5933968	5935076	5936183	5937290
393	5943925	5945030	5946135	5947239	5948344
394	5954962	5956064	5957166	5958268	5959369
395	5965971	5967070	5968169	5969268	5970367
396	5976952	5978048	5979145	5980241	5981336
397	5987905	5988999	5990092	5991186	5992279
398	5998831	5999922	6001013	6002103	6003193

Natural Numbers.	5	6	7	8	9
355	5508396	5509618	5510839	5512059	5513280
356	5520595	5521813	5523031	5524248	5525465
357	5532760	5533975	5535189	5536403	5537617
358	5544892	5546103	5547314	5548524	5549735
359	5556989	5558197	5559404	5560612	5561818
360	5569053	5570257	5571461	5572665	5573869
361	5581083	5582284	5583485	5584686	5585886
362	5593080	5594278	5595476	5596673	5597870
363	5605044	5606239	5607433	5608627	5609820
364	5616975	5618167	5619358	5620548	5621739
365	5628875	5630062	5631250	5632437	5633624
366	5640740	5641925	5643109	5644293	5645477
367	5652573	5653755	5654936	5656117	5657298
368	5664375	5665553	5666731	5667909	5669087
369	5676144	5677320	5678494	5679669	5680843
370	5687882	5689054	5690226	5691397	5692568
371	5699588	5700757	5701926	5703094	5704262
372	5711263	5712428	5713594	5714759	5715924
373	5722906	5724069	5725231	5726393	5727555
374	5734518	5735678	5736837	5737996	5739154
375	5746099	5747256	5748412	5749568	5750723
376	5757650	5758803	5759956	5761109	5762261
377	5769169	5770320	5771470	5772620	5773769
378	5780659	5781806	5782953	5784100	5785246
379	5792118	5793262	5794406	5795550	5796693
380	5803547	5804688	5805829	5806969	5808110
381	5814945	5816084	5817222	5818359	5819497
382	5826314	5827450	5828585	5829719	5830854
383	5837654	5838786	5839918	5841050	5842181
384	5848963	5850093	5851222	5852351	5853479
385	5860244	5861370	5862496	5863622	5864748
386	5871495	5872618	5873742	5874865	5875987
387	5882717	5883838	5884958	5886078	5887198
388	5893910	5895028	5896145	5897262	5898379
389	5905075	5906189	5907304	5908418	5909532
390	5916210	5917322	5918434	5919546	5920657
391	5927318	5928427	5929536	5930644	5931753
392	5938397	5939503	5940609	5941715	5942820
393	5949447	5950551	5951654	5952757	5953860
394	5960470	5961571	5962671	5963771	5964871
395	5971465	5972563	5973660	5974758	5975855
396	5982432	5983527	5984622	5985717	5986811
397	5993371	5994464	5995556	5996648	5997739
398	6004283	6005373	6006462	6007551	6008640

Natural Numbers.	0	1	2	3	4
399	6009729	6010817	6011905	6012993	6014081
400	6020600	6021685	6022771	6023856	6024941
401	6031444	6032527	6033609	6034692	6035774
402	6042261	6043341	6044421	6045500	6046580
403	6053050	6054128	6055205	6056282	6057359
404	6063814	6064888	6065963	6067037	6068111
405	6074550	6075622	6076694	6077766	6078837
406	6085260	6086330	6087399	6088468	6089537
407	6095944	6097011	6098078	6099144	6100210
408	6106602	6107666	6108730	6109794	6110857
409	6117233	6118295	6119356	6120417	6121478
410	6127839	6128898	6129957	6131015	6132073
411	6138418	6139475	6140531	6141587	6142643
412	6148972	6150026	6151080	6152133	6153187
413	6159501	6160552	6161603	6162654	6163705
414	6170003	6171052	6172101	6173149	6174197
415	6180481	6181527	6182573	6183619	6184665
416	6190933	6191977	6193021	6194064	6195107
417	6201360	6202402	6203443	6204484	6205524
418	6211763	6212802	6213840	6214879	6215917
419	6222140	6223177	6224213	6225149	6226284
420	6232493	6233527	6234560	6235594	6236627
421	6242821	6243852	6244884	6245915	6246945
422	6253124	6254153	6255182	6256211	6257239
423	6263404	6264430	6265457	6266483	6267509
424	6273659	6274683	6275707	6276730	6277754
425	6283889	6284911	6285933	6286954	6287975
426	6294096	6295115	6296134	6297153	6298172
427	6304279	6305296	6306312	6307329	6308345
428	6314438	6315452	6316467	6317481	6318495
429	6324573	6325585	6326597	6327609	6328620
430	6334685	6335694	6336704	6337713	6338723
431	6344773	6345780	6346788	6347795	6348801
432	6354837	6355843	6356848	6357852	6358857
433	6364879	6365882	6366884	6367887	6368889
434	6374897	6375898	6376898	6377898	6378898
435	6384893	6385891	6386889	6387887	6388884
436	6394865	6395861	6396857	6397852	6398847
437	6474814	6405808	6406802	6407795	6408788
438	6414741	6415733	6416724	6417715	6418705
439	6424645	6425634	6426623	6427612	6428601
440	6434527	6435514	6436500	6437487	6438473
441	6444386	6445371	6446355	6447339	6448323
442	6454223	6455205	6456187	6457169	6458151

Natural Numbers.	5	6	7	8	9
399	6015168	6016255	6017341	6018428	6019514
400	6026025	6027109	6028193	6029277	6030361
401	6036855	6037937	6039018	6040099	6041180
402	6047659	6048738	6049816	6050895	6051973
403	6058435	6059512	6060587	6061663	6062738
404	6069185	6070259	6071332	6072405	6073478
405	6079909	6080979	6082050	6083120	6084190
406	6090605	6091674	6092742	6093809	6094877
407	6101276	6102342	6103407	6104472	6105537
408	6111921	6112948	6114046	6115109	6116171
409	6122539	6123599	6124660	6125720	6126779
410	6133132	6134189	6135247	6136304	6137361
411	6143698	6144754	6145809	6146863	6147918
412	6154240	6155292	6156345	6157397	6158449
413	6164755	6165805	6166855	6167905	6168954
414	6175245	6176293	6177340	6178387	6179434
415	6185710	6186755	6187800	6188845	6189889
416	6196150	6197193	6198235	6199277	6200319
417	6206565	6207605	6208645	6209684	6210724
418	6216955	6217992	6219030	6220067	6221104
419	6227320	6228355	6229300	6230424	6231459
420	6237660	6238693	6239725	6240757	6241789
421	6247976	6249006	6250036	6251066	6252095
422	6258267	6259295	6260322	6261350	6262377
423	6268534	6269559	6270585	6271610	6272634
424	6278777	6279800	6280823	6281845	6282867
425	6288996	6290016	6291036	6292057	6293076
426	6299190	6300208	6301226	6302244	6303262
427	6309361	6310377	6311392	6312408	6313423
428	6319508	6320522	6321535	6322548	6323560
429	6329632	6330643	6331653	6332664	6333674
430	6339732	6340740	6341749	6342757	6343765
431	6349808	6350814	6351820	6352826	6353832
432	6359861	6360865	6361869	6362872	6363876
433	6369891	6370893	6371894	6372895	6373896
434	6379898	6380897	6381896	6382895	6383894
435	6389882	6390879	6391876	6392872	6393869
436	6399842	6400837	6401832	6402826	6403820
437	6409781	6410773	6411765	6412758	6413749
438	6419696	6420686	6421676	6422666	6423656
439	6429589	6430577	6431565	6432552	6433540
440	6439459	6440445	6441430	6442416	6443401
441	6449307	6450291	6451274	6452257	6453240
442	6459133	6460114	6461095	6462076	6453057

Natural Numbers.	0	1	2	3	4
443	6464037	6465017	6465997	6466977	6467957
444	6473830	6474808	6475785	6476763	6477740
445	6483600	6484576	6485552	6486527	6487502
446	6493349	6494322	6495296	6496269	6497242
447	6503075	6504047	6505018	6505989	6506960
448	6512780	6513749	6514719	6515687	6516656
449	6522463	6523430	6524397	6525364	6526331
450	6532125	6533090	6534055	6535019	6535984
451	6541765	6542728	6543691	6544653	6545616
452	6551384	6552345	6553306	6554266	6555226
453	6560982	6561941	6562899	6563857	6564815
454	6570559	6571515	6572471	6573427	6574383
455	6580114	6581068	6582023	6582976	6583930
456	6589648	6590601	6591553	6592505	6593456
457	6599162	6600112	6601062	6602012	6602962
458	6608655	6609603	6610551	6611499	6612446
459	6618127	6619073	6620019	6620964	6621910
460	6627578	6628522	6629466	6630410	6631353
461	6637009	6637951	6638893	6639835	6640776
462	6646420	6647360	6648299	6649239	6650178
463	6655810	6656748	6657685	6658623	6659560
464	6665180	6666116	6667051	6667987	6668922
465	6674530	6675463	6676397	6677331	6678264
466	6683859	6684791	6685723	6686654	6687585
467	6693169	6694099	6695028	6695958	6696887
468	6702459	6703386	6704314	6705242	6706169
469	6711728	6712654	6713580	6714506	6715431
470	6720979	6721903	6722826	6723750	6724673
471	6730209	6731131	6732053	6732974	6733896
472	6739420	6740340	6741260	6742179	6743099
473	6748611	6749529	6750447	6751365	6752283
474	6757783	6758700	6759615	6760531	6761447
475	6766936	6767850	6768764	6769678	6770592
476	6776069	6776982	6777894	6778806	6779718
477	6785184	6786094	6787004	6787914	6788824
478	6794279	6795187	6796096	6797004	6797912
479	6803355	6804262	6805168	6806074	6806980
480	6812412	6813317	6814222	6815126	6816030
481	6821451	6822354	6823256	6824159	6825061
482	6830470	6831371	6832272	6833173	6834073
483	6839471	6840370	6841269	6842168	6843066
484	6848454	6849351	6850248	6851145	6852041
485	6857417	6858313	6859208	6860103	6860998
486	6866353	6867256	6868149	6869043	6869936

Natural Numbers.	5	6	7	8	9
443	6468936	6469915	6470894	6471873	6472851
444	6478718	6479695	6480671	6481648	6482624
445	6488477	6489452	6490426	6491401	6492375
446	6498215	6499187	6500160	6501132	6502104
447	6507930	6508901	6509871	6510841	6511811
448	6517624	6518593	6519561	6520528	6521496
449	6527297	6528263	6529229	6530195	6531160
450	6536948	6537912	6538876	6539839	6540802
451	6546578	6547539	6548501	6549462	6550423
452	6556186	6557145	6558105	6559064	6560023
453	6565773	6566730	6567688	6568645	6569602
454	6575339	6576294	6577250	6578205	6579159
455	6584884	6585837	6586790	6587743	6588696
456	6594408	6595359	6596310	6597261	6598212
457	6603911	6604860	6605809	6606758	6607706
458	6613393	6614340	6615287	6616234	6617181
459	6622855	6623800	6624745	6625690	6626634
460	6632296	6633239	6634182	6635125	6636067
461	6641717	6642658	6643599	6644539	6645480
462	6651117	6652056	6652995	6653933	6654872
463	6660497	6661434	6662371	6663307	6664244
464	6669857	6670792	6671727	6672661	6673595
465	6679197	6680130	6681062	6681995	6682927
466	6688516	6689447	6690378	6691308	6692239
467	6697816	6698745	6699674	6700602	6701530
468	6707096	6708023	6708950	6709876	6710802
469	6716356	6717281	6718206	6719130	6720054
470	6725596	6726519	6727442	6728365	6729287
471	6734817	6735738	6736659	6737574	6738500
472	6744018	6744937	6745856	6746775	6747693
473	6753200	6754117	6755034	6755951	6756867
474	6762362	6763277	6764192	6765107	6766022
475	6771505	6772418	6773332	6774244	6775157
476	6780629	6781540	6782452	6783362	6784273
477	6789734	6790643	6791552	6792461	6793370
478	6798819	6799727	6800634	6801541	6802448
479	6807886	6808792	6809697	6810602	6811507
480	6816934	6817838	6818741	6819645	6820548
481	6825963	6826865	6827766	6828668	6829569
482	6834973	6835873	6836773	6837673	6838572
483	6843965	6844863	6845761	6846659	6847556
484	6852938	6853834	6854730	6855626	6856522
485	6861892	6862787	6863681	6864575	6865469
486	6870828	6871721	6872613	6873506	6874398

Natural Numbers.	0	1	2	3	4
487	6875290	6876181	6877073	6877964	6878855
488	6884198	6885088	6885978	6886867	6887757
489	6893089	6893977	6894864	6895752	6896640
490	6901961	6902847	6903733	6904616	6905505
491	6910815	6911699	6912584	6913468	6914352
492	6919651	6920534	6921416	6922298	6923180
493	6928469	6929350	9930231	6931111	6931991
494	6937269	6938148	6939027	6939906	6940785
495	6946052	6946929	6947806	6948683	6949560
496	6954817	6955692	6956568	6957443	6958318
497	6963564	6964438	6965311	6966185	6967058
498	6972293	6973165	6974037	6974909	6975780
499	6981005	6981876	6982746	6983616	6984485
500	6989700	6990569	6991437	6992305	6993173
501	6998377	6999244	7000111	7000977	7001843
502	7007037	7007902	7008767	7009632	7010496
503	7015680	7016543	7017406	7018269	7019132
504	7024305	7025167	7026028	7026890	7027751
505	7032914	7033774	7034633	7035493	7036352
506	7041505	7042363	7043221	7044079	7044937
507	7050080	7050936	7051792	7052649	7053505
508	7058637	7059492	7060347	7061201	7062055
509	7067178	7068031	7068884	7069737	7070589
510	7075702	7076553	7077405	7078256	7079107
511	7084206	7085059	7085908	7086758	7087607
512	7092700	7093548	7094396	7095244	7096091
513	7101174	7102020	7102866	7103713	7104559
514	7109631	7110476	7111321	7112165	7113010
515	7118072	7118915	7119759	7120601	7121444
516	7126497	7127339	7128180	7129021	7129862
517	7134905	7135745	7136585	7137425	7138264
518	7143298	7144136	7144974	7145812	7146650
519	7151674	7152510	7153347	7154183	7155019
520	7160033	7160869	7161703	7162538	7163373
521	7168377	7169211	7170044	7170877	7171710
522	7176705	7177537	7178369	7179200	7180032
523	7185017	7185847	7186677	7187507	7188337
524	7193313	7194142	7194970	7195799	7196627
525	7201593	7202420	7203247	7204074	7204901
526	7209857	7210683	7211508	7212334	7213159
527	7218106	7218930	7219754	7220578	7221401
528	7226339	7227162	7227984	7228806	7229628
529	7234557	7235378	7236198	7237019	7237839
530	7242759	7243578	7244397	7245216	7246035

Natural Numbers.	5	6	7	8	9
487	6879746	6880637	6881528	6882418	6883308
488	6888646	6889535	6890423	6891312	6892200
489	6897527	6898414	6899301	6900188	6901074
490	6906390	6907275	6908161	6909046	6909930
491	6915235	6916119	6917002	6917885	6918768
492	6924062	6924944	6925826	6926707	6927588
493	6932872	6933752	6934631	6935511	6936390
494	6941663	6942541	6943419	6944297	6945174
495	6950437	6951313	6952189	6953065	6953941
496	6959193	6960067	6960942	6961816	6962690
497	6967931	6968804	6969676	6970549	6971421
498	6976652	6977523	6978394	6979264	6980135
499	6985355	6986224	6987093	6987963	6988831
500	6994041	6994908	6995776	6996643	6997510
501	7002709	7003575	7004441	7005307	7006172
502	7011361	7012225	7013089	7013953	7014816
503	7019995	7020857	7021719	7022582	7023444
504	7028612	7029472	7070333	7031193	7032054
505	7037212	7038071	7038929	7039788	7040647
506	7045793	7046652	7047509	7048366	7049223
507	7054360	7055216	7056072	7056927	7057782
508	7062910	7063764	7064617	7065471	7066324
509	7071442	7072294	7073146	7073998	7074850
510	7079957	7080808	7081659	7082509	7083359
511	7088456	7089305	7090154	7091003	7091851
512	7096939	7097786	7098633	7099480	7100327
513	7105404	7106250	7107096	7107941	7108786
514	7113854	7114698	7115542	7116385	7117229
515	7122287	7123129	7123971	7124813	7125655
516	7130703	7131544	7132385	7133225	7134065
517	7139104	7139943	7140782	7141620	7142459
518	7147488	7148325	7149162	7150000	7150837
519	7155856	7156691	7157527	7158363	7159198
520	7164207	7165042	7165876	7166710	7167544
521	7172543	7173376	7174208	7175041	7175873
522	7180863	7181694	7182525	7183356	7184186
523	7189167	7189996	7190826	7191655	7192484
524	7197455	7198283	7199111	7199938	7200766
525	7205727	7206554	7207380	7208206	7209032
526	7213984	7214809	7215633	7216458	7217282
527	7222225	7223048	7223871	7224694	7225517
528	7230450	7231272	7232093	7232914	7233736
529	7238660	7239480	7240300	7241120	7241939
530	7246854	7247672	7248491	7249309	7250127

Natural Numbers.	0	1	2	3	4
531	7250945	7251763	7252581	7253398	7254215
532	7259116	7259933	7260749	7261565	7262380
533	7267272	7268087	7268901	7269716	7270531
534	7275413	7276226	7277039	7277852	7278664
535	7283538	7284349	7285161	7285972	7286784
536	7291648	7292458	7293268	7294078	7294888
537	7299743	7300551	7301360	7302168	7302977
538	7307823	7308630	7309437	7310244	7311051
539	7315888	7316693	7317499	7318304	7319109
540	7323938	7324749	7325546	7326350	7327153
541	7331973	7332775	7333578	7334380	7335182
542	7339993	7340794	7341595	7342396	7343197
543	7347998	7348798	7349598	7350397	7351196
544	7355989	7356787	7357585	7358383	7359181
545	7363965	7364762	7365558	7366355	7367151
546	7371926	7372722	7373517	7374312	7375107
547	7379873	7380667	7381461	7382254	7383048
548	7387806	7388598	7389390	7390182	7390974
549	7395723	7396514	7397305	7398096	7398886
550	7403627	7404416	7405206	7405995	7406784
551	7411516	7412304	7413092	7413880	7414668
552	7419391	7420177	7420964	7421750	7422537
553	7427251	7428037	7428822	7429607	7430392
554	7435098	7435881	7436665	7437449	7438232
555	7442930	7443712	7444495	7445277	7446059
556	7450748	7451529	7452310	7453091	7453871
557	7458552	7459332	7460111	7460890	7461670
558	7466342	7467120	7467898	7468676	7469454
559	7474118	7474895	7475672	7476448	7477225
560	7481880	7482656	7483431	7484206	7484981
561	7489629	7490403	7491177	7491950	7492724
562	7497363	7498136	7498908	7499681	7500453
563	7505087	7505855	7506626	7507398	7508168
564	7512791	7513561	7514331	7515100	7515870
565	7520484	7521253	7522022	7522790	7523558
566	7528164	7528932	7529699	7530466	7531232
567	7535831	7536596	7537362	7538128	7538893
568	7543483	7544248	7545012	7545777	7546541
569	7551123	7551886	7552649	7553412	7554178
570	7558749	7559510	7560279	7561034	7561795
571	7566361	7567122	7567882	7568642	7569402
572	7573960	7574719	7575479	7576237	7576996
573	7581546	7582304	7583062	7583819	7584577
574	7589119	7589875	7590632	7591388	7592144

Natural Numbers.	5	6	7	8	9
531	7255033	7255850	7256667	7257483	7258300
532	7263196	7264012	7264827	7265642	7266457
533	7271344	7272158	7272972	7273786	7274599
534	7279477	7280290	7281101	7281914	7282726
535	7287595	7288406	7289216	7290027	7290838
536	7295697	7296506	7297316	7298125	7298934
537	7303785	7304593	7305400	7306208	7307015
538	7311857	7312663	7313470	7314276	7315082
539	7319914	7320719	7321524	7322329	7323133
540	7327957	7328760	7329564	7330367	7331170
541	7335985	7336787	7337588	7338390	7339191
542	7343997	7344798	7345598	7346398	7347198
543	7351995	7352794	7353593	7354392	7355191
544	7359979	7360776	7361574	7362371	7363168
545	7367948	7368744	7369540	7370335	7371131
546	7375902	7376696	7377491	7378285	7379079
547	7383841	7384634	7385427	7386220	7387013
548	7391766	7392558	7393350	7394141	7394932
549	7399677	7400467	7401257	7402047	7402837
550	7407573	7408362	7409151	7409939	7410728
551	7415455	7416283	7417030	7417817	7418604
552	7423323	7424109	7424895	7425680	7426466
553	7431176	7431961	7432745	7433530	7434314
554	7439015	7439799	7440582	7441365	7442147
555	7446841	7447622	7448404	7449187	7449967
556	7454652	7455432	7456212	7456992	7457772
557	7462449	7463228	7464006	7464785	7465564
558	7470232	7471009	7471787	7472564	7473341
559	7478001	7478777	7479553	7480329	7481105
560	7485756	7486531	7487306	7488080	7488854
561	7493498	7494271	7495044	7495817	7496590
562	7501225	7501997	7502769	7503541	7504312
563	7508939	7509710	7510480	7511251	7512021
564	7516639	7517409	7518178	7518947	7519716
565	7524326	7525094	7525862	7526629	7527397
566	7531999	7532766	7533532	7534298	7535065
567	7539659	7540424	7541189	7541954	7542719
568	7547305	7548069	7548832	7549596	7550359
569	7554937	7555700	7556462	7557224	7557987
570	7562556	7563318	7564079	7564840	7565600
571	7570162	7570922	7571682	7572441	7573201
572	7577755	7578513	7579272	7580030	7580788
573	7585334	7586091	7586848	7587605	7588362
574	7592900	7593656	7594412	7595168	7595923

Natural Numbers.	0	1	2	3	4
575	7596678	7597434	7598189	7598944	7599699
576	7604225	7604979	7605733	7609486	7607240
577	7611758	7612511	7613263	7614016	7614768
578	7619278	7620030	7620781	7621532	7622283
579	7626786	7627536	7628286	7629035	7629785
580	7634280	7635029	7635777	7636526	7637274
581	7641761	7642509	7643256	7644003	7644750
582	7649230	7649976	7650722	7651468	7652214
583	7656686	7657430	7658175	7658920	7659664
584	7664128	7664872	7665616	7666359	7667102
585	7671559	7672301	7673043	7673785	7674527
586	7678976	7679717	7680458	7681199	7681940
587	7686381	7687121	7687860	7688600	7689339
588	7693773	7694512	7695250	7695988	7696727
589	7701153	7701890	7702627	7703364	7704101
590	7708520	7709256	7709992	7710728	7711463
591	7715875	7716610	7717344	7718079	7718813
592	7723217	7723951	7724684	7725417	7726150
593	7730547	7731279	7732011	7732743	7733475
594	7737864	7738596	7739326	7740057	7740788
595	7745170	7745899	7746629	7747359	7748088
596	7752463	7753191	7753920	7754648	7755376
597	7759743	7760471	7761198	7761925	7762652
598	7767012	7767738	7768464	7769190	7769916
599	7774268	7774993	7775718	7776443	7777167
600	7781513	7782236	7782960	7783983	7784407
601	7788745	7789467	7790190	7790912	7791634
602	7795965	7796686	7797408	7798129	7798850
603	7803173	7803893	7804613	7805333	7806053
604	7810369	7811088	7811807	7812526	7813245
605	7817554	7818272	7818989	7819707	7820424
606	7824726	7825443	7826159	7826876	7827592
607	7831887	7832602	7833318	7834033	7834748
608	7839036	7839750	7840464	7841178	7841892
609	7846173	7846886	7847599	7848312	7849024
610	7853298	7854010	7854722	7855434	7856145
611	7860412	7861123	7861833	7862544	7863254
612	7867514	7868224	7868933	7869643	7870352
613	7874605	7875313	7876021	7876730	7877438
614	7881684	7882391	7883098	7883805	7884512
615	7888751	7889457	7890163	7890869	7891575
616	7895807	7896512	7897217	7897922	7898626
617	7902852	7903555	7904259	7904963	7905666
618	7909885	79.0587	0911290	7911992	7912695

Natural Numbers.	5	6	7	8	9
575	7600453	7601208	7601962	7602717	7603471
576	7607993	7608746	7609500	7610253	7611005
577	7615520	7616272	7617024	7617775	7618527
578	7623034	7623784	7624535	7625285	7626035
579	7630534	7631284	7632033	7632782	7633531
580	7638022	7638770	7639518	7640266	7641014
581	7645497	7646244	7646991	7647737	7648484
582	7652959	7653705	7654450	7555195	7655941
583	7660409	7661153	7661897	7662641	7663385
584	7667845	7668588	7669331	7670074	7670816
585	7675269	7676011	7676752	7677494	7678235
586	7682680	7683421	7684161	7684901	7685641
587	7690079	7690818	7691557	7692296	7693035
588	7697465	7698203	7698940	7699678	7700416
589	7704838	7705575	7706311	7707048	7707784
590	7712199	7712934	7713670	7714405	7715140
591	7719547	7720282	7721016	7721750	7722483
592	7726884	7727616	7728349	7729082	7729814
593	7734207	7734939	7735670	7736402	7737133
594	7741519	7742249	7742979	7743710	7744440
595	7748818	7749547	7750276	7751005	7751734
596	7756104	7756832	7757560	7758288	7759016
597	7763379	7764106	7764833	7765559	7766286
598	7770642	7771367	7772093	7772818	7773543
599	7777892	7778616	7779340	7780065	7780789
600	7785130	7785853	7786576	7787299	7788022
601	7792356	7793078	7793800	7794522	7795243
602	7799571	7800291	7801012	7801732	7802453
603	7806773	7807492	7808212	7808931	7809650
604	7813963	7814681	7815400	7816118	7816836
605	7821141	7821859	7822576	7823293	7824010
606	7828308	7829024	7829740	7830456	7831171
607	7835463	7836178	7836892	7837607	7838321
608	7842606	7843319	7844033	7844746	7845460
609	7849737	7850450	7851162	7851874	7852586
610	7856857	7857568	7858279	7858990	7859701
611	7863965	7864675	7865385	7866095	7866805
612	7871061	7871770	7872479	7873188	7873896
613	7878146	7878853	7879561	7880269	7880976
614	7885219	7885926	7886632	7887339	7888045
615	7892281	7892986	7893691	7894397	7895102
616	7899331	7900035	7900739	7901444	7902148
617	7906370	7907073	7907776	7908479	7909182
618	7913397	7914099	7914801	7915503	7916205

Natural Numbers.	0	1	2	3	4
619	7916906	7917608	7918309	7919011	7919712
620	7923917	7924617	7925318	7926018	7926718
621	7930916	7931615	7932314	7933014	7933712
622	7937904	7938602	7939300	7939998	7940696
623	7944880	7945578	7946274	7946971	7947668
624	7951846	7952542	7953238	7953933	7954629
625	7958800	7959495	7960190	7960884	7961578
626	7965743	7966437	7967131	7967824	7968517
627	7972675	7973368	7974060	7974753	7975445
628	7979596	7980288	7980979	7981671	7982362
629	7986506	7987197	7987887	7988577	7982267
630	7993405	7994097	7994784	7995473	7996162
631	8000294	8000982	8001670	8002358	8003046
632	8007171	8007858	8008545	8009232	8009919
633	8014037	8014723	8015409	8016095	8016781
634	8020893	8021578	8022262	8022947	8023632
635	8027737	8028421	8029105	8029789	8030472
636	8034571	8035254	8035937	8036619	8037302
637	8041394	8042076	8042758	8043439	8044121
638	8048207	8048887	8049568	8050248	8050929
639	8055009	8055688	8056368	8057047	8057726
640	8061800	8062478	8063157	8063835	8064513
641	8068580	8069258	8069935	8070612	8071290
642	8075350	8076027	8076703	8077379	8078055
643	8082110	8082785	8083460	8084136	8084811
644	8088859	8089533	8090207	8090881	8091555
645	8095597	8096270	8096944	8097617	8098290
646	8102325	8102997	8103670	8104342	8105013
647	8109043	8109714	8110385	8111056	8111727
648	8115750	8116420	8117090	8117760	8118430
649	8122447	8123116	8123785	8124454	8125123
650	8129134	8129802	8130470	8131138	8131805
651	8135810	8136477	8137144	8137811	8138478
652	8142476	8143142	8143808	8144474	8145140
653	8149132	8149797	8150462	8151127	8151791
654	8155777	8156441	8157105	8157769	8158433
655	8162413	8163076	8163739	8164402	8165064
656	8169038	8169700	8170362	8171024	8171686
657	8175654	8176315	8176976	8177636	8178297
658	8182259	8182919	8183579	8184239	8184898
659	8188854	8189513	8190172	8190831	8191489
660	8195439	8196097	8196755	8197413	8198071
661	8202015	8202672	8203328	8203987	8204642
662	8208580	8209236	8209892	8210548	8211203

Natural Numbers.	5	6	7	8	9
619	7920413	7921114	7921815	7922516	7923216
620	7927418	7928118	7928817	7929517	7930217
621	7934411	7935110	7935809	7936507	7937206
622	7941394	7942091	7942789	7943486	7944183
623	7948365	7949061	7949757	7950454	7951150
624	7955324	7956020	7956715	7957410	7958105
625	7962273	7962967	7963662	7964356	7965050
626	7969211	7969904	7970597	7971290	7971983
627	7976137	7976829	7977521	7978213	7978905
628	7983053	7983744	7984435	7985125	7985816
629	7989957	7990647	7991337	7992027	7992716
630	7996851	7997540	7998228	7998917	7999605
631	8003734	8004421	8005109	8005796	8006484
632	8010605	8011292	8011978	8012665	8013351
633	8017466	8018152	8018837	8019522	8020208
634	8024316	8025001	8025685	8026369	8027053
635	8031156	8031839	8032522	8033205	8033888
636	8037984	8038666	8039348	8040031	8040712
637	8044802	8045483	8046164	8046845	8047526
638	8051609	8052289	8052969	8053649	8054329
639	8058405	8059085	8059763	8060442	8061121
640	8065191	8065869	8066547	8067225	8067903
641	8071967	8072643	8073320	8073997	8074874
642	8078731	8079407	8080083	8080759	8081434
643	8085485	8086160	8086835	8087510	8088184
644	8092229	8092903	8093577	8094250	8094924
645	8098962	8099635	8100308	8100980	8101653
646	8105685	8106357	8107029	8107700	8108371
647	8112398	8113068	8113739	8114409	8115080
648	8119100	8119769	8120439	8121108	8121778
649	8125792	8126460	8127129	8127797	8128465
650	8132473	8133141	8133808	8134475	8135143
651	8139144	8139811	8140477	8141144	8141810
652	8145805	8146471	8147136	8147801	8148467
653	8152456	8153120	8153785	8154449	8155113
654	8159096	8159760	8160423	8161087	8161750
655	8165727	8166389	8167052	8167714	8168376
656	8172347	8173009	8173670	8174331	8174993
657	8178958	8179618	8180278	8180939	8181599
658	8185558	8186217	8186877	8187536	8188195
659	8192146	8192806	8193465	8194123	8194781
660	8198728	8199386	8200043	8200700	8201358
661	8205298	8205955	8206611	8207268	8207924
662	8211859	8212514	8213170	8213825	8214480

Natural Numbers.	0	1	2	3	4
663	8215135	8215790	8216445	8217100	8217755
664	8221681	8222335	8222989	8223643	8224296
665	8228216	8228869	8229522	8230175	8230828
666	8234742	8235394	8236046	8236698	8237350
667	8241258	8241909	8242560	8243211	8243862
668	8247765	8248415	8249065	8249715	8250364
669	8254261	8254910	8255559	8256208	8256857
670	8260748	8261396	8262044	8262692	8263340
671	8267225	8267872	8268519	8269166	8269813
672	8273693	8274339	8274985	8275631	8276277
673	8280151	8280796	8281441	8282086	8282731
674	8286599	8287243	8287887	8288532	8289176
675	8293038	8293681	8294324	8294967	8295611
676	8299467	8300109	8300752	8301394	8302036
677	8305887	8306528	8307169	8307811	8308452
678	8312297	8312937	8313578	8314218	8314858
679	8318698	8319337	8319977	8320616	8321255
680	8325089	8325728	8326366	8327005	8327643
681	8331471	8332109	8332746	8333384	8334021
682	8337844	8338480	8339117	8339754	8340390
683	8344207	8344843	8345479	8346114	8346750
684	8350561	8351196	8351831	8352465	8353100
685	8356906	8357540	8358174	8358807	8359441
686	8363241	8363874	8364507	8365140	8365773
687	8369567	8370199	8370832	8371463	8372095
688	8375884	8376516	8377147	8377778	8378409
689	8382192	8382822	8383453	8384083	8384713
690	8388491	8389120	8389750	8390379	8391008
691	8394780	8395409	8396037	8396666	8397294
692	8401061	8401688	8402316	8402943	8403571
693	8407332	8407959	8408586	8409212	8409838
694	8413595	8414220	8414846	8415472	8416097
695	8419848	8420473	8421098	8421722	8422347
696	8426092	8426716	8427340	8427964	8428588
697	8432328	8432951	8433574	8434197	8434819
698	8438554	8439176	8439798	8440420	8441042
699	8444772	8445393	8446014	8446635	8447256
700	8450980	8451601	8452221	8452841	8453461
701	8457180	8457800	8458419	8459038	8459658
702	8463371	8463990	8464608	8465227	8465845
703	8469553	8470171	8470789	8471406	8472024
704	8475727	8476343	8476960	8477577	8478193
705	8481891	8482507	8483123	8483739	8484355
706	8488047	8488662	8489277	8489892	8490507

Natural Numbers.	5	6	7	8	9
663	8218409	8219064	8219718	8220372	8221027
664	8224950	8225603	8226257	8226910	8227563
665	8231481	8232133	8232786	8233438	8234090
666	8238002	8238653	8239305	8239956	8240607
667	8244513	8245163	8245814	8246464	8247114
668	8251014	8251664	8252313	8252963	8253612
669	8257506	8258154	8258803	8259451	8260100
670	8263988	8264635	8265283	8265931	8266578
671	8270460	8271107	8271753	8272400	8273046
672	8276923	8277569	8278214	8278860	8279505
673	8283376	8284021	8284665	8285310	8285955
674	8289820	8290463	8291107	8291751	8292394
675	8296254	8296896	8297539	8298182	8298824
676	8302678	8303320	8303962	8304603	8305245
677	8309093	8309734	8310375	8311016	8311656
678	8315499	8316139	8316778	8317418	8318058
679	8321895	8322534	8323173	8323812	8324450
680	8328281	8328919	8329558	8330195	8330833
681	8334659	8335296	8335933	8336570	8337207
682	8341027	8341663	8342299	8342937	8343571
683	8347385	8348021	8348656	8349291	8349926
684	8353735	8354369	8355003	8355638	8356272
685	8360075	8360708	8391341	8361975	8362608
686	8366405	8367038	8367670	8368303	8368935
687	8372727	8373359	8373990	8374622	8375253
688	8379039	8379670	8380301	8380931	8381562
689	8385343	8385973	8386602	8387232	8387861
690	8391637	8392266	8392895	8393523	8394152
691	8397922	8398550	8399178	8399806	8400433
692	8404198	8404825	8405452	8406079	8406706
693	8410465	8411091	8411717	8412343	8412969
694	8416722	8417348	8417973	8418598	8419223
695	8422971	8423596	8424220	8424844	8425468
696	8429211	8429735	8430458	8431081	8431705
697	8435442	8436065	8436687	8437310	8437932
698	8441664	8442286	8442907	8443529	8444150
699	8447877	8448498	8449119	8449739	8450360
700	8454081	8454701	8455321	8455941	8456561
701	8460277	8460896	8461515	8462134	8462752
702	8466463	8467081	8467700	8468318	8468935
703	8472641	8473258	8473876	8474493	8475110
704	8478810	8479426	8480043	8480659	8481275
705	8484970	8485586	8486201	8486817	8487432
706	8491122	8491736	8492351	8492965	8493580

Natural Numbers.	0	1	2	3	4
707	8494194	8494808	8495423	8496037	8496651
708	8500333	8500946	8501559	8502172	8502786
709	8506462	8507075	8507687	8508300	8508912
710	8512583	8513195	8513807	8514418	8515030
711	8518696	8519307	8519917	8520528	8521139
712	8524800	8525410	8526020	8526629	8527239
713	8530895	8531504	8532113	8532722	8533331
714	8536982	8537590	8538198	8538806	8539414
715	8543060	8543668	8544275	8544882	8545489
716	8549130	8549737	8550343	8550949	8551556
717	8555192	8555797	8556403	8557008	8557614
718	8561244	8561849	8562454	8563059	8563663
719	8567289	8567893	8568497	8569101	8569704
720	8573325	8573928	8574531	8575134	8575737
721	8579353	8579955	8580557	8581159	8581761
722	8585372	8985973	8586575	8587176	8587777
723	8591383	8591984	8592584	8593185	8593785
724	8597386	8597985	8598585	8599185	8599784
725	8603380	8603979	8604578	8605177	8605776
726	8609366	8609964	8610562	8611160	8611758
727	8615344	8615941	8616539	8617136	8617733
728	8621314	8621910	8622507	8623103	8623699
729	8627275	8627871	8628467	8629062	8629658
730	8633229	8633823	8634418	8635013	8635608
731	8639174	8639768	8640362	8640956	8641550
732	8645111	8645704	8646297	8646890	8647483
733	8651040	8651632	8652225	8652817	8653409
734	8656961	8657552	8658144	8658735	8659327
735	8662873	8663464	8664055	8664646	8665236
736	8668778	8669368	8669958	8670548	8671138
737	8674675	8675264	8675853	8676442	8677031
738	8680564	8681152	8681740	8682329	8682917
739	8686444	8687032	8687620	8688207	8688794
740	8692317	8692904	8693491	8694077	8694664
741	8698182	8698768	8699354	8699940	8700526
742	8704039	8704624	8705209	8705795	8706380
743	8709888	8710473	8711057	8711641	8712226
744	8715729	8716313	8716897	8717480	8718064
745	8721563	8722146	8722728	8723311	8723894
746	8727388	8727970	8728552	8729134	8729716
747	8733206	8733788	8734369	8734950	8735531
748	8739016	8739597	8740177	8740757	8741338
749	8744818	8745398	8745978	8746557	8747137
750	8750613	8751192	8751771	8752349	8752928

Natural Numbers.	5	6	7	8	9
707	8497264	8497878	8498492	8499106	8499719
708	8503399	8504011	8504624	8505237	8505850
709	8509524	8510136	8510748	8511360	8511972
710	8515641	8516252	8516863	8517474	8518085
711	8521749	8522359	8522970	8523580	8524190
712	8527849	8528458	8529068	8529677	8530286
713	8533940	8534548	8535157	8535765	8536374
714	8540022	8540630	8541238	8541845	8542453
715	8546096	8546703	8547310	8547917	8548524
716	8552162	8552768	8553374	8553980	8554586
717	8558219	8558824	8569429	8560035	8560640
718	8564268	8564872	8555476	8566081	8566685
719	8570308	8570912	8571515	8572118	8572722
720	8576340	8576943	8577545	8578148	8578750
721	8582363	8582965	8583567	8584169	8584770
722	8588379	8588980	8589581	8590181	8590782
723	8594385	8594986	8595586	8596186	8596786
724	8600384	8600983	8601583	8602182	8602781
725	8606374	8606973	8607571	8608170	8608768
726	8612356	8612954	8613552	8614149	8614747
727	8618330	8618927	8619524	8620120	8620717
728	8624296	8624892	8625488	8626084	8626679
729	8630253	8630848	8631443	8632039	8732634
730	8636202	8636797	8637391	8637985	8638580
731	8642143	8642737	8643331	8643924	8644517
732	8648076	8648669	8649262	8649855	8650447
733	8654001	8654593	8655185	8655777	8656369
734	8659918	8660509	8661100	8661691	8662282
735	8665827	8666417	8667008	8667598	8668188
736	8671728	8672317	8672907	8673496	8674086
737	8677620	8678209	8678798	8679387	8679975
738	8683505	8684093	8684681	8685269	8685857
739	8689382	8689969	8690556	8691143	8691730
740	8695251	8695837	8696423	8697010	8697596
741	8701112	8701697	8702283	8702868	8703454
742	8706965	8707549	8708134	8708719	8709304
743	8712810	8713394	8713978	8714562	8715146
744	8718647	8719230	8719814	8720397	8720980
745	8724476	8725059	8725641	8726224	8726806
746	8730298	8730880	8731461	8732043	9732625
747	8736112	8736693	8737274	8737855	8738435
748	8741918	8742498	8743078	8743658	8744238
749	8747716	8748296	8748875	8749454	8750034
750	8753507	8754086	8754664	8755243	8755821

Natural Numbers.	0	1	2	3	4
751	8756399	8756978	8757556	8758134	8758712
752	8762178	8762756	8763333	8763911	8764488
753	8767950	8768526	8769103	8769680	8770256
754	8773713	8774289	8774865	8775441	8776017
755	8779469	8780045	8780620	8781195	8781770
756	8785218	8785792	8786367	8786941	8787515
757	8790959	8791532	8792106	8792680	8793253
758	8796692	8797265	8797838	8798411	8798983
759	8802418	8802990	8803562	8804134	8804706
760	8808136	8808707	8809279	8809850	8810421
761	8813847	8814417	8814988	8815558	8816129
762	8819550	8820120	8820689	8821259	8821829
763	8825245	8825815	8826384	8826953	8827522
764	8830934	8831502	8832070	8832639	8833207
765	8836614	8837182	8837750	8838317	8838885
766	8842288	8842855	8843421	8843988	8844555
767	8847954	8848520	8849086	8849652	8850218
768	8853612	8854178	8854743	8855308	8855874
769	8859263	8859828	8860393	8860957	8861522
770	8864907	8865471	8866035	8866599	8867163
771	8870544	8871107	8871670	8872233	8872796
772	8876173	8876736	8877298	8877860	8878423
773	8881795	8882357	8882918	8883480	8884042
774	8887410	8887971	8888532	8889093	8889653
775	8893017	8893577	8894138	8894698	8895258
776	8898617	8899177	8899736	8900296	8900855
777	8904210	8904769	8905328	8905887	8906445
778	8909796	8910354	8910912	8911470	8912028
779	8915375	8915932	8916489	8917047	8917604
780	8920946	8921503	8922059	8922616	8923173
781	8926510	8927066	8937622	8928178	8928734
782	8932068	8932623	8933178	8933733	8934288
783	8937618	8938172	8938727	8939281	8939836
784	8943161	8943715	8944268	8944822	8945376
785	8948697	8949250	8949803	8950356	8950909
786	8954225	8954778	8955330	8955883	8956435
787	8959747	8960299	8960851	8961403	8961954
788	8965262	8965813	8966364	8966915	8967466
789	8978770	8971320	8971871	8972421	8972971
790	8976271	8976821	8977370	8977920	8978469
791	8981765	8982314	8982863	8983412	8983960
792	8987252	8987800	8988348	8988897	8989445
793	8992732	8993279	8993827	8994375	8994922
794	8998205	8998752	8999299	8999846	9000392

Natural Numbers.	5	6	7	8	9
751	8759290	8759868	8760445	8761023	8761601
752	8765065	8765642	8766219	8766796	8767373
753	8770833	8771409	8771985	8772561	8773137
754	8776592	8777168	8777743	8778319	8778894
755	8782345	8782919	8783494	8784069	8784643
756	8788089	8788663	8789237	8789811	8790385
757	8793826	8794400	8794973	8795546	8796119
758	8799556	8800128	8800701	8801273	8801846
759	8805278	8805850	8806421	8806993	8807564
760	8810992	8811563	8812134	8812705	8813276
761	8816699	8817269	8817840	8818410	8818980
762	8822398	8822968	8823537	8824107	8824676
763	8828090	8828659	8829228	8829797	8830365
764	8833775	8834343	8834911	8835479	8836047
765	8839452	8840019	8840586	8841154	8841721
766	8845122	8845688	8846255	8846821	8847387
767	8850784	8851350	8851915	8852481	8853047
768	8856439	8857004	8857569	8858134	8858699
769	8866086	8862651	8863215	8863779	8863343
770	8867726	8868290	8868854	8869417	8869980
771	8873359	8873922	8874485	8875048	8875610
772	8878985	8879547	8880109	8880671	8881233
773	8884603	8885165	8885726	8886287	8886848
774	8890214	8890775	8891336	8891896	8892457
775	8895818	8896378	8896935	8897498	8898058
776	8901415	8901974	8902533	8903092	8903651
777	8907004	8907562	8908121	8908679	8909238
778	8912586	8913144	8913702	8914259	8914817
779	8918161	8918718	8919275	8919832	8920389
780	8923729	8924285	8924842	8925398	8925954
781	8929290	8929846	8930401	8930957	8931512
782	8934843	8935398	8935953	8936508	8937063
783	8940390	8940944	8941498	8942053	8942607
784	8945929	8946483	8947037	8947590	8948143
785	8951462	8952015	8952567	8953120	8953673
786	8956987	8957539	8958092	8958644	8959195
787	8962506	8963057	8963608	8964160	8964711
788	8968017	8968568	8969118	8969669	8970219
789	8973521	8974071	8974621	8975171	8975721
790	8979019	8979568	8980117	8980667	8981216
791	8984509	8985058	8985606	8986155	8986703
792	8989993	8990541	8991089	8991636	8992184
793	8995469	8996017	8996564	8997111	8997658
794	9000939	9001486	9002032	9002579	9003125

Natural Numbers.	0	1	2	3	4
795	9003671	9004218	9004764	9005310	9005856
796	9009131	9009676	9010222	9010767	9011313
797	9014583	9015128	9015673	9016218	9016762
798	9020029	9020573	9021117	9021661	9022205
799	9025468	9026011	9026555	9027098	9027641
800	9030900	9031443	9031985	9032528	9033071
801	9036325	9036867	9037409	9037951	9038493
802	9041744	9042285	9042827	9043368	9043909
803	9047155	9047696	9048237	9048778	9049318
804	9052560	9053101	9053641	9054181	9054721
805	9057960	9058498	9059038	9059577	9060116
806	9063351	9063889	9064428	9064967	9065505
807	9068735	9069273	9069812	9070350	9070887
808	9074114	9074651	9075188	9075726	9076263
809	9079485	9080022	9080559	9081095	9081632
810	9084850	9085386	9085922	9086458	9086994
811	9090209	9090744	9091279	9091815	9092350
812	9095560	9096095	9096630	9097165	9097699
813	9100905	9101440	9101974	9102508	9103042
814	9106244	9106778	9107311	9107844	9108378
815	9111576	9112109	9112642	9113174	9113707
816	9116902	9117434	9117966	9118498	9119030
817	9122220	9122752	9123234	9123815	9124346
818	9127533	9128064	9128595	9129126	9129656
819	9132839	9133369	9133899	9134430	9134960
820	9138139	9138668	9139198	9139727	9140257
821	9143432	9143961	9144489	9145018	9145547
822	9148718	9149246	9149775	9150303	9150831
823	9153998	9154526	9155054	9155581	9156109
824	9159272	9159799	9160326	9160853	9161380
825	9164539	9165066	9165592	9166118	9166645
826	9169800	9170326	9170852	9171378	9171903
827	9175055	9175580	9176105	9176630	9177155
828	9180303	9180828	9181352	9181877	9182401
829	9185545	9186069	9185593	9187117	9187640
830	9190781	9191304	9191827	9192350	9192873
831	9196010	9196533	9197055	9197578	9198100
832	9201233	9201755	9202277	9202799	9203321
833	9206450	9206971	9207493	9208014	9208535
834	9211661	9212181	9212702	9213222	9213743
835	9216865	9217385	9217905	9218425	9218945
836	9222063	9222582	9223102	9223621	9224140
837	9227255	9227773	9228292	9228811	9229330
838	9232440	9232958	9233477	9233995	9234513

Natural Numbers.	5	6	7	8	9
795	9006402	9006948	9007494	9008039	9008585
796	9011858	9012403	9012948	9013493	9014038
797	9017307	9017851	9018396	9018940	9019485
798	9022749	9023293	9023837	9024381	9024924
799	9028185	9028728	9029271	9029814	9030357
800	9033613	9034156	9034698	9035241	9035783
801	9039035	9039577	9040119	9040661	9041202
802	9044450	9044992	9045533	9046073	9046615
803	9049859	9050399	9050940	9051480	9052020
804	9055261	9055800	9056340	9056880	9057419
805	9060655	9061195	9061734	9062274	9062812
806	9066044	9066582	9067121	9067659	9068197
807	9071425	9071963	9072501	9073038	9073576
808	9076800	9077337	9077874	9078411	9078948
809	9082169	9082705	9083241	9083778	9084314
810	9087530	9088066	9088602	9089137	9089673
811	9092885	9093420	9093955	9094490	9095025
812	9098234	9098768	9099303	9099837	9100371
813	9103576	9104109	9104643	9105177	9105710
814	9108911	9109444	9109977	9110510	9111043
815	9114240	9114772	9115305	9115837	9116369
816	9119562	9120094	9120626	9121157	9121689
817	9124878	9125409	9125940	9126471	9127002
818	9130187	9130717	9131248	9131778	9132309
819	9135490	9136019	9136549	9137079	9137609
820	9140786	9141315	9141844	9142373	9142903
821	9146076	9146604	9147133	9147661	9148190
822	9151359	9151887	9152415	9152943	9153471
823	9156636	9157163	9157691	9158218	9158745
824	9161907	9162433	9162960	9163487	9164013
825	9167171	9167697	9168223	9168749	9169275
826	9172429	9172954	9173479	9174005	9174530
827	9177680	9178205	9178730	9179254	9179779
828	9182925	9183449	9183973	9184497	9185021
829	9188164	9188687	9189211	9189734	9190258
830	9193396	9193919	9194442	9194965	9195488
831	9198623	9199145	9199667	9200189	9200711
832	9203842	9204364	9204886	9205407	9205929
833	9209056	9209577	9210098	9210619	9211140
834	9214263	9214784	9215304	9215824	9216345
835	9219465	9219984	9220504	9221024	9221543
836	9224659	9225179	9225698	9226217	9226736
837	9229848	9230367	9230885	9231404	9231922
838	9235031	9235549	9236066	9236584	9237102

Natural Numbers.	0	1	2	3	4
839	9237620	9238137	9238655	9239172	9239690
840	9242793	9243310	9243827	9244344	9244860
841	9247960	9248476	9248993	9249509	9250025
842	9253121	9253637	9254152	9254668	9255184
843	9258276	9258791	9259306	9259821	9260336
844	9263424	9263939	9264453	9264968	9265482
845	9268567	9269081	9269595	9270109	9270622
846	9273704	9274217	9274730	9275243	9275757
847	9278834	9279347	9279859	9280372	9280885
848	9283959	9284471	9284983	9285495	9286007
849	9289077	9289588	9290100	9290611	9291123
850	9294189	9294700	9295211	9295722	9296233
851	9299296	9299806	9300316	9300826	9301336
852	9304396	9304906	9305415	9305925	9306434
853	9309490	9309999	9310508	9311017	9311526
854	9314579	9315087	9315596	9316104	9316612
855	9319661	9320169	9320677	9321185	9321692
856	9324738	9325245	9325752	9326259	9326767
857	9329808	9330315	9330822	9331328	9331835
858	9334873	9335379	9335885	9336391	9336897
859	9339932	9340437	9340943	9341448	9341953
860	9344984	9345489	9345994	9346499	9347004
861	9350032	9350536	9351040	9351544	9352049
862	9355073	9355576	9356080	9356584	9357087
863	9360108	9360611	9361114	9361617	9362120
864	9365137	9365640	9366143	9366645	9367148
865	9370161	9370663	9371165	9371667	9372169
866	9375179	9375680	9376182	9376683	9377184
867	9380191	9380692	9381193	9381693	9382194
868	9385197	9385697	9386198	9386698	9387198
869	9390198	9390697	9391197	9391697	9392196
870	9395193	9395692	9396191	9396690	9397189
871	9400182	9400680	9401179	9401677	9402176
872	9405165	9405663	9406161	9406659	9407157
873	9410142	9410640	9411137	9411635	9412132
874	9415114	9415611	9416108	9416605	9417101
875	9420081	9420577	9421073	9421569	9422065
876	9425041	9425537	9426032	9426528	9427024
877	9429996	9430491	9430986	9431481	9431976
878	9434945	9435440	9435934	9436429	9436923
879	9439889	9440383	9440877	9441371	9441865
880	9444827	9445320	9445814	9446307	9446800
881	9449759	9450252	9450745	9451238	9451730
882	9454686	9455178	9455671	9456163	9456655

Natural Numbers.	5	6	7	8	9
839	9240208	9240724	9241246	9241759	9242276
840	9245377	9245894	9246410	9246927	9247444
841	9250541	9251057	9251573	9252689	9252605
842	9255699	9256215	9256730	9257245	9257761
843	9260851	9261366	9261880	9262395	9262910
844	9265995	9266511	9267025	9267539	9268053
845	9271136	9271650	9272163	9272677	9273190
846	9276270	9276783	9277296	9277808	9278321
847	9281397	9281909	9282422	9282934	9283446
848	9286518	9287030	9287542	9288054	9288565
849	9291634	9292145	9292656	9293167	9293678
850	9296743	9297254	9297764	9298275	9298785
851	9301847	9302357	9302866	9303376	9303886
852	9306944	9307453	9307963	9308472	9308981
853	9312035	9312544	9313053	9313561	9314070
854	9317121	9317629	9318137	9318645	9319153
855	9322200	9322708	9323215	9323723	9324230
856	9327274	9327781	9328288	9328795	9329301
857	9332341	9332848	9333354	9333860	9334367
858	9337403	9337909	9338415	9338920	9339426
859	9342459	9342964	9343469	9343974	9344479
860	9347509	9348013	9348518	9349022	9349527
861	9352553	9353057	9353561	9354065	9354569
862	9357591	9358095	9358598	9359101	9359605
863	9362623	9363126	9363629	9364132	9364635
864	9367650	9368152	9368655	9369157	9369659
865	9372671	9373172	9373674	9374176	9374677
866	9377686	9378187	9378688	9379189	9379690
867	9382695	9383195	9383696	9384196	9384697
868	9387698	9388198	9388698	9389198	9389698
869	9392696	9393195	9393695	9394194	9394693
870	9397688	9398187	9398685	9399184	9399683
871	9402674	9403172	9403670	9404169	9404667
872	9407654	9408152	9408650	9409147	9409645
873	9412629	9413126	9413623	9414120	9414617
874	9417598	9418095	9418591	9419088	9419584
875	9422561	9423058	9423554	9424049	9424545
876	9427519	9428015	9428510	9429005	9429501
877	9432471	9432966	9433461	9433956	9434450
878	9437418	9437912	9438406	9438900	9439395
879	9442358	9442852	9443346	9443840	9444333
880	9447294	9447787	9448280	9448773	9449266
881	9452223	9452716	9453208	9453701	9454193
882	9457147	9457639	9458131	9458623	9459115

Natural Numbers.	0	1	2	3	4
883	9459607	9460099	9460591	9461082	9461574
884	9464523	9465014	9465505	9465996	9466487
885	9469433	9469923	9470414	9470905	9471395
886	9474337	9474827	9475317	9475807	9476297
887	9479236	9479726	9480215	9480705	9481194
888	9484130	9484619	9485108	9485597	9486085
889	9489018	9489506	9489994	9490483	9490971
890	9493900	9494388	9494876	9495364	9495852
891	9498777	9499264	9499752	9500239	9500726
892	9503649	9504135	9504622	9505109	9505595
893	9508515	9509001	9509487	9509973	9510459
894	9513375	9513861	9514347	9514832	9515318
895	9518230	9518716	9519201	9519686	9520171
896	9523080	9523565	9524049	9524534	9525018
897	9527924	9528409	9528893	9529377	9529861
898	9532763	9533247	9533730	9534214	9534697
899	9537597	9538080	9538563	9539046	9539529
900	9542425	9542908	9543390	9543872	9544355
901	9547248	9547730	9548212	9548694	9549176
902	9552065	9552547	9553028	9553510	9553991
903	9556877	9557358	9557839	9558320	9558801
904	9561684	9562165	9562645	9563125	9563605
905	9566486	9566966	9567445	9567925	9568405
906	9571282	9571761	9572241	9572720	9573199
907	9576073	9576552	9577030	9577509	9577988
908	9580858	9581337	9581815	9582293	9582771
909	9585639	9586117	9586594	9587072	9587549
910	9590414	9590891	9591368	9591845	9592322
911	9595184	9595660	9596137	9596614	9597090
912	9599948	9600425	9600901	9601377	9601853
913	9604708	9605183	9605659	9606135	9606610
914	9609462	9609937	9610412	9610887	9611362
915	9614211	9614686	9615160	9615635	9616109
916	9618955	9619429	9619903	9620377	9620851
917	9623693	9624167	9624640	9625114	9625587
918	9628427	9628900	9629373	9629846	9630319
919	9633155	9633628	9634100	9634573	9635045
920	9637878	9638350	9638822	9639294	9639766
921	9642596	9643068	9643539	9644011	9644482
922	9647309	9647780	9648251	9648722	9647193
923	9652017	9652488	9652958	9653428	9653899
924	9656720	9657190	9657660	9658130	9658599
925	9661417	9661887	9662356	9662826	9663295
926	9666110	9667579	9667048	9667517	9667985

Natural Numbers.	5	6	7	8	9
883	9462066	9462557	9463048	9463540	9464031
884	9466978	9467469	9467960	9468451	9468942
885	9471886	9472376	9472866	9473357	9473847
886	9476787	9477277	9477767	9478257	9478747
887	9481684	9482173	9482662	9483151	9483641
888	9486574	9487063	9487552	9488040	9488529
889	9491460	9491948	9492436	9492924	9493412
890	9496330	9496827	9497314	9497802	9498290
891	9501213	9501701	9502188	9502675	9503162
892	9506082	9506569	9507055	9507542	9508028
893	9510946	9511432	9511918	9512404	9512889
894	9515803	9516289	9516774	9517260	9517745
895	9520656	9521141	9521626	9522111	9522595
896	9525503	9525987	9526472	2526956	9527440
897	9530345	9530828	9531312	9531796	9532280
898	9535181	9535664	9536147	9536631	9537114
899	9540012	9540494	9540977	9541460	9541943
900	9544837	9545319	9545802	9546284	9546766
901	9549657	9550139	9550621	9551102	9551584
902	9554472	9554953	9555434	9555915	9556397
903	9559282	9559762	9560243	9560723	9561204
904	9564086	9564566	9565046	9565526	9566006
905	9568885	9569364	9569844	9570323	9570803
906	9573678	9574157	9574636	9575115	9575594
907	9578466	9578945	9579423	9579902	9580380
908	9583249	9583727	9584205	9584683	9585161
909	9588027	9588505	9588982	9589459	9589937
910	9592799	9593276	9593754	9594230	9594707
911	9597567	9598043	9598520	9598996	9599472
912	9602329	9602805	9603280	9603756	9604232
913	9607086	9607561	9608036	9608511	9608987
914	9611837	9612312	9612787	9613261	9613736
915	9516583	9617058	9617532	9618006	9618481
916	9621325	9621799	9622272	9622746	9623220
917	9626061	9626534	9627007	9627481	9627954
918	9630792	9631264	9631737	9632210	9632683
919	9635517	9635990	9636462	9636934	9637406
920	9640238	9640710	9641181	9641653	9642125
921	9644953	9645425	9645896	9646367	9646838
922	9649664	9650134	9650605	9651076	9651546
923	9654369	9654839	9655309	9655780	9656250
924	9659069	9659539	9660009	9660478	9660948
925	9663764	9664233	9664703	9665172	9665641
926	9668454	9668923	9669392	9669860	9670329

Natural Numbers.	0	1	2	3	4
927	9670797	9671266	9671734	9672203	9672671
928	9675480	9675948	9676416	9676883	9677351
929	9680157	9680625	9681092	9681559	9682027
930	9684829	9585296	9685763	9686230	9686697
931	9689497	9689963	9690430	9690896	9691362
932	9694159	9694625	9695091	9695557	9696023
933	9698816	9699282	9699747	9700213	9700678
934	9703469	9703934	9704399	9704863	9705328
935	9708116	9708581	9709045	9709509	9709974
936	9712758	9713222	9713686	9714150	9714614
937	9717396	9717859	9718323	9718786	9719249
938	9722028	9722491	9722954	9723417	9723880
939	9726656	9727118	9727581	9728043	9728506
940	9731278	9731741	9732202	9732664	9733126
941	9735896	9736358	9736819	9737281	9737742
942	9740509	9740970	9741431	9741892	9742353
943	9745117	9745577	9746038	9746498	9746959
944	9749720	9750180	9750640	9751100	9751560
945	9754318	9754778	9755237	9755695	9756156
946	9758911	9759370	9759829	9760288	9760747
947	9763500	9763958	9764417	9764875	9765334
948	9768083	9768541	9768999	9769457	9769915
949	9772662	9773120	9773577	9774035	9774492
950	9777236	9777693	9778150	9778607	9779064
951	9781805	9782262	9782718	9783175	9783631
952	9786369	9786826	9787282	9787738	9788194
953	9790929	9791385	9791840	9792296	9792751
954	9795484	9795939	9796394	9796849	9797304
955	9800034	9800488	9800943	9801398	9801852
956	9804579	9805033	9805487	9805942	9806396
957	9809119	9809573	9810027	9810481	9810934
958	9813655	9814108	9814562	9815015	9815468
959	9818186	9818639	9819092	9819544	9819997
960	9822712	9823165	9823617	9824069	9824522
961	9827234	9827686	9828138	9828589	9829041
962	9831751	9832202	9832654	9833105	9833556
963	9836263	9836714	9837165	9837616	9838066
964	9840770	9841221	9841671	9842122	9842572
965	9845273	9845723	9846173	9846623	9847073
966	9849771	9850221	9850670	9851120	9851569
967	9854265	9854714	9855163	9855612	9856061
968	9858754	9859202	9859651	9860099	9860548
969	9863238	9863686	9864134	9864582	9865030
970	9867717	9868165	9868613	9869060	9869508

Natural Numbers.	5	6	7	8	9
927	9673139	9673607	9674076	9674544	9675012
928	9677819	9678287	9678754	9679222	9679690
929	9682494	9682961	9683428	9683895	9684362
930	9687164	9687630	9688097	9688564	9689030
931	9691829	9692295	9692761	9693227	9693693
932	9696488	9696954	9697420	9697885	9698351
933	9701143	9701608	9702074	9702539	9703004
934	9705793	9706258	9706722	9707187	9707652
935	9710438	9710902	9711366	9711830	9712294
936	9715078	9715542	9716005	9716469	9716932
937	9719713	9720176	9720639	9721102	9721565
938	9724343	9724805	9725268	9725731	9726193
939	9728968	9729430	9729892	9730354	9730816
940	9733588	9734050	9734511	9734973	9735435
941	9738203	9738664	9739126	9739587	9740048
942	9742814	9743274	9743735	9744196	9744656
943	9747419	9747879	9748340	9748800	9749260
944	9752020	9752479	9752939	9753399	9753858
945	9756615	9757075	9757534	9757993	9758452
946	9761206	9761665	9762124	9762582	9763041
947	9765792	9766251	9766709	9767167	9767625
948	9770373	9770831	9771289	9771747	9772204
949	9774950	9775407	9775864	9776322	9776779
950	9779521	9779978	9780435	9780892	9781348
951	9784088	9784544	9785001	9785457	9785913
952	9788650	9789106	9789562	9790017	9790473
953	9793207	9793662	9794118	9794573	9795028
954	9797759	9798214	9798669	9799124	9799579
955	9802307	9802761	9803216	9803670	9804125
956	9806850	9807304	9807758	9808212	9808666
957	9811388	9811841	9812295	9812748	9813202
958	9815921	9816374	9816827	9817280	9817733
959	9820450	9820902	9821355	9821807	9822260
960	9824974	9825426	9825878	9826330	9826782
961	9829493	9839945	9830396	9830848	9831299
962	9834007	9834459	9834910	9835361	9835812
963	9838517	9838968	9839419	9839869	9840320
964	9843022	9843473	9843923	9844373	9844823
965	9847523	9847973	9848422	9848872	9849322
966	9852019	9852468	9852917	9853366	9853816
967	9856510	9856959	9857407	9857856	9858305
968	9860996	9861445	9861893	9862341	9862790
969	9865478	9865926	9866374	9866822	9867270
970	9869955	9870403	9870850	9871298	9871745

Natural Numbers.	0	1	2	3	4
971	9872192	9872640	9873087	9873534	9873981
972	9876663	9877109	9877556	9878003	9878449
973	9881128	9881575	9882021	9882467	9882913
974	9885590	9886035	9886481	9886927	9887373
975	9890046	9890492	9890937	9891382	9891828
976	9894498	9894943	9895388	9895833	9896278
977	9898946	9899390	9899835	9900279	9900723
978	9903389	9903833	9904277	9904721	9905164
979	9907827	9908270	9908714	9909158	9909601
980	9912261	9912704	9913147	9913590	9914033
981	9916690	9917133	9917575	9918018	9918461
982	9921115	9921557	9921999	9922441	9922884
983	9925535	9925977	9926419	9926860	9927302
984	9929951	9930392	9930834	9931275	9931716
985	9934362	9934803	9935244	9935685	9936126
986	9938769	9939210	9939650	9940090	9940531
987	9943172	9943612	9944051	9944491	9944931
988	9947569	9948009	9948448	9948888	9949327
989	9951963	9952402	9952841	9953280	9953719
990	9956352	9956791	9957229	9957668	9958106
991	9960737	9961175	9961613	9962051	9962489
992	9965117	9965554	9965992	9966430	9966868
993	9969492	9969930	9970367	9970804	9971242
994	9973864	9974301	9974738	9975174	9975611
995	9978231	9978667	9979104	9979540	9979976
996	9982593	9983029	9983465	9983901	9984337
997	9986952	9987387	9987823	9988258	9988694
998	9991305	9991740	9992176	9992611	9993046
999	9995655	9996090	9996524	9996959	9997393

Natural Numbers.	5	6	7	8	9
971	9874428	9874875	9875322	9875769	9876216
972	9878896	9879343	9879789	9880236	9880682
973	9883360	9883806	9884252	9884698	9885144
974	9887818	9888264	9888710	9889155	9889601
975	9892273	9892718	9893163	9893608	9894050
976	9896722	9897167	9897612	9898056	9898501
977	9901168	9901612	9902056	9902500	9902944
978	9905608	9906052	9906496	9906940	9907383
979	9910044	9910488	9910931	9911374	9911818
980	9914476	9914919	9915362	9915805	9916247
981	9918903	9919345	9919788	9920230	9920673
982	9923326	9923768	9924210	9924651	9925093
983	9927744	9928185	9928627	9929068	9929510
984	9932157	9932598	9933039	9933480	9933921
985	9936566	9937007	9937448	9937888	9938329
986	9940971	9941411	9941851	9942291	9942731
987	9945371	9945811	9946251	9946690	9947130
988	9949767	9959206	9950645	9951085	9951524
989	9954158	9954597	9955036	9955474	9955913
990	9958545	9958983	9959422	9959860	9960298
991	9962927	9963365	9963803	9964241	9964679
992	9967305	9967743	9968180	9968618	9969055
993	9971679	9972116	9972553	9972990	9973427
994	9976048	9976485	9976921	9977358	9977794
995	9980413	9980849	9981285	9981721	9982157
996	9984773	9985209	9985645	9986080	9986516
997	9989129	9989564	9990000	9990435	9990870
998	9993481	9993916	9994350	9994785	9995220
999	9997828	9998262	9998697	9999131	9999566

10000 it's Log.=4.0000000

The End of the Table of Logarithms.

A

TABLE

OF THE

Proportional Parts

OF THE

DIFFERENCES

OF

LOGARITHMS.

By which the foregoing TABLE is made to extend to the Logarithm of 100000, &c.

Differences of Logarithms.	*Tenth Parts of those Differences.* (VIZ.)								
	1	2	3	4	5	6	7	8	9
435	44	87	130	174	218	261	305	348	392
436	44	87	131	174	218	262	305	349	392
438	44	88	131	175	219	263	307	350	394
440	44	88	132	176	220	264	308	352	396
442	44	88	133	177	221	265	309	354	398
444	44	89	133	178	222	266	311	355	400
446	45	89	134	178	223	268	312	357	401
448	45	90	134	179	224	269	314	358	403
450	45	90	135	180	225	270	315	360	405
452	45	90	136	181	226	271	316	362	407
454	45	91	136	182	227	272	318	363	409
456	46	91	137	182	228	274	319	365	410
458	46	92	137	183	229	275	321	366	412

Diff. of Logar.	Tenth Parts of those Differences. 1	2	3	4	5	6	7	8	9
460	46	92	138	184	230	276	322	368	414
462	46	92	139	185	231	277	323	370	416
464	46	93	139	186	232	278	325	371	418
466	47	93	140	186	233	280	326	373	419
468	47	94	140	187	234	281	328	374	421
470	47	94	141	188	235	282	329	376	423
472	47	94	142	189	236	283	330	378	425
474	47	95	142	190	237	284	332	379	427
476	48	95	143	190	238	286	333	381	428
478	48	96	143	191	239	287	335	382	430
480	48	96	144	192	240	288	336	384	432
482	48	96	145	193	241	289	337	386	434
484	48	97	145	194	242	290	339	387	436
486	49	97	146	194	243	292	340	389	437
488	49	98	146	195	244	293	342	390	439
490	49	98	147	196	245	294	343	392	441
492	49	98	148	197	246	295	344	394	443
494	49	99	148	198	247	296	346	395	445
496	50	99	149	198	248	298	347	397	446
498	50	100	149	199	249	299	348	398	448
500	50	100	150	200	250	300	350	400	450
502	50	100	151	201	251	301	351	402	452
504	50	101	151	202	252	302	353	403	454
506	51	101	152	202	253	304	354	405	455
508	51	102	152	203	254	305	356	406	457
510	51	102	153	204	255	306	357	408	459
512	51	102	154	205	256	307	358	410	461
514	51	103	154	206	257	308	360	411	463
516	52	103	155	206	258	310	361	413	464
518	52	104	155	207	259	311	363	414	466
520	52	104	156	208	260	312	364	416	468
522	52	104	157	209	261	313	365	418	470
524	52	105	157	210	262	314	367	419	472
526	53	105	158	210	263	316	368	421	473
528	53	106	158	211	264	317	370	422	475
530	53	106	159	212	265	318	371	424	477
532	53	106	160	213	266	319	372	426	479
534	53	107	160	214	267	320	374	427	481
536	54	107	161	214	268	322	375	429	482
538	54	108	161	215	269	323	377	430	484

A TABLE of Proportional Parts.

Diff. of Logar.	Tenth Parts of those Differences. 1	2	3	4	5	6	7	8	9
540	54	108	162	216	270	324	378	432	486
542	54	108	163	217	271	325	379	434	488
544	54	109	163	218	272	326	381	435	490
546	55	109	164	218	273	328	382	437	491
548	55	110	164	219	274	329	384	438	493
550	55	110	165	220	275	330	385	440	495
552	55	110	166	221	276	331	386	442	497
554	55	111	166	222	277	332	388	443	499
556	56	111	167	222	278	334	389	445	500
558	56	112	167	223	279	335	391	446	502
560	56	112	168	224	280	336	392	448	504
562	56	112	169	225	281	337	393	450	506
564	56	113	169	226	282	338	395	451	508
566	57	113	170	226	283	340	396	453	509
568	57	114	170	227	284	341	398	454	511
570	57	114	171	228	285	342	399	456	513
572	57	114	172	229	286	343	400	458	515
574	57	115	172	230	287	344	401	459	516
576	58	115	173	230	288	346	403	461	518
578	58	116	173	231	289	347	405	462	520
580	58	116	174	232	290	348	406	464	522
582	58	116	175	233	291	349	407	466	524
584	58	117	175	234	292	350	409	467	526
586	59	117	176	234	293	352	410	469	527
588	59	118	176	235	294	353	412	470	529
590	59	118	177	236	295	354	413	472	531
592	59	118	178	237	295	355	414	474	533
594	59	119	178	238	297	356	416	475	535
596	60	119	179	238	298	358	417	477	536
598	60	120	179	239	299	359	419	478	538
600	60	120	180	240	300	360	420	480	540
602	60	120	181	241	301	361	421	482	542
604	60	121	181	242	302	362	423	483	544
606	61	121	182	242	303	364	424	485	545
608	61	122	182	243	304	365	426	486	547
610	61	122	183	244	305	366	427	488	549
612	61	122	184	245	306	367	428	490	551
614	61	123	184	246	307	368	430	492	553
616	62	123	185	246	308	370	431	493	554
618	62	124	185	247	309	371	433	494	556

Diff. of Logar.	Tenth Parts of those Differences.								
	1	2	3	4	5	6	7	8	9
620	62	124	186	248	310	372	434	496	558
622	62	124	187	249	311	373	435	498	560
624	62	125	187	250	312	374	437	499	562
626	63	125	188	250	313	376	438	501	563
628	63	126	188	251	314	377	440	502	565
630	63	126	189	252	315	378	441	504	567
632	63	126	190	253	316	379	442	506	569
634	63	127	190	254	317	380	444	507	571
636	64	127	191	254	318	382	445	509	572
638	64	128	191	255	319	383	447	510	574
640	64	128	192	256	320	384	448	512	576
642	64	128	193	257	321	385	449	514	578
644	64	129	193	258	322	386	451	515	580
646	65	129	194	258	323	388	452	517	581
648	65	130	194	259	324	389	454	518	583
650	65	130	195	260	325	390	455	520	585
652	65	130	196	261	326	391	456	522	587
654	65	131	196	262	327	392	458	523	589
656	66	131	197	262	328	394	459	525	590
658	66	132	197	263	329	395	461	526	592
660	66	132	198	264	330	396	462	528	594
662	66	132	199	265	331	397	463	530	596
664	66	133	199	266	332	398	465	531	598
666	67	133	200	266	333	400	466	533	599
668	67	134	200	267	334	401	468	534	601
670	67	134	201	268	335	402	469	536	603
672	67	134	202	269	336	403	470	538	605
674	67	135	202	270	337	404	472	539	607
676	68	135	203	270	338	406	473	541	608
678	68	136	203	271	339	407	475	542	610
680	68	136	204	272	340	408	476	544	612
682	68	136	205	273	341	409	477	546	614
684	68	137	205	274	342	410	479	547	616
686	69	137	206	274	343	412	480	549	617
688	69	138	206	275	344	413	482	550	619
690	69	138	207	276	345	414	483	552	621
692	69	138	208	277	346	415	484	554	623
694	69	139	208	278	347	416	486	555	625
696	70	139	209	278	348	417	487	557	626
698	70	140	209	279	349	419	489	558	628

A TABLE of Proportional Parts.

	Tenth Parts of those Differences.								
Diff. of Logar.	1	2	3	4	5	6	7	8	9
700	70	140	210	280	350	420	490	560	630
702	70	140	211	281	351	421	491	562	632
704	70	141	211	282	352	422	492	563	634
706	71	141	212	282	353	424	494	565	635
708	71	142	212	283	354	425	496	566	637
710	71	142	213	284	355	426	497	568	639
712	71	142	214	285	356	427	498	570	641
714	71	143	214	286	357	428	500	571	643
716	72	143	215	286	358	430	501	573	644
718	72	144	215	287	359	431	503	574	646
720	72	144	216	288	360	432	504	576	648
722	72	144	217	289	361	433	505	578	650
724	72	145	217	290	362	434	507	579	652
726	73	145	218	290	363	436	508	581	653
728	73	146	218	291	364	437	510	582	655
730	73	146	219	292	365	438	511	584	657
732	73	146	220	293	366	439	512	586	659
734	73	147	220	294	367	440	514	587	661
736	74	147	221	294	368	442	515	589	662
738	74	148	221	295	369	443	517	590	664
740	74	148	222	296	370	444	518	592	666
742	74	148	223	297	371	445	519	594	668
744	74	149	223	298	372	446	521	595	670
746	75	149	224	298	373	448	522	597	671
748	75	150	224	299	374	449	524	598	673
750	75	150	225	300	375	450	525	600	675
752	75	150	226	301	376	451	526	602	677
754	75	151	226	302	377	452	528	603	679
756	76	151	227	302	378	454	529	605	680
758	76	152	227	303	379	455	531	606	682
760	76	152	228	304	380	456	532	608	684
762	76	152	229	305	381	457	533	610	686
764	76	153	229	306	382	458	535	611	688
766	77	153	230	306	383	460	536	613	689
768	77	154	230	307	384	461	538	614	691
770	77	154	231	308	385	462	539	616	693
772	77	154	232	309	386	463	540	618	695
774	77	155	232	310	387	464	542	619	697
776	78	155	233	310	388	466	543	621	699
778	78	156	233	311	389	467	545	622	700

A TABLE of Proportional Parts.

Diff. of Logar.	Tenth Parts of those Differences. 1	2	3	4	5	6	7	8	9
780	78	156	234	312	390	468	546	624	702
782	78	156	235	313	391	469	547	626	704
784	78	157	235	314	392	470	549	627	706
786	79	157	236	314	393	472	550	629	707
788	79	158	236	315	394	473	552	630	709
790	79	158	237	316	395	474	553	632	711
792	79	158	238	317	396	475	554	634	713
794	79	159	238	318	397	477	556	635	715
796	80	159	239	318	398	478	557	637	717
798	80	160	239	319	399	479	559	638	718
800	80	160	240	320	400	480	560	640	720
802	80	160	241	321	401	481	561	642	722
804	80	161	241	322	402	482	563	643	724
806	81	161	242	322	403	484	564	645	725
808	81	162	242	323	404	485	566	646	727
810	81	162	243	324	405	486	567	648	729
812	81	162	244	325	406	487	568	650	731
814	81	163	244	326	407	488	570	651	733
816	82	163	245	326	408	490	571	653	734
818	82	164	245	327	409	491	572	654	736
820	82	164	246	328	410	492	574	656	738
822	82	164	247	329	411	493	575	658	740
824	82	165	247	330	412	494	577	659	742
826	83	165	248	330	413	496	578	661	743
828	83	166	248	331	414	497	580	662	745
830	83	166	249	332	415	498	581	664	747
832	83	166	250	333	416	499	582	666	749
834	83	167	250	334	417	500	584	667	751
836	84	167	251	334	418	502	585	669	753
838	84	168	251	335	419	503	587	670	754
840	84	168	252	336	420	504	588	672	756
842	84	168	253	337	421	505	589	674	758
844	84	169	253	338	422	506	591	675	760
846	85	169	254	338	423	508	592	677	761
848	85	170	254	339	424	509	594	678	763
850	85	170	255	340	425	510	595	680	765
852	85	170	256	341	426	511	596	682	767
854	85	171	256	342	427	512	598	683	769
856	86	171	257	342	428	514	599	685	771
858	86	172	257	343	429	515	601	686	773

A TABLE of Proportional Parts.

Diff. of Logar.	Tenth Parts of those Differences.								
	1	2	3	4	5	6	7	8	9
860	86	172	258	344	430	516	602	688	774
862	86	172	259	345	431	517	603	690	776
864	86	173	259	346	432	518	605	691	778
866	87	173	260	346	433	520	606	693	779
868	87	174	260	347	434	521	608	694	781
870	87	174	261	348	435	522	609	696	783
872	87	174	262	349	436	523	611	698	785
874	87	175	262	350	437	524	612	699	787
876	88	175	263	350	438	526	613	701	789
878	88	176	263	351	439	527	615	702	791
880	88	176	264	352	440	528	616	704	793
882	88	176	265	353	441	529	617	706	795
884	88	177	265	354	442	530	619	707	796
886	89	177	266	354	443	532	620	709	797
888	89	178	266	355	444	533	622	710	799
890	89	178	267	356	445	534	623	712	801
892	89	178	268	357	446	535	624	714	803
894	89	179	268	358	447	536	626	715	805
896	90	179	269	358	448	538	627	717	806
898	90	180	269	359	449	539	629	718	808
900	90	180	270	260	450	540	630	720	810
903	90	181	271	361	451	542	632	722	813
906	91	181	272	363	453	544	634	725	815
909	91	182	273	364	454	545	636	727	818
912	91	182	274	365	456	547	638	730	821
915	91	183	274	366	457	549	640	732	823
918	92	184	275	367	459	551	643	734	826
921	92	184	276	368	460	553	645	737	829
924	92	185	277	370	462	554	647	739	832
927	93	185	278	371	463	556	649	742	834
930	93	186	279	372	465	558	651	744	837
933	93	187	280	373	466	560	653	746	839
936	94	187	281	374	468	562	655	749	842
939	94	188	282	376	469	563	657	751	845
942	94	188	283	377	471	565	659	754	848
945	94	189	283	378	472	567	661	756	850
948	95	190	284	379	474	569	664	758	853
951	95	190	285	380	475	571	666	761	856
954	95	191	286	382	477	572	668	763	859
957	96	191	287	383	478	574	670	766	861

A TABLE of Proportional Parts.

Diff. of Logar.	Tenth Parts of those Differences. 1	2	3	4	5	6	7	8	9
960	96	192	288	384	480	576	672	768	864
963	96	193	289	385	481	578	674	770	867
966	97	193	290	386	483	580	676	773	869
969	97	194	291	388	484	581	678	775	872
972	97	194	292	389	486	583	680	778	875
975	98	195	292	390	487	585	682	780	878
978	98	196	293	391	489	587	685	782	880
981	98	196	294	392	490	589	687	785	883
984	98	197	295	394	492	590	689	787	886
987	99	197	296	395	493	592	691	790	888
990	99	198	297	396	495	594	693	792	891
993	99	199	298	397	496	596	695	794	894
996	100	199	299	398	498	598	697	797	897
999	100	200	300	400	499	599	699	799	899
1005	101	201	301	402	502	603	703	804	904
1009	101	202	303	404	504	605	706	807	908
1013	101	203	304	405	506	608	709	810	912
1017	102	203	305	407	508	610	712	814	915
1021	102	204	306	408	510	613	715	817	919
1025	103	205	307	410	512	615	717	820	923
1029	103	206	309	412	514	617	720	823	926
1033	103	207	310	413	516	620	723	826	930
1037	104	207	311	415	518	622	726	830	933
1041	104	208	312	416	520	625	729	833	937
1045	105	209	313	418	522	627	731	836	941
1049	105	210	315	420	524	629	734	839	944
1053	105	211	316	421	526	632	737	842	948
1057	106	211	317	423	528	634	740	846	951
1061	106	212	318	424	530	637	743	849	955
1065	107	213	319	426	532	639	745	852	959
1069	107	214	321	428	534	641	748	855	962
1073	107	215	322	429	536	644	751	858	966
1077	108	215	323	431	538	646	754	862	969
1081	108	216	324	432	540	649	757	865	973
1085	109	217	325	434	542	651	759	868	977
1089	109	218	327	436	544	653	762	871	980
1093	110	219	328	437	546	656	765	874	984
1097	110	219	329	439	548	658	768	878	987
1101	110	220	330	440	550	661	771	881	991
1105	111	221	331	442	552	663	773	884	994

A TABLE of Proportional Parts.

Diff. of Logar.	Tenth Parts of those Differences. 1	2	3	4	5	6	7	8	9
1109	111	222	333	444	554	665	776	887	998
1113	111	223	334	445	556	668	779	890	1002
1117	112	223	335	447	558	670	782	894	1005
1121	112	224	336	448	560	673	785	897	1009
1125	113	225	337	450	562	675	787	900	1013
1129	113	226	339	452	564	677	790	903	1016
1133	113	227	340	453	566	680	793	906	1020
1137	114	227	341	455	568	682	796	910	1023
1141	114	228	342	456	570	685	799	913	1027
1145	115	229	343	458	572	687	801	916	1031
1149	115	230	345	460	574	689	804	919	1034
1153	115	231	346	461	576	692	807	922	1038
1157	116	231	347	463	578	694	810	926	1041
1161	116	232	348	464	580	697	813	929	1045
1165	117	233	349	466	582	699	815	932	1049
1169	117	234	351	468	584	701	818	935	1052
1173	117	235	352	469	586	704	821	938	1056
1177	118	235	353	471	588	706	824	942	1059
1181	118	236	354	472	590	709	827	945	1063
1185	118	237	355	474	592	711	829	948	1067
1189	119	238	357	476	594	713	832	951	1070
1193	119	239	358	477	596	716	835	954	1074
1197	120	239	359	479	598	718	838	958	1077
1201	120	240	360	480	600	721	841	961	1081
1205	120	241	361	482	602	723	843	964	1084
1209	121	242	363	484	604	725	846	967	1088
1213	121	243	364	485	606	728	849	970	1092
1217	122	243	365	487	608	730	852	974	1095
1221	122	244	366	488	610	732	855	977	1099
1225	122	245	367	490	612	735	857	980	1102
1229	123	246	369	492	614	737	860	983	1106
1233	123	247	370	493	616	740	863	986	1110
1237	124	247	371	495	618	742	866	990	1113
1241	124	248	372	496	620	745	869	993	1117
1245	124	249	373	498	622	747	871	996	1120
1249	125	250	375	500	624	749	874	999	1124
1253	125	251	376	501	626	752	877	1002	1128
1257	126	251	377	503	628	754	880	1006	1131
1260	126	252	378	504	630	756	882	1008	1134
1263	126	253	379	585	631	758	884	1010	1137

Diff. of Logar.	Tenth Parts of those Differences. 1	2	3	4	5	6	7	8	9
1267	127	253	380	507	633	760	887	1014	1140
1271	127	254	381	508	635	763	890	1017	1144
1275	127	255	382	510	637	765	892	1020	1147
1279	128	256	384	512	639	767	895	1023	1151
1283	128	257	385	513	641	770	898	1026	1155
1287	129	257	386	515	643	772	901	1030	1158
1291	129	258	387	516	645	775	904	1033	1162
1295	129	259	388	518	647	777	906	1036	1165
1299	130	260	389	520	649	779	909	1039	1169
1303	130	261	391	521	651	782	912	1042	1173
1307	131	261	392	523	653	784	915	1046	1176
1311	131	262	393	524	655	787	918	1049	1180
1315	131	263	394	526	657	789	920	1052	1183
1319	132	264	396	528	659	791	923	1055	1187
1323	132	265	397	529	661	794	926	1058	1191
1327	133	265	398	531	663	796	929	1062	1194
1331	133	266	399	532	665	799	932	1065	1198
1335	133	267	400	534	667	801	934	1068	1201
1339	134	268	402	536	669	803	937	1071	1205
1343	134	269	403	537	671	866	940	1074	1209
1347	135	269	404	539	673	808	943	1078	1212
1351	135	270	405	540	675	811	946	1081	1216
1355	135	271	406	542	677	813	948	1084	1219
1359	136	272	408	544	679	815	951	1087	1223
1363	136	273	409	545	681	818	954	1090	1227
1367	137	273	410	547	683	820	957	1094	1230
1371	137	274	411	548	685	823	960	1097	1234
1375	137	275	412	550	687	825	962	1100	1237
1379	138	276	414	552	689	827	965	1103	1241
1383	138	277	415	553	691	830	968	1106	1245
1387	139	277	416	555	693	832	971	1110	1248
1391	139	278	417	556	695	835	974	1113	1252
1395	140	279	418	558	697	837	976	1116	1255
1399	140	280	420	559	699	839	979	1119	1259
1403	140	281	421	561	701	842	982	1122	1263
1407	141	281	422	563	703	844	985	1126	1266
1411	141	282	423	564	705	847	988	1129	1270
1415	141	283	424	566	707	849	990	1132	1273
1420	142	284	426	568	710	852	994	1136	1278
1425	142	285	427	570	712	855	997	1140	1282

A TABLE of Proportional Parts.

Diff. o Logar.	Tenth Parts of those Differences. 1	2	3	4	5	6	7	8	9
1430	143	286	429	572	715	858	1001	1144	1287
1435	143	287	430	574	717	861	1004	1148	1291
1440	144	288	432	576	720	864	1008	1152	1296
1445	144	289	433	578	722	867	1011	1156	1300
1450	145	290	435	580	725	870	1015	1160	1305
1455	145	291	436	582	727	873	1018	1164	1309
1460	146	292	438	584	730	876	1022	1168	1314
1465	146	293	439	586	732	879	1025	1172	1318
1470	147	294	441	588	735	882	1029	1176	1323
1475	147	295	442	590	737	885	1032	1180	1327
1480	148	296	444	592	740	888	1036	1184	1332
1485	148	297	445	594	742	891	1039	1188	1336
1490	149	298	447	596	745	894	1043	1192	1341
1495	149	299	448	598	747	897	1046	1196	1345
1500	150	300	450	600	750	900	1050	1200	1350
1505	150	301	451	602	752	903	1053	1204	1354
1510	151	302	453	604	755	906	1057	1208	1359
1515	151	303	454	606	757	909	1060	1212	1363
1520	152	304	456	608	760	912	1064	1216	1368
1525	152	305	457	610	762	915	1067	1220	1372
1530	153	306	459	612	765	918	1071	1224	1377
1535	153	307	460	614	767	921	1074	1228	1381
1540	154	308	462	616	770	924	1078	1232	1386
1545	154	309	463	618	772	927	1081	1236	1390
1550	155	310	465	620	775	930	1085	1240	1395
1555	155	311	460	622	777	933	1088	1244	1399
1560	156	312	468	624	780	936	1092	1248	1404
1565	156	313	469	626	782	939	1095	1252	1408
1570	157	314	471	628	785	942	1099	1256	1413
1575	157	315	472	630	787	945	1102	1260	1417
1580	158	316	474	632	790	948	1106	1264	1422
1585	158	317	475	634	792	951	1109	1268	1426
1590	159	318	477	636	795	954	1113	1272	1431
1595	159	319	478	638	797	957	1116	1276	1435
1600	160	320	480	640	800	960	1120	1280	1440
1605	160	321	481	642	802	963	1123	1284	1444
1610	161	322	483	644	805	966	1127	1288	1449
1615	161	323	484	646	807	969	1130	1292	1453
1620	162	324	486	648	810	972	1134	1296	1458
1625	162	325	487	650	812	975	1137	1300	1462

A TABLE of Proportional Parts.

	Tenth Parts of those Differences.								
Diff. of Logar.	1	2	3	4	5	6	7	8	9
1630	163	326	489	652	815	978	1141	1304	1467
1635	163	327	490	654	817	981	1144	1308	1471
1640	164	328	492	656	820	984	1148	1312	1476
1645	164	329	493	658	822	987	1151	1316	1480
1650	165	330	495	660	825	990	1155	1320	1485
1655	165	331	496	662	827	993	1158	1324	1489
1660	166	332	498	664	830	996	1162	1328	1494
1665	167	333	499	666	832	999	1166	1332	1498
1670	167	334	501	668	835	1002	1169	1336	1503
1675	167	335	502	670	837	1005	1172	1340	1507
1680	168	336	504	672	840	1008	1176	1344	1512
1685	168	337	505	674	842	1011	1179	1348	1516
1690	169	338	507	676	845	1014	1183	1352	1521
1695	169	339	508	678	847	1017	1186	1356	1525
1700	170	340	510	680	850	1020	1190	1360	1530
1705	170	341	511	682	852	1023	1193	1364	1534
1710	171	342	513	684	855	1026	1197	1368	1539
1715	171	343	514	686	857	1029	1200	1372	1543
1720	172	344	516	688	860	1032	1204	1376	1548
1725	172	345	517	690	862	1035	1207	1380	1552
1730	173	346	519	692	865	1038	1211	1384	1557
1735	173	347	520	694	867	1041	1214	1388	1561
1740	174	348	522	696	870	1044	1218	1392	1566
1745	174	349	523	698	872	1047	1221	1396	1570
1750	175	350	525	700	875	1050	1225	1400	1575
1755	175	351	526	702	877	1053	1228	1404	1579
1760	176	352	528	704	880	1056	1232	1408	1584
1765	176	353	529	706	882	1059	1235	1412	1588
1770	177	354	531	708	885	1062	1239	1416	1593
1775	178	355	532	710	887	1065	1242	1420	1597
1780	178	356	534	712	890	1068	1246	1424	1602
1785	179	357	535	714	892	1071	1249	1428	1606
1790	179	358	537	716	895	1074	1253	1432	1611
1795	180	359	538	718	897	1077	1256	1436	1615
1800	180	360	540	720	900	1080	1260	1440	1620
1805	181	361	541	722	902	1083	1263	1444	1624
1810	181	362	543	724	905	1086	1267	1448	1629
1815	182	363	544	726	907	1089	1270	1452	1633
1820	182	364	546	728	910	1092	1274	1456	1638
1825	183	365	547	730	912	1095	1277	1460	1642

A TABLE of Proportional Parts.

Diff. of Logar.	Tenth Parts of those Differences. 1	2	3	4	5	6	7	8	9
1830	183	366	549	732	915	1098	1281	1464	1647
1835	184	367	550	734	917	1101	1284	1468	1651
1840	184	368	552	736	920	1104	1288	1472	1656
1845	185	369	553	738	922	1107	1291	1476	1660
1850	185	370	555	740	925	1110	1295	1480	1665
1855	186	371	556	742	927	1113	1298	1484	1669
1860	186	372	558	744	930	1116	1302	1488	1674
1865	187	373	559	746	932	1119	1305	1492	1678
1870	187	374	561	748	935	1122	1309	1496	1683
1875	188	375	562	750	937	1125	1312	1500	1687
1880	188	376	564	752	940	1128	1316	1504	1692
1885	189	377	565	754	942	1131	1319	1508	1696
1890	189	378	567	756	945	1134	1323	1512	1701
1895	190	379	568	758	947	1137	1326	1516	1705
1900	190	380	570	760	950	1140	1330	1520	1710
1905	191	381	571	762	952	1143	1333	1524	1714
1910	191	382	573	764	955	1146	1337	1528	1719
1915	192	383	574	766	957	1149	1340	1532	1723
1920	192	384	576	768	960	1152	1344	1536	1728
1925	193	385	577	770	962	1155	1347	1540	1732
1930	193	386	579	772	965	1158	1351	1544	1737
1935	194	387	580	774	967	1161	1354	1548	1742
1940	194	388	582	776	970	1164	1358	1552	1746
1945	195	389	583	778	972	1167	1361	1556	1750
1950	195	390	585	780	975	1170	1365	1560	1755
1955	196	391	586	782	977	1173	1368	1564	1759
1960	196	392	588	784	980	1176	1372	1568	1764
1965	197	393	589	786	982	1179	1375	1572	1768
1970	197	394	591	788	985	1182	1379	1576	1773
1975	198	395	592	790	987	1185	1382	1580	1777
1980	198	396	594	792	990	1188	1386	1584	1782
1985	199	397	595	794	992	1191	1389	1588	1786
1990	199	398	597	796	995	1194	1393	1592	1791
1995	200	399	598	798	997	1197	1396	1596	1795
2000	200	400	600	800	1000	1200	1400	1600	1800
2010	201	402	603	804	1005	1206	1407	1608	1809
2020	202	404	606	808	1010	1212	1414	1616	1818
2030	203	406	609	812	1015	1218	1421	1624	1827
2040	204	408	612	816	1020	1224	1428	1632	1836
2050	205	410	615	820	1025	1230	1435	1640	1845

Diff. of Logar.	Tenth Parts of those Differences. 1	2	3	4	5	6	7	8	9
2060	206	412	618	824	1030	1236	1442	1648	1854
2070	207	414	621	828	1035	1242	1449	1656	1863
2080	208	416	624	832	1040	1248	1456	1664	1872
2090	209	418	627	836	1045	1254	1463	1672	1881
2100	210	420	630	840	1050	1260	1470	1680	1890
2110	211	422	633	844	1055	1266	1477	1688	1899
2120	212	424	636	848	1060	1272	1484	1696	1908
2130	213	426	639	852	1065	1278	1491	1704	1917
2140	214	428	642	856	1070	1284	1498	1712	1926
2150	215	430	645	860	1075	1290	1505	1720	1935
2160	216	432	648	864	1080	1296	1512	1728	1944
2170	217	434	651	868	1085	1302	1519	1736	1953
2180	218	436	654	872	1090	1308	1526	1744	1962
2190	219	438	657	876	1095	1314	1533	1752	1971
2200	220	440	660	880	1100	1320	1540	1760	1980
2210	221	442	663	884	1105	1326	1547	1768	1989
2220	222	444	666	888	1110	1332	1554	1776	1998
2230	223	446	669	892	1115	1338	1561	1784	2007
2240	224	448	672	896	1120	1344	1568	1792	2016
2250	225	450	675	900	1125	1350	1575	1800	2025
2260	226	452	678	904	1130	1356	1582	1808	2034
2270	227	454	681	908	1135	1362	1589	1816	2043
2280	228	456	684	912	1140	1368	1596	1824	2052
2290	229	458	687	916	1145	1374	1603	1832	2061
2300	230	460	690	920	1150	1380	1610	1840	2070
2310	231	462	693	924	1155	1386	1617	1848	2079
2320	232	464	696	928	1160	1392	1624	1856	2088
2330	233	466	699	932	1165	1398	1631	1864	2097
2340	234	468	702	936	1170	1404	1638	1872	2106
2350	235	470	705	940	1175	1410	1645	1880	2115
2360	236	472	708	944	1180	1416	1652	1888	2124
2370	237	474	711	948	1185	1422	1659	1896	2133
2380	238	476	714	952	1190	1428	1666	1904	2142
2390	239	478	717	956	1195	1434	1673	1912	2151
2400	240	480	720	960	1200	1440	1680	1920	2160
2410	241	482	723	964	1205	1446	1687	1928	2169
2420	242	484	726	968	1210	1452	1694	1936	2178
2430	243	486	729	972	1215	1458	1701	1944	2187
2440	244	488	732	976	1220	1464	1708	1952	2196
2450	245	490	735	980	1225	1470	1715	1960	2205

Diff. of Logar.	Tenth Parts of those Differences. 1	2	3	4	5	6	7	8	9
2460	246	492	738	984	1230	1476	1722	1968	2214
2470	247	494	741	988	1235	1482	1729	1976	2223
2480	248	496	744	992	1240	1488	1736	1984	2232
2490	249	498	747	996	1245	1494	1743	1992	2241
2500	250	500	750	1000	1250	1500	1750	2000	2250
2510	251	502	753	1004	1255	1506	1757	2008	2259
2520	252	504	756	1008	1260	1512	1764	2016	2268
2530	253	506	759	1012	1265	1518	1771	2024	2277
2540	254	508	762	1016	1270	1524	1778	2032	2286
2550	255	510	765	1020	1275	1530	1785	2040	2295
2560	256	512	768	1024	1280	1536	1792	2048	2304
2570	257	514	771	1028	1285	1542	1799	2056	2313
2580	258	516	774	1032	1290	1548	1806	2064	2322
2590	259	518	777	1036	1295	1554	1813	2072	2331
2600	260	520	780	1040	1300	1560	1820	2080	2340
2610	261	522	783	1044	1305	1566	1827	2088	2349
2620	262	524	785	1048	1310	1572	1834	2096	2358
2630	263	526	789	1052	1315	1578	1841	2104	2367
2640	264	528	792	1056	1320	1584	1848	2112	2376
2650	265	530	795	1060	1325	1590	1855	2120	2385
2660	266	532	798	1064	1330	1596	1862	2128	2394
2670	267	534	801	1068	1335	1602	1869	2136	2403
2680	268	536	804	1072	1340	1608	1876	2144	2412
2690	269	538	807	1076	1345	1614	1883	2152	2421
2700	270	540	810	1080	1350	1620	1890	2160	2430
2710	271	542	813	1084	1355	1626	1897	2168	2439
2720	272	544	816	1088	1360	1632	1904	2176	2448
2730	273	546	819	1092	1365	1638	1911	2184	2457
2740	274	548	822	1096	1370	1644	1918	2192	2466
2750	275	550	825	1100	1375	1650	1925	2200	2475
2760	276	552	828	1104	1380	1656	1932	2208	2484
2770	277	554	831	1108	1385	1662	1939	2216	2493
2780	278	556	834	1112	1390	1668	1946	2224	2502
2790	279	558	837	1116	1395	1674	1953	2232	2511
2800	280	560	840	1120	1400	1680	1960	2240	2520
2810	281	562	843	1124	1405	1686	1967	2248	2529
2820	282	564	846	1128	1410	1692	1974	2256	2538
2830	283	566	849	1132	1415	1698	1981	2264	2547
2840	284	568	852	1136	1420	1704	1988	2272	2556
2850	285	570	855	1140	1425	1710	1995	2280	2565

Diff. of Logar.	1	2	3	4	5	6	7	8	9
	Tenth Parts of those Differences.								
2860	286	572	858	1144	1430	1716	2002	2288	2574
2870	287	574	861	1148	1435	1722	2009	2296	2583
2880	288	576	864	1152	1440	1728	2016	2304	2592
2890	289	578	867	1156	1445	1734	2023	2312	2601
2900	290	580	870	1160	1450	1740	2030	2320	2610
2910	291	582	873	1164	1455	1746	2037	2328	2619
2920	292	584	876	1168	1460	1752	2044	2336	2628
2930	293	586	879	1172	1465	1758	2051	2344	2637
2940	294	588	882	1176	1470	1764	2058	2352	2646
2950	295	590	885	1180	1475	1770	2065	2360	2655
2960	296	592	888	1184	1480	1776	2072	2368	2664
2970	297	594	891	1188	1485	1782	2079	2376	2673
2980	298	596	894	1192	1490	1788	2086	2384	2682
2990	299	598	897	1196	1495	1794	2093	2392	2691
3000	300	600	900	1200	1500	1800	2100	2400	2700
3010	301	602	903	1204	1505	1806	2107	2408	2709
3020	302	604	906	1208	1510	1812	2114	2416	2718
3030	303	606	909	1212	1515	1818	2121	2424	2727
3040	304	608	912	1216	1520	1824	2128	2432	2736
3050	305	610	915	1220	1525	1830	2135	2440	2745
3060	306	612	918	1224	1530	1836	2142	2448	2754
3070	307	614	921	1228	1535	1842	2149	2456	2763
3080	308	616	924	1232	1540	1848	2156	2464	2772
3090	309	618	927	1236	1545	1854	2163	2472	2781
3100	310	620	930	1240	1550	1860	2170	2480	2790
3110	311	622	933	1244	1555	1866	2177	2488	2799
3120	312	624	936	1288	1560	1872	2184	2496	2808
3130	313	626	939	1252	1565	1878	2191	2504	2817
3140	314	628	942	1256	1570	1884	2198	2512	2826
3150	315	630	945	1260	1575	1890	2205	2520	2835
3160	316	632	948	1264	1580	1896	2212	2528	2844
3170	317	634	951	1268	1585	1902	2219	2536	2853
3180	318	636	954	1272	1590	1908	2226	2544	2862
3190	319	638	957	1276	1595	1914	2233	2552	2871
3200	320	640	960	1280	1600	1920	2240	2560	2880
3210	321	642	963	1284	1605	1926	2247	2568	2889
3220	322	644	966	1288	1610	1932	2254	2576	2898
3230	323	646	969	1292	1615	1938	2261	2584	2907
3240	324	648	972	1296	1620	1944	2268	2592	2916
3250	325	650	975	1300	1625	1950	2275	2600	2925

A TABLE of Proportional Parts.

Diff. of Logar.	Tenth Parts of those Differences. 1	2	3	4	5	6	7	8	9
3260	326	652	978	1304	1630	1956	2282	2608	2934
3270	327	654	981	1308	1635	1962	2289	2616	2943
3280	328	656	984	1312	1640	1968	2296	2624	2952
3290	329	658	987	1316	1645	1974	2303	2632	2961
3300	330	660	990	1320	1650	1980	2310	2640	2970
3310	331	662	993	1324	1655	1986	2317	2648	2979
3320	332	664	996	1328	1660	1992	2324	2656	2988
3330	333	666	999	1332	1665	1998	2331	2664	2997
3340	334	668	1002	1336	1670	2004	2338	2672	3006
3350	335	670	1005	1340	1675	2010	2345	2680	3015
3360	336	672	1008	1344	1680	2016	2352	2688	3024
3370	337	674	1011	1348	1685	2022	2359	2696	3033
3380	338	676	1014	1352	1690	2028	2366	2704	3042
3390	339	678	1017	1356	1695	2034	2373	2712	3051
3400	340	680	1020	1360	1700	2040	2380	2720	3060
3410	341	682	1023	1364	1705	2046	2387	2728	3069
3420	342	684	1026	1368	1710	2052	2394	2736	3078
3430	343	686	1029	1372	1715	2058	2401	2744	3087
3440	344	688	1032	1376	1720	2064	2408	2752	3096
3450	345	690	1035	1380	1725	2070	2415	2760	3105
3460	346	692	1038	1384	1730	2076	2422	2768	3114
3470	347	694	1041	1388	1735	2082	2429	2776	3123
3480	348	696	1044	1392	1740	2088	2436	2784	3132
3490	349	698	1047	1396	1745	2094	2443	2792	3141
3500	350	700	1050	1400	1750	2100	2450	2800	3150
3510	351	702	1053	1404	1755	2106	2457	2808	3159
3520	352	704	1056	1408	1760	2112	2464	2816	3168
3530	353	706	1059	1412	1765	2118	2471	2824	3177
3540	354	708	1062	1416	1770	2124	2478	2832	3186
3550	355	710	1065	1420	1775	2130	2485	2840	3195
3560	356	712	1068	1424	1780	2136	2492	2848	3204
3570	357	714	1071	1428	1785	2142	2499	2856	3213
3580	358	716	1074	1432	1790	2148	2506	2864	3222
3590	359	718	1077	1436	1795	2154	2513	2872	3231
3600	360	720	1080	1440	1800	2160	2520	2880	3240
3610	361	722	1083	1444	1805	2166	2527	2888	3249
3620	362	724	1086	1448	1810	2172	2534	2896	3258
3630	363	726	1089	1452	1815	2178	2541	2904	3267
3640	364	728	1092	1456	1820	2184	2548	2912	3276
3650	365	730	1095	1460	1825	2190	2555	2920	3285

A TABLE of Proportional Parts.

Diff. of Logar.	Tenth Parts of those Differences.								
	1	2	3	4	5	6	7	8	9
3660	366	732	1098	1464	1830	2196	2562	2928	3294
3670	367	734	1101	1468	1835	2202	2569	2936	3303
3680	368	736	1104	1472	1840	2208	2576	2944	3312
3690	369	738	1107	1476	1845	2214	2583	2952	3321
3700	370	740	1110	1480	1850	2220	2590	2960	3330
3710	371	742	1113	1484	1855	2226	2597	2968	3339
3720	372	744	1116	1488	1860	2232	2604	2976	3348
3730	373	746	1119	1492	1865	2238	2611	2984	3357
3740	374	748	1122	1496	1870	2244	2618	2992	3366
3750	375	750	1125	1500	1875	2250	2625	3000	3375
3760	376	752	1128	1504	1880	2256	2632	3008	3384
3770	377	754	1131	1508	1885	2262	2639	3016	3393
3780	378	756	1134	1512	1890	2268	2646	3024	3402
3790	379	758	1137	1516	1895	2274	2653	3032	3411
3800	380	760	1140	1520	1900	2280	2660	3040	3420
3810	381	762	1143	1524	1905	2286	2667	3048	3429
3820	382	764	1146	1528	1910	2292	2674	3056	3438
3830	383	766	1149	1532	1915	2298	2681	3064	3447
3840	384	768	1152	1536	1920	2304	2688	3072	3456
3850	385	770	1155	1540	1925	2310	2695	3080	3465
3860	386	772	1158	1544	1930	2316	2702	3088	3474
3870	387	774	1161	1548	1935	2322	2709	3096	3483
3880	388	776	1164	1552	1940	2328	2716	3104	3492
3890	389	778	1167	1556	1945	2334	2723	3112	3501
3900	390	780	1170	1560	1950	2340	2730	3120	3510
3910	391	782	1173	1564	1955	2346	2737	3128	3519
3920	392	784	1176	1568	1960	2352	2744	3136	3528
3930	393	786	1179	1572	1965	2358	2751	3144	3537
3940	394	788	1182	1576	1970	2364	2758	3152	3546
3950	395	790	1185	1580	1975	2370	2765	3160	3555
3960	396	792	1188	1584	1980	2376	2772	3168	3564
3970	397	794	1191	1588	1985	2382	2779	3176	3573
3980	398	796	1194	1592	1990	2388	2786	3184	3582
3990	399	798	1197	1596	1995	2394	2793	3192	3591
4000	400	800	1200	1600	2000	2400	2800	3200	3600
4010	401	802	1203	1604	2005	2406	2807	3208	3609
4020	402	804	1206	1608	2010	2412	2814	3216	3618
4030	403	806	1209	1612	2015	2418	2821	3224	3627
4040	404	808	1212	1616	2020	2424	2828	3232	3636
4050	405	810	1215	1620	2025	2430	2835	3240	3645

A TABLE of Proportional Parts.

Diff. of Logar.	Tenth Parts of those Differences. 1	2	3	4	5	6	7	8	9
4060	406	812	1218	1624	2030	2436	2842	3248	3654
4070	407	814	1221	1628	2035	2442	2849	3256	3663
4080	408	816	1224	1632	2040	2448	2856	3264	3672
4090	409	818	1227	1636	2045	2454	2863	3272	3681
4100	410	820	1230	1640	2050	2460	2870	3280	3690
4110	411	822	1233	1644	2055	2466	2877	3288	3699
4120	412	824	1236	1648	2060	2472	2884	3296	3708
4130	413	826	1239	1652	2065	2478	2891	3304	3717
4140	414	828	1242	1656	2070	2484	2898	3312	3726
4150	415	830	1245	1660	2075	2490	2905	3320	3735
4160	416	832	1248	1664	2080	2496	2912	3328	3744
4170	417	834	1251	1668	2085	2502	2919	3336	3753
4180	418	836	1254	1672	2090	2508	2926	3344	3762
4190	419	838	1257	1676	2095	2514	2933	3352	3771
4200	420	840	1260	1680	2100	2520	2940	3360	3780
4210	421	842	1263	1684	2105	2526	2947	3368	3789
4220	422	844	1266	1688	2110	2532	2954	3376	3798
4230	423	846	1269	1692	2115	2538	2961	3384	3807
4240	424	848	1272	1696	2120	2544	2968	3392	3816
4250	425	850	1275	1700	2125	2550	2975	3400	3825
4260	426	852	1278	1704	2130	2556	2982	3408	3834
4270	427	854	1281	1708	2135	2562	2989	3416	3843
4280	428	856	1284	1712	2140	2568	2996	3424	3852
4290	429	858	1287	1716	2145	2574	3003	3432	3861
4300	430	860	1290	1720	2150	2580	3010	3440	3870
4310	431	862	1293	1724	2155	2586	3017	3448	3879
4320	432	864	1296	1728	2160	2592	3024	3456	3888

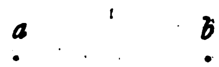
The End of the Table of Parts Proportional.

CHAP. VIII.

Lineal Arithmetic. (See Plate *A*.)

THIS fourth Species of Arithmetic is performed by Right Lines, so that Magnitude being here considered, it may be called *Geometrical Arithmetic.*

2. A Right Line is properly defined to be the nearest Distance between two Points, as the Line (*a c b*, *Fig.* 1.) is the nearest Distance between the Points *a* and *b*.



3. The Disposition of the two principal Lines, by which this kind of Arithmetic is performed, are either in a Right Angle or an Oblique.

4. A Right Angle is when one Line falls Perpendicular on another, as the Line *a b* (*Fig.* 2.) does on *b c*; but when two Lines meet not so, they form an Angle called *Oblique*, as the Angle *d e f* (*Fig.* 3) by both which, Questions are answered.

As in Logarithms those Artificial Numbers are not employed about the most easy and self-evident Rules of *Addition* and *Substraction* of Natural Numbers: So here by Lines I shall only shew how *Multiplication*, *Division*, the *Rule of Proportion*, with *Extraction of Roots*, are performed thereby.

SECT. II. *Multiplication by Lines, three Ways.*

1. What is the Product of $17\frac{1}{2}$ by $12\frac{1}{2}$? See *Fig.* 4. in the folded Sheet (*A*.)

To answer this, draw a Line at pleasure, and set off $12\frac{1}{2}$ equal Parts as from *a* to *b*, from the Line of equal Parts.

Then draw a perpendicular Line at pleasure as *b c*, and set off from *b* to *c* $17\frac{1}{2}$.

Lastly, Draw Lines between each equal Part, and the $\frac{1}{2}$, Parallel to (*a b*) and to (*b c*), then telling the several little Squares each of which is a Unit, you'll find 204 intire, and 17 and 12 Halves, or $14\frac{1}{2}$ which together, and the $\frac{1}{4}$ at the Angle (*c*) make the Product $218\frac{3}{4}$.

If you would multiply 175 by 125, that is, if you suppose every Unit in the Line (*a b*) to be 10, then $12\frac{1}{2}$ will be 125, and by

the ſame Rule the Line *b c* will be 175, and by ſuppoſing each little Square 100 (or 10 times 10, the Lines *a b* and *b c* being 10 times as many as before) you'll find 20400 little Squares or Units, and 17 and 12 Fiftie (or 1450) and 25 the 4th of 100 at the Angle (*c*), which added together, makes 21875 = the Product required.

A ſecond Way to multiply by Lines.

But before I proceed therein, it will be neceſſary to ſhew how to draw a Line Parallel to another, and through any Point aſſigned: So to draw a Line through the Point (*a*) (See *Fig.* 5. *Plate A.*) and Parallel to (*m n*), ſet the Foot of your Compaſſes in the Point, and extend the other to the Line *m n*, ſo as to deſcribe the Arc *c c* juſt to touch the Line *m n*; and with that Diſtance of the Feet ſet one in the Point (*e*) (or thereabout) and deſcribe the Arc (*d d*); then lay a Ruler from that Arc to the Point *a*, and draw the Line *p q*, which is the Anſwer.

To multiply by 2 Lines, including an Oblique Angle as that (e d n,) *Fig.* 6.

What is the Product of 3.5 by 2.2? For anſwer, having made an Angle at pleaſure, not too acute.

Rule.] Take a Line of equal Parts, and ſet off 1 from the Angle *d* to 1. Then ſet the Multiplicand from *d* to 3.5 on the other Leg (*d n*). 3*dly*, Set the Multiplier from 1 to 2.2. 4*thly*, Draw the Line 1 to 3.5, produced to about (*m*), for the better drawing the Line (2.2; 7.7) Parallel to (1, *m*); ſo is the Diſtance between 3.5 and 7.7 the Anſwer, which meaſured upon the ſame Line of equal Parts, is found 7.7 = the Product.

2. Or if you ſet the Multiplicand from 1 to *b*; and the Multiplier from *d* to *a*, a Line (*b c*) drawn Parallel to (1, *a*) cutteth the Product (*a c*) 7.7 as before.

3. Or if you ſuppoſe the Factors 10 times as much, the Anſwer will be by the ſame Method 770.

A third Way to multiply by Lines.

This ſhews the *Uſe of a Line of Numbers*, or as 'tis often called *Gunter*'s Line; being a Line of *Logarithms*, as will appear by the *Uſe*, *Addition performing Multiplication*; *Subſtraction*, the *Work of Diviſion*; *Diviſion*, the *Extraction of Roots*, &c.

This

This useful line was the Invention of Mr *Edmund Gunter*, Professor of *Astronomy* at *Gresham-College* about the Year 1620; and it is a line divided into 10 unequal Parts, and those 10 Parts repeated are called 20, 30, 40, *&c.* when the former Parts are called 1, 2, 3, *&c.*

If the first 10 are Tens,	The 2d ten Parts are 100's
If the first 10 are Hundreds,	The 2d Radius are so many (Thousands.
If you multiply 1 place by 2 places,	The Product is so many 10's (as the Figure denotes.

As your own Experience will easily discover to you. See the making of this line near the End of *Sect.* 4th following.

To multiply any 2 Numbers, as those in the last Case 2.2 by 3.5. See *Fig.* 7. *Plate A.*

Rule.] Extend the Foot of a Pair of Compasses (or other Measure) from 1 to 2.2, and that Extent reaches from 3.5 to 7.7;

2dly, Or the Extent from 1 to 3.5, reaches from 2.2 to 7.7.

3dly, Or if you multiply 35 by 22: In this case, what is called 1, 2, *&c.* in the 1st part, must be supposed 10, 20, *&c.* and what are 10th parts must be called Units, and if both Factors be 10 or more each, the Product is Hundreds. So the Extent as before from 10 to 22, reaches from 35 to 770; for here 35 being multiplied by a Number which is above 10, is so many 100's and 10's as the Measure extends to; for your Inspection shews, that 35 by 10 is 350, which is 3 places; and if for any Number of Tens, the like: And if the 2d Extent be beyond 10 on the line, it is still so many Hundreds, as 30 by 50 is 1500, *&c.*

And *Note*, That whatever you call the larger Divisions, the smaller in the same Radius are a 10th thereof, as is before hinted, and the contrary.

SECT. III. *Divisions by Lines, three Ways.*

As I have shewed *Multiplication* by three different lines, and by each two ways; so I intend to shew how to divide.

Case 1.] What is the Quotient of 218¾ by 17½? See *Fig.* 4.

The Dividend is represented by the several small Squares and Parts in the Superficies *a c*, which if you divide by the side (*a* 17½) the Quotient will be one of the smaller sides *a b* or (17½ *c*).

For having divided a Line *(a, 17½)* into 17½, parts, your Divisor, and made a little Square upon each part, and half of one for the half; if you add to those Squares at Right Angles so many as will make them up 218¾, and as many in one Row of Squares as another; you will then find that the Line including a Right Angle with *(a 17½)* will be *(a b)* which will contain 12½ of those small Squares, and is consequently the Quotient, because that Quote multiplied in the Divisor *(a, 17½)* produceth the Dividend or Number of small Squares *(a c)* = 218¾. And the same may be said of other like Superficies and their Sides, whatever they be for Numbers.

A second Way to divide Lines.

What is the Quote of 7.7 divided by 3.5? (*Fig.* 6. for I make use of the same Numbers as in *Multiplication*, to prevent Confusion by a Multitude of prickt Lines.)

1st, Take 1 from any Line of equal Parts, and set off (*d* 1).

2dly, Set the Divisor (*d*, 3.5) off from the same Line of Parts.

3dly, Draw the Line (1, 3.5) and produce it to *m*, &c.

4thly, Set your Dividend (7.7) from 3½ towards *n*, which extends to 7.7.

5thly, Through that Point draw a Line parallel to (1 *m*) and it will pass through the Point 2.2 in the Line (*d e*).

6thly, Take the Distance from 1 to 2.2, and apply it to your equal Parts, and you'll find it 2.2 = the Quote.

Or if you were to divide 7.7 by 2.2, the Divisor set from *d* to *a*, and the Dividend from *a* to *c*: If a Line be drawn (1 *a*), and another parallel thereto through the Point *c*, that Line will pass thro' *b* in the Line *d e*, and the distance (1 *b*) measured on the same Line of equal Parts, will be 3.5 = the Quote.

Or if you divide 770 by 35, the Method is the same as the first of these, and 77 by 22 the same as the second or last, *viz.* 35. And the like of other Numbers.

A third Way to divide by Lines.

Suppose by the Line of Numbers, *Fig.* 7. *Plate A.* you would divide 770 by 22. Extend the Compasses from 10 to 22 (calling those 10, &c. in the first Radius, which are 1, 2, &c. and those 100's in the second, which are 10, 20, 30, &c.) and the same

same Extent reaches from 770 in the second Radius, backward to 35.

Or to divide 770 by 35, the Extent from 10 to 35, extends backward from 770 to 22. And the like of other Numbers.

SECT. IV. *The direct Rule of Proportion by Lines.*

If 10 ℔ of Sugar cost 22 *s.* what will 35 ℔ cost?

1st, By the lines, *Fig.* 6. set 10 from the Angle to 1 (which suppose 10, as if the 1 was not there;) then set the 22 *s.* from 1 to 2.2, that is, from 10 to 22. *3dly*, Set the third Number from the Angle at *d* to 3.5, or to 35. Then draw a line from 10 to 35 = the third Number given, and a line parallel to that through the Point 2.2 (here 22 will pass through, or cut the line *d n* in 7.7) so the Distance 7.7 to 3.5 (measured upon the line whereby *d* 1 is 10) is 77 *s.* = the value of the 35 ℔ of Sugar. I have done this (as I said before) by the same parallel lines, to prevent Confusion by a Multiplicity.

2dly, *By the line of Numbers*, Fig. 7. If 12 C. of Sugar cost 25 *l.* 4 *s.* what will 1 Ton or 20 C. cost?

Extend the Compasses from 12 C. to 25.2, and the same Extent reaches from 20 C. to 42 l. the Answer.

Quest. 2. If you would know the Interest of 90 *l.* for a Year at 6 *per Cent.* extend the Compasses from 100 backward to 6; and the same Extent reaches from 90 to 5.4 *l.* or *l.* 5 : 8 : = the Answer.

3dly, To Extract the Square Root, take half the Distance between the Square Number given and 1. And 1 third of the Distance from a Cube Number to 1 is the Cube Root, taking the third next to 1.

And now I might end this part of Arithmetic, but before I do I shall shew the making of a line of Chords of Numbers, and a line of half Tangents with something of their Use, and that of equal Parts, which will be a Preparative for those who would know how to perform Trigonometry Geometrically.

The line of Chords is 90 Degrees (or equal Parts in the fourth Part of a Circle) set off upon a straight line, as *Fig.* 8. *Plate A.* the Degree (*e m*) are set upon the Diameter (*e c n*) where (*e* 60) of the Circle, is equal to Radius or the Semi-diameter of that Circle, and *e m* equal to *e n*, &c.

Now to measure any Angle by this line extend the Compasses from *e* to 60 in the line (*e n*), and with that (suppose in *Fig.* 9.) set

ſet one Foot in *h*, and deſcribe the Arc *(r o)*, then take the Diſtance *(r o)* in your Compaſſes, and apply it to your line of Chords, and you'll find it 33 Degrees: That is to ſay, the line or leg *(h f)* is diſtant from that *g h* 33 Degrees or equal Parts of the Circle *(d m e*, &c.*)* as thoſe are transferred upon the line *(e d)*; ſo the Angle *(h f g)* or at *f*, is found 58 Degrees, and the Sum of thoſe two Angles deducted from 180 leaves the Angle *(f g h*, or) at *g*. And the Sides of this Triangle are meaſured by applying them between the Compaſſes to *the line of equal Parts* (*y z*), *Fig*. 10.

The way to make a Line of Numbers or Logarithms (commonly called Gunter's *Line).*

The line (1, 10, *Plate A.*) is thus made: *Firſt*, Make a line of equal Parts acccording to the Length you intend your line of Numbers, as here (0, 1000) is divided actually into 100 equal Parts, and each Part ſuppoſed to be divided into 10, makes 1000; and if your line were deſigned to be much longer, you might divide your equal Parts into 10000.

2dly, Lay before you a Table of Logarithms, and from that take the Logarithms (omitting the Index) of 2, 3, 4, *&c.* to 10, the largeſt Diviſions of your line: As, for 2 I find the Logarithm .3010300, but I take Notice only of the three next places to the Point agreeable to my line of equal Parts, as 301 taken from that line, and ſet off upon the Line (1, 10,) from 1 to 2 gives the firſt of the firſt 9 unequal Parts of the line of Numbers.

Then I find the Logarithms of	3 to be	.477	Which Logarithms taken from the line of equal Parts, and ſet on that of Numbers from	1 to 3	Gives you the other unequal Parts of the line of Numbers.
	4	.602		1 to 4	
	5	.698		1 to 5	
	6	.778		1 to 6	
	7	.845		1 to 7	
	8	.903		1 to 8	
	9	.954		1 to 9	

Then for the Sub-diviſions or ſmaller unequal Parts; I proceed in the very ſame manner 'till I have ſet them all off from 1 (the beginning of the line of Numbers): As.

The

The Logarithm	.041		1 to 1.1	
	.079		1 to 1.2	
	.113		1 to 1.3	
	.146	Taken from the	1 to 1.4	On the Line of
	.176	Line of equal	1 to 1.5	Numbers.
	.204	Parts, extends	1 to 1.6	
	.230	from	1 to 1.7	
	.255		1 to 1.8	
	.278		1 to 1.9	
	.322		1 to 2.1	
	.342, *&c.*		1 to 2.2, *&c.*	

I have been more particular in this Affair than has been shewed before, because I find most Persons, and some even the Instrument-Makers, ignorant, how this very useful line is made originally to any Radius, which depends on the Magnitude of the equal Parts.

But *Note*, That the line of Numbers consisteth of two such lines, as (1, 10; on *Plate A. Fig.* 10.) next the *Fig.* 7. (or Numbers) and so from 10 is numbered 20, 30, *&c.* to 100 on the 2d Part, which is made 1 line with the first, and must so be made by the Artist.

To make a Line of equal Parts.

To make the line (0, 1000, *Fig.* 10.) imaginarily divided into 1000 Parts, or actually into 100.

1st, I divide it into 5 with my Compasses.

2dly, I divide each 5th into 2, which makes the whole into 10.

3dly, I divide 1 tenth Part into 10 (as 0, 100).

4thly, I take (0, 100) between my Compasses, and setting 1 Foot in (*i*), the other extends to (*i*), or a 10th of the next 10th, and so forward divides the whole line into 100, and each 100 must be supposed 10, in taking off the 1000th Parts to make the line of Numbers by, or in other Uses.

The line of half Tangents, is the line (*c m*, *Fig.* 8.) made by laying a Ruler to (*d*), and the several Degrees in the Quadrant (*e m*), and is used in measuring such straight lines as pass through the Center of a Circle, and do represent Circles, though they appear Right lines, as (*c m*), (*c e*), (*c d*), &c. The Use of the line of equal Parts has been fully shewn before in measuring Right lines, and

and there is nothing more falls within the Compass of Lineal or Geometrical Arithmetic; which will be of great Use to such as intend to study Geometry, to which I recommend the reading of *Euclid*, lib. 2. I shall therefore proceed to

CHAP. IX.

INSTRUMENTAL ARITHMETIC.

INSTRUMENTAL Arithmetic is that Part which is performed by Instruments, accommodated with Figures or Numbers for that purpose. Of this kind, that of *Neper*'s Bones justly claims the first Consideration, as being the most plain and easy Instrument for all Operations, whether in Multiplication, Division, or Extraction of Roots.

A Description of Neper'*s Bones*, (See *Fig.* 1. *Plate B.*) As to their Matter, they are sometimes made of Ivory, and then called Bones, sometimes of Box-Wood, and then called *Virgulæ, or little Rods*; for as to *their Magnitude and Form*, they may be made as large as you please: But the common Sort are each Rod about 2.5 Inches long, and .32 Inches broad, and are in Figure what Geometricians call a Parallelopipedon, which is a long Square solid, bounded with 6 Superficies, (or like 7, 8 *ec* Dice joined exactly one to another.)

The Number of these Rods in a Set are 10, that is, the 9 Digits and Cypher, besides the Index-Bone which is fixed to the Box, and hath only one Face divided into 9 equal Parts, whereon the 9 Digits are placed as you see *Fig.* 1. *Plate B.* But the other 10 have 4 Faces each, whereon are placed the 9 Digits at the top, and an Arithmetical Progression of those Digits added as 2, 4, 6; or 3, 9, 12, *&c.* And whatever the first Figure of any Rod is, that of the opposite Side is the Complement that makes it up 9. And the 10 times 4 Faces or Sides being 40, contain 4 of each of the 9 Digits, and their Series's Arithmetical, and 4 more Faces are of Cyphers.

How to add the Figures on the Rods. See *Fig.* 12. *Plate A.*

To reduce any line of Figures on the Rods which stand against any Digit on the Index-Face to a true legible Number, you must

must begin at the right hand: Thus to add the Digits standing against 6 in the Set, *Fig.* 1. *Plate B.* You see there is nothing to do but to add the two Digits together, which stand between the same Slant or Diagonal lines, as 4 is 4, then 5 and 8 is 13; 1 and 4 is 5, and 2 is 7; 4 and 6 is 10; 1 and 3 is 4; 3 and 4 is 7; 2 and 8 is 10; 1 and 1 is 2, and 2 is 4; 1 and 6 is 7: So that six times 123456789, is found 740740734.

SECT. II. *To multiply by* Neper's *Rods or Bones.*

Quest. 1. What is the Product of 34729 by 23795; See *Fig.* 13. *Plate A.*

First, tabulate the Multiplicand, (or either Factor, but the larger is most usual;) that is, take so many Rods out of your Set, that the first Figures (or those at the upper End of each Rod) will compose your Multiplicand in order, as you see done, *Fig.* 13. of *Plate A.*

Then looking for the Figures of your Multiplier severally in the Index-Rod next the left hand, you find against each its respective Product in the Multiplicand, as *Fig.* 13. aforesaid.

Quest. 2. What is the Product of 3040999 by 2341?

In this Example there being 3 Nines and 2 Cyphers, on the opposite Sides, whereof are 2 Nines makes 5 Nines, which is one more than is in a set of these Rods; therefore, if you have

3040999 = the Multiplicand.

Multiplied by 1 = 3040999
by 4 = 12163996
by 3 = 9122997
by 2 = 6081998

7118978659 = Product.

not another Set, you may easily supply the Defect by observing the Product of each Figure (as you go on) in the Multiplier, and those other Nines, and so making one of those extraordinary: Thus in Multiplying by the 4 there (as *Fig.* 14.) and if there were another Rod for the third Nine in the Multiplier, I can easily see there would be more (as *Fig.* 15. *Plate A.*) which therefore, I suppose, and so put down 6, and then two Nines instead of the one Nine in the Rods; so when I multiply by the 3, I find in the Tabulet toward the right hand against 3, (see *Fig.* 16. *Plate A.*) But I know if I had another Rod of the

 same

ſame Digit 9, there would have been another, as *Fig.* 17. *Plate A.* which ſuppoſing there, it would make 2997, that is, a 9 more than I find in the Tabulet, and ſo the reſt.

SECT. III. *Diviſion by* Neper's *Bones or Rods.*

What is the Quote of 23734.691084 divided by 253.24?

An Example of the whole Proceſs, and of what help you have in dividing from the Rods. See *Fig.* 18. *Plate A.*

Hence it appears, that the Rods are only uſeful in exhibiting the Products of the Diviſor by each Figure put in the Quotient, which knowing, you proceed in all other Reſpects as in Diviſion of Integers, *Sect.* 5. *Chap.* I. *e c.* firſt Tabulating the Diviſor, and then whatſoever the Quote be, you have each Figure multiplied in the Diviſor, although I have only placed in the Index-Rod the Figures that fall in the Quote above, and that in the Order they happen therein, for the more orderly ſhewing the Nature of the Work, and how it agrees with common Diviſion; ſo that in doing this Queſtion above, you finding that 227916 is the next Number on the Rods to, and leſs than, 237346 (the firſt Part of the Dividend) looking againſt that in the Index-Rod you find 9; therefore put 9 in the Quote, and deduct as in the Example, and the Remainer (with 9 brought down to it) is 94309; which found in the Rods, or what is next leſs, I obſerve 3 to ſtand againſt it, which I put in the Quote, and ſo proceed with the reſt as in the Example, which is ſo plain, that it will ſerve for all. I have put the Figures of the Quote againſt their reſpective Dividuals, which added and placed as above in its proper place, is the general Quotient required 93.7241.

SECT. IV. *The Uſe of* Neper's *Bones or Rods in Extraction of the Square Root.*

Square.	*Square Root.*
54756	(234
4	
147	
129	
1856	
1856	
0 Reſts.	

What is the Square Root of 54756? (See *Fig.* 3. *Plate B.*)

Rule.] Having pointed the Square Number given, as taught in intire Numbers; by the three Points, it appears, there will be Hundreds, Tens, and Units in the Root.

2*dly*, Having put the Rod of the Square of the 9 Digits next towards the right hand of the Index-Rod, you'll find that againſt 4, the

next

next Square to 5 (the first pointed towards the left hand) there stands its Root upon the said fixed Rod, *viz.* 2; therefore put 2 in the Root, and deduct the said Square 4 from 5, and there rests 1.

3dly, Having brought as usual the next Branch down, it makes 147. Then take a Rod, whose uppermost Number is the double of the Figure last put in that Root (here 4); and place the Rod of Squares to the right hand, that (as you see *Fig.* 3. *Plate B.* in the middle Part over 30) then look in the two Rods next the right hand for 147, and the next less is 129, right against which on the Index-Rod stands 3; therefore put 3 in the Root, and deduct 129 from 147, and the Remainer is 18.

4thly, Having placed the next two Figures (or last Binary of the Square given) toward the right hand of 18, they make 1856. Then place two Rods (between the Rods of the Index, and that of Squares) whose uppermost Digits are the double of the 2 in the Root (as you see done in *Fig.* 3. aforesaid next the right hand) and look in the 3 Rods next that hand for your 1856, which you find just, and towards the left hand thereof, on the Index-Rod stands 4; therefore put 4 in the Root, and deducting, you see nothing remains, and that 234 is the Root required. *This I have made much more plain than has been done before, by* Fig. 3. *which shews the Position of the Rods upon each Digit put in the Root, and doubt not but it is sufficiently intelligible.*

But *Note*, That if the double of the Figure put in the Root be more than 9, you must put two Rods next the Index, which begin with the Digits of that double; and three Rods put when the double of two places is above 99. Thus, if 9 is the first in the Root, the double is 18; so I put two Rods between the Index and the Rod of Squares, beginning with 1 and 8. So also, if 99 were the two first in the Root, the double is 198; I put therefore three Rods, beginning with 1, 9, and 8, between the Index-Rod and that of Squares, *&c.*

For a farther Exemplification; extract the Square Root by the Bones of 97535376. See the Work in the Margin.

1st, Here you find 81 the next less Square to 97, and against it is 9 in the fixed Rod; so you put 9 in the Root, and deducting 81, there rests 16, and the next Branch or Period is 1653.

2dly, Doubling 9 is 18, which two Rods placed next that fixed, and that of Squares next the right hand; the top Digits will stand in this Order, see *Fig.* 19. *Plate A.* and in the Progression lower you'll find against 1504, the next less, to 1653, stands 8, which put in the Root, and deduct 1504, there rests 149; which with the next Period makes 14953.

```
                 Square.
                 . . . .          Square
Example 2 = 97535376 (9876 = Root
            81
            ——
            1653
            1504
            ————
            14953
            13769
            —————
            118476
            118476
            ——————
                 0
```

3dly, Doubling 98 is 196; which three Rods placed next that fixed, and that of Squares towards the right, the first Figures on the Tabulet Rods will stand thus, see *Fig.* 20. *Plate A.* And in the Progression thereof lower you'll find 13769; against which is 7, which put in the Root, and the 13769 deducted from 14953, there rests 1184; to which bringing down the next Branch, makes 118476.

4thly, Doubling 987 in the Root, makes 1974; therefore taking 6 out of the last Tablet, and putting the Rods beginning with 7 and 4, the top of the six Rods will stand thus, see *Fig.* 21. And in the Progression lower you'll find on the 5 Rods next the right hand 118476, against which towards the left hand upon the fixed Rod is 6: So there Rests (0), and the Operation is finished.

SECT. V. *The Use of* Neper's *Bones in the Extraction of the Cube Root.*

The Rods make this (which is reckoned the most difficult Rule) very easy, so soon as you know how to place your *Virgulæ*, which I shall therefore illustrate by an Example of their Position at every Figure put in the Root, as I have done in the preceeding Parts of Arithmetic by the Rods.

The

The Cube Number given must be prepared for Extraction by pointing over the Units, and every third afterwards, as is taught in extracting Roots of Vulgar intire Numbers, *&c.* for Example: What is the Cube Root of 182284628? See *Figures* 22, 23, 24. *Plate A.*

1*st*, The first Figure in the Root is found by placing your Cube-rod next the Index-rod; for looking on the former for 182 (the first Point next the left) you find the next less to be 125.

```
182284628 (567
125
 57
```

And right against that 125, stands 5 on the fixed Rod, which 5 I therefore put in the Root, and substracting 125 from 182, there Rests 57. To which bringing down the next Point or Branch, it makes 57284.

```
45216
  540
50616
 6668628
```

Then for a Divisor to divide the 57284: Take treble the Square of the 5 (in the Root) which is 75, see *Fig.* 23. *Plate A.* And having tabulated that, and the Rod of Cubes, as you see, I find on those three Rods 45216 the next less to be taken, against which in the Index is 6, which I put in the Root: To which 45216 I add 540 (which is the Square of 6 in the Root multiplied in three times the 5 there) and the Sum is 50616; which substracted from the Dividend 57284, there rests 6668, to which I bring down 628 = the third Branch.

```
9408) 6668628
      6585943
        8232
      6668263
          365
```

And it makes the last Dividend 6668628; and treble the Square of the Root (56) makes 9408 (a Divisor) which Tabulated with the Rod of Cubes, as you see *Fig.* 24. *Plate A.* I look upon the Rods, and find 6585943 to be the next less Number than the 6668628;

6668628; and against it is 7 in the Index, which 7 I put in the Root: Then for the 8232 to be added to the Number next above it, I multiply 3 times 56 (in the Root) by the Square of 7 there; and the Sum is 6668263, which I take from the last Dividend 6668628, and the Remainer is 365; to which, if you desire Decimal Places, you may put three Cyphers towards the Right, and so proceed as before. I have made this Example so demonstrative, that I shall need to give no more, for by the very same Method you may do any other.

Note, That in the Division (by 75) the next of all on the Rods, and less than the Dividend 57284 is 52843, against which stands 7 on the Index, but then had you put 7 in the Root instead of 6, you would have found a Number too great to be deducted from the said Dividend or Resolvend 57284; and therefore I take still the next less Number which stands against 6.

2*d Note*, That the 540 and the 8232 are placed 1 Degree towards the left hand, because they would fall so if put under the Tens places of the Cube of the last Figure put in the Root; as is done in the common way of Extraction.

Note also, That if you have not the Rods of Squares and Cubes, you may do without, either by placing in the room a Rod, whose first Number is the same with that which falls in the Root (which you may know by comparing the Figures on the Rods toward the left hand with those of the Dividend) or by making them (for present Use) of Paper, according to the many Examples above given of them, till you get the Rods of Squares and Cubes.

SECT. VI. *Notation and Numeration of the Diagonal Circular Instrument*, (See Plate *B.*)

This Instrument was by me contrived chiefly for Reduction of Money, Weight, or Measure, into Decimals; or for finding with Ease and Speed the Value of those Fractions, and for Multiplication, Division, and Evolution. It is marked on *Plate* B. with *Fig*. 5. and consisteth of seven Concentric Circles (besides the two outermost, which are Lines of Numbers) as

1. The outward or largest Circle (save two) is of equal Parts, to which all the rest that are lesser Circles refer in their Use; it is first divided into 10 Parts, each of which being subdivided into 10, makes a 100 Parts; and each 100 Part again divided into 10 by the

the Diagonal Line (1000, *a*) *&c.* divides the whole Circle into 1000 equal Parts. So that for Example: To find in that Circle 767 thousandth Parts, you have 760 in the Arch of that Circle, and 7 in the Diagonal Line, telling the Parts upward from 760.

2. The second Circle is of *English* Coin, the whole representing 1 *l.* divided into 20 *s.* each Shilling in 12 Pence, and each Penny by the Diagonal into 4 Farthings. So that to set off 13 *s.* 7 *d.* $\frac{1}{4}$ *q.* look for 13 *s.* 7 *d.* in the Ambit of the Circle, and then from 7 *d.* telling upwards, you see 1 Farthing in the Diagonal.

3. The next inner Circle (which is the third from those of Numbers) is of Beer Measure, being one Barrel divided into 4 Firkins, each Firkin into 9 Gallons, and each Gallon into Quarts and Pints as you see.

4. The next inward is a Barrel of Ale divided into 4 Firkins, those into each 8 Gallons, and those again into Quarts and Pints.

5. The fifth Circle inward from those of Logarithms is the Pound Averdupois divided into 16 Ounces, and those each Ounce into 16 Drams.

6. The next inward Circle is the Hundred Averdupois divided into four Quarters, and each Quarter into 28 ℔. And

7. The innermost Circle is Foot Measure, being 1 Foot divided into 12 Inches, and each Inch into 4 Quarters.

And all these Divisions and Subdivisions are actually done, and not imagined only as some Scales are, which are of little or no Use. And any Quantity in the seven inner Circles is easily pointed to by the same Rule as given for the two first; from those of Numbers, only in these three are no Diagonals, there being no need of them, the Parts being sufficiently expressed without.

8. The two outermost Circles are Lines of Numbers, whose Uses follow after 10th.

Reduction of Decimals by the Circular Instrument.

Quest. 1.] What Decimal of a Pound is 13 *s.* 6 *d.* $\frac{1}{4}$ *q?*

Rule.] Lay a Ruler from the Center to 13 *s.* 6 *d.* $\frac{1}{4}$ *q.* in the second Circle, and that will cut in the Diagonal Circle .676, which is the Answer.

Quest. 2.] What is the Value of .876543 of a Pound Sterling?

Rule.] Lay a Ruler to the Center, and to 876, and half in the Diagonal Circle (or equal Parts) and you'll find it to cut in the Circle of Money 17 *s.* 6 *d.* $\frac{1}{2}$ *q.*

Quest.

Quest. 3.] What Decimal of a Barrel of Beer is 2 Firkins, 7 Gallons, 3 Quarts, and a Pint?

Rule.] Lay the Ruler to the Center, and the Quantity given in the third Circle from the Line of Numbers, and it will shew in the first Circle the Decimal required to be .7187, or apparently more than .718 and ½.

Quest. 4.] What is the Value of .876543 of a Barrel of Ale?

Rule.] Lay a Ruler to the Center, and the Decimal given in the Diagonal Circle (I mean the four first Figures next the Point) and it will cut in the fourth Circle 3 Firkins, 4 Gallons, and about half a Pint.

Quest. 5.] In 3 Firkins, 5 Gallons, 3 Quarts of Ale, how much Beer?

Rule.] Lay a Ruler from the Center to the Quantity given in the Circle of Ale Measure, and it will give in the Circle of Beer Measure 3 Firkins, 6 Gallons, 2 Quarts Beer. And in like manner Beer is reduced to Ale Measure.

Quest. 6.] What Decimal of a Pound is 13 Ounces 11 Drams?

Rule.] Lay a Ruler from the Center to 13 Ounces and 11 Drams, and it will cut in the Diagonal Circle .855.

Quest. 7.] What is the Value of .876543 of a Pound Averdupois?

Rule.] By laying a Ruler to the Center, and the Decimal given, it will shew the Value in the fifth Circle from those of Numbers to be 14 Ounces and near half a Dram.

Quest. 8.] What Decimal of a hundred Weight is 2 Quarters 21 Pound?

Rule.] Lay a Ruler to the Center, and the given Quantity in the sixth Circle, and it will cut in the Diagonal Circle .687, or .6875.

Quest. 9.] What is the Value of .876543 of an hundred Weight?

Rule.] Laying a Ruler as in the 7th Question, it will cut in the 6th Circle 3 Quarters 14 Pound.

Quest. 10.] What Decimal of a Foot is 10 Inches 3 Quarters?

Rule.] Laying a Ruler to the Center, and the Quantity given in the 7th Circle, it will cut in the equal Parts .8958, or visibly .895 and more.

And a Line of Numbers being delineated without the largest or diagonal Circle (as by Rules given for drawing that Line, setting the Parts off from the Center) and another to turn round within it; you may by them do most Questions in Multiplication, Division, and Extraction of Roots very accurately, as by Rules in *Chap.* VIII. Or thus, Suppose I would do the second Question in

Sect

Sect. 4. of *Chap.* VIII. *i. e.* If 12 C. of Sugar cost *l.* 25 : 4 : 0 what will 1 Ton cost?

Turn 12 in the inner Circle, to 20 in the outer Circle; and then against 25.2 in the inner, will stand 42 in the outer.

Or in Case you have only a Cut of this Instrument pasted on a strong Board that will not warp; you may do these Questions by the Compasses and one of the Lines (or the outmost Circle); as if 20 require 30, what will 50 require? In the second half Circle, the Extent of a pair of Compasses from 20 to 30, will extend from 50 to 75, the fourth Proportional or Answer.

The compound Interest of any Sum for Years, is found by the two Lines of Numbers: as that of *l.* 650 for 5 Years at 5 *per Cent.* the fifth Power of 1.05 in the Amount of *l.* 1 for five Years, which multiplied by *l.* 650 gives *l.* 829 : 11 : 0 the Answer: For

Turn 1 in the inner Circle, to 1.05 in the outer Circle, then against	1.05	In the inner Circle stands	1.1	In the outer Circle.
	1.1		1.16	
	1.16		1.22	
	1.22		1.28	= the Amount of 1 *l.* for 5 Years.

And turning 1 in the inner Circle to 1.28 in the innermost Circle; against 650 (the Principal) in the inner Circle, you will find 829½ in the outer Circle, the Amount required.

Many other Questions might be proposed, and Uses shewn of this Instrument; but I leave it to the Reader to find out, of his own accord, to exercise his Parts.

The End of Instrumental Arithmetic.

CHAP. X.

ALGEBRAICAL ARITHMETIC.

I. ALGEBRA being a Word derived from the *Arabic Algiabr*, makes it probable, that the *Arabs* were the most ancient Professors, and great Proficients in this Art. Although some have ascribed the first Invention to the *Chinese*, *Indians*, or *Persians*, some to the *Grecians*, and that it was first propagated by *Plato*; though it was kept a Secret, from the Principle, that the

the divulging a Truth was prophaning of it. The first Treatise of this Art is said to be wrote by *Diophantus*, extant in *Latin* and *Greek*; but the first *European* Author, was, most probably, *Lucas de Burgo*, a Friar, who publish'd a Book of this Subject in *Italian*, printed at *Venice*, *Anno* 1494; yet *Scipio*, *Cardan*, and others about their Time, first shewed the Solution of Cubical Equations. The famous *Vieta* was the Inventor of *Specious Algebra* about 1590, which was improv'd by *Oughtred*; and the Methods of solving Quadratics, by changing the second Term, and by compleating of the Square, were, with many other excellent Improvements, introduced by the celebrated *Harriot*, who was contemporary with *Oughtred*, and flourished about the Year 1630. And so much may suffice for the Invention and Improvement of this great Art.

II. Some define *Algebra* to be, *The Art of Restitution and Comparison*. It is sometimes called, *The Analitick, or the Art of Enquiring into the fundamental Nature and Reason of Propositions, and of composing and solving Equations, and discovering Canons and Theorems accordingly.*

III. *Algebra* is either Numeral or Literal; the former, when a Question is solved by the Numbers themselves; and Literal, when done by Symbols, Letters, or Species, representing the Numbers in a Question.

What I have proposed for this first Section, I shall give the Reader in this Method.

1. Shew him the Characters and Signs used in *Algebra*.
2. Say what is necessary of the Symbols or Letters, representing Numbers in this Case.
3. Of the Notation, Numeration, or way of reading Compound *e c* Quantity.
4. Of the Definitions and Explanations of Terms peculiar to, and used in *Algebra*.

IV. The learned *Algebraists*, for the more commodious and expeditious Way of Process in solving Problems, have very wisely agreed upon *certain Characters*, whereby Quantities, or the Powers of them, are expressed and connected, and the Mind of the Artist is briefly and methodically explained, which would appear inconveniently confused, should Words at length be used: Thus,

Signs or Characters.	*Their Signification.*	*Literal Examples; as,*	*Numeral Examples; or,*
$+$	More.	$b+a.$	$3+2=5.$
$-$	Less.	$b-a.$	$3-2=1.$
$=$	Equal to	$b+a=c.$	$3+2=5.$
⊹	Transpos'd to the Right.	$a+b=c$, is $a=c-b.$	$2+3=5$, is $2=5-3.$
$\times$	Multiply'd by	$b\times a=ba.$	$3\times 2=6.$
⊹	Transposed to the Left.	$b=c-a$, is $a+b=c.$	$3=5-2$, is $2+3=5.$
$\div$	Divided by	$b\div a=\frac{b}{a}$	$3\div 2=\frac{3}{2}=1.5.$
$\backsim$	Difference between	$b \backsim a$	$3 \backsim 2=1.$
∟ (or ⊐)	Lesser than	a ∟ $b.$	2 ⊐ $3.$
⌐ (or ⊏)	Greater than	b ⌐ $a.$	3 ⊏ $2.$
$::$	*So is* (when in the middle of 4 Geometrical Proportionals.)	$b.\ a :: c.\frac{ac}{b}$	$3.\ 2 :: 5.\frac{10}{3}.$
$\div\!\div$	Continual Geometrical Proportionals.	$a.b.d.b.r.\ \div\!\div$	$1.3.9.27.81\ \div\!\div$
$\sqrt{}$	The Square Root of	$\sqrt{bb}=b.$	$\sqrt{9}=3.$
$\sqrt[3]{}$	The Cube Root of	$\sqrt[3]{aaa}=a.$	$\sqrt[3]{8}=2.$
$\sqrt[4]{}$	The Biquadrat Root of	$\sqrt[4]{aaaa}=a.$	$\sqrt[4]{16}=2.$
$\sqrt[5]{}$	The Root of the 5th Power, or Surfolid Root.	$\sqrt[5]{bbbbb}=b.$	$\sqrt[5]{243}=3.$ for
aa (or a^2)	The Square of a	$a^2=aa.$	$2^2=4.$
aaa (or a^3)	The Cube of a	$a^3=aaa.$	$2^3=8.$
$aaaa$ (or a^4)	The Biquadrat of a	$a^4=aaaa.$	$2^4=16.$
$aaaaa$ (or a^5)	The 5th Power, or Surfolid of a	$a^5=aaaaa.$	$2^5=32.$

Other Characters less used are; $a\sqrt{b}$ used in Surds, signifies a is multiplied in the Square Root of b.

(∟) Right Angle. (▭) Square. ($<$) Angle. ($\triangle$) Triangle. ($\underline{\underline{1}}$) Equilateral. (Ꙩ) Involution, obviating that such a Quantity is squared; and (*ω*) Evolution (or that the Root is extracted).

V. *The Species or Symbols* used in *Algebra*, are Letters, which are placed in an Operation to represent the Numbers given in any Question; and these are commonly Consonants, as *b*, *c*, *d*, &c. But the Letters put in an Operation by *Algebra* to stand for the unknown Numbers (or those sought for) are usually Vowels. And I shall observe this way in the following Work, but with those Limitations which I think are necessary.

1. I shall put such Consonants to stand for the known Numbers as are most expressive of the Things to which those Numbers relate; and the like for unknown, or if there is but one unknown Number sought in a Question, I shall always put *u* for it.

2. I shall not, as has been usual, put the Letters in a Product promiscuously, and without Order, but shall place them as they have Order or Priority in the Alphabet; whereby one and the same Quantity is easier seen in contracting manifold compound Quantities, and several other good Uses may be made thereof.

VI. *Of the Notation, Numeration*, or way of reading Algebraical Quantities, it may be observed from the foregoing Signs.

$+$ 1. Such as are connected with the Affirmative Sign $+$, as $a+b+c+d=p$, are to be read, *a* more *b*, more *c*, more *d*, equal to *p*.

Note, Which *p*, though it hath no Sign before, is (and the like is always to be observed) supposed to have $+$, as being always affirmative.

$-$ 2. Such Quantities as are connected with the Negative Sign $-$, as, $a-b-c$, are to be read thus; *a* less *b*, less *c*. And as in the first Example, the Numbers signified by *b*, *c*, and *d*, are to be added together to make the Number represented by *p*: So in this second Example, either the Sum of the Numbers represented by *b* and *c*, are to be deducted from *a*; or else *b* and *c* taken from *a* singly, *b* from *a*, and then *c* from the Remainer.

$\backsim$ Or suppose $b+c-d=a\backsim p$; that is, the Number represented by *d*, being deducted from the Sum of those represented by *b* and *c*; the Remainer is equal to the difference between the Numbers represented by *p*, and that unknown Number by *a*.

Note, That such Quantities as have no Numbers standing to the left hand, are supposed to have a Unit or 1; thus *b* is 1 *b*, *c* is 1 *c*, &c.

In $c-\overline{b+d}-p\backsim a$: Here the Difference between *p* and *a*, is to be deducted from the Remainer, when *b* more *d* is taken from *c*; for 'tis read, *c* less *b* more *d*, less the difference between *p* and *a*;

for

for a being unknown, it is so likewise whether the Excess lieth in a or p, therefore the $\backsim$ to represent the Difference is necessary.

3. Such Quantities as have the Sign $\times$ between them, are commonly expressed without it, and the Letters put as in a Word; thus $a \times b$, is, a multiplied in b, or ab; $c \times b \times p$, is c multiplied in b, and that Product in p, but is mostly put thus bcp; and $b \times$ in b is bb, $c \times c \times d \times d$ is $ccdd$, or c squared in d squared. $\times$

Or $a \backsim c + b \times p \times r$ is $a \backsim c + bpr$, that is the Product of b, p, and r added to the difference between a and c.

4. So all Quantities having the Sign $\div$ may be, and are most commonly expressed by way of Fraction, the Quantity to be divided being placed over the other thus, $a - b \div c + p$ will stand $\frac{a-b}{c+p}$; *i. e.* a less b, divided by c more p. $\div$

5. Surd Quantities are expressed with their proper Marks before them, as $\sqrt{g-a}$ is the Square Root of the Remainer, when a is deducted from g. So also $\sqrt[3]{p+q-r}$ is the Cube Root of the Remainer, when r is deducted from the Sum of p and q. Lastly, $\sqrt[4]{b-c+\frac{d}{q}}$, which is the biquadrat Root of the Sum when c is taken from b, and that Remainer added to the Quote of d, divided by q. $\sqrt{}$ $\sqrt[3]{}$ $\sqrt[4]{}$

But some Examples admit of a double Entendre.

As under the 2d of this 6th Head, wherein $c - \overline{b+d}$: If it were not for the Line over b and d, instead of deducting $b+d$ from c, b would only be shewed to be deducted from the Sum of c and d.

So also in the 4th above, if there were a Line over $b \div c + p$, it would stand thus $a - \overline{b \div c + p}$, which without the Line (supposing $a = 40$, $b = 20$, $c = 6$, and $p = 4$) would be but 2; whereas with the Line 'tis $= 38$, for

$a - b = 20$ And that divided by $c + p = 10$ is $= 2$	But (with the Line) $a = 40$ Less (b) Divided by $c+p$ $\} = 2$ Which deducted is $= 38$

so there is 36 difference. According to the 5th Example above, there is nothing more frequent in solving Quadratic Equations, than to have occasion in the Canon or Answer to use Lines to distinguish what Quantities are to have their Roots extracted, and which

which are not, as $u = \sqrt{\overline{bc+p}} - d$; that is, ($u$) is equal to the Remainer, when (d) is deducted from the Square Root of the Sum of (p), and (b) multiplied in (c), as if b be 16.

The Numbers to which Equation discover, from the Letters, the Value of (u), and so you must unfold any Canon of Species.

But had not the Line been drawn over $bc+p$, it would have shewed that the Root is to be extracted of $bc+p-d$, which would give too much.

$c = 3$
Product $= 48$
More $p = 16$
Sum $= 64$
Square Root of which is 8
Less $d = 3$, so ($u = 5$.)

7. In the new Notation $\frac{1}{a}$ is often expressed a^{-1}; $\frac{1}{a^2}$ is a^{-2}; $\frac{1}{a^3}$ is a^{-3}; $\frac{1}{a^4} = a^{-4}$, &c. And is read 1 divided by a; 1 divided by a squared; 1 by a cubed, &c.

Also $a^7 + b^8$, &c. is the 7th Power of a, plus, the 8th Power of b. And $\frac{1}{a+b}$ is wrote $\overline{a+b}^{-1}$, &c. $a^7+b^7 \times \frac{1}{a+b}$ is $a^7+b^7 \times \overline{a+b}^{-1}$ that is, 1 divided by a more b; and the 7th Power of a more the 7th Power of b multiplied in 1 divided by a more b.

Likewise $\frac{1}{a^2}b$, $\frac{1}{a^3}b^2$, &c. is in the new Notation $a^{-2}b$; $a^{-3}b^2$, &c. that is, 1 divided by the Square of a multiplied in b; 1 divided by the Cube of a multiplied in b squared.

And $\frac{a^5+b^5}{a+b} = a^5+b^5 \times \frac{1}{a+b} = a^5+b^5 \times \overline{a+b}^{-1}$; *i.e.* the Sursolid of a, more that of b, divided by a more b; is expressed also $a^5+b^5 \times \frac{1}{a+b}$. Or rather $a^5+b^5 \times \overline{a+b}^{-1}$. And for Proof suppose $a=10$, and $b=20$; then is $a^5+b^5 = 3300000$; and $a+b$ being $= 30$, $3300000 \div 30, = 110000$. And so likewise is $3300000 \times 1 \div 30$: or $3300000 \times .3\,r = 109999.9\,r$ or 110000: which plainly proves the Truth of the manner of new Notation.

VII. An Alphabetical Explanation and Definition of most Terms used in *Algebra*, which the Reader may have recourse to as he finds occasion; for having well understood what is above in this Section, he may pass this Head, and proceed to *Addition*, &c.

Absolute

Absolute Quantity, or *Absolute Number*,] Is such a known one as possesseth one Side of an Equation, as $u + cu = b$, here b is the Absolute Quantity, whose Value is known, and is free from any Dependance or Power of any other. Or it is 1 marked with the Sign $+$, contrary to Negative marked $-$.

Affirmative Quantities.] Such as have either no Sign before them, or are marked with $+$; so $a + b$ are both Affirmative Quantities, as by the last.

Algorism,] Is the Operations in the several Parts of *Algebra* practically handled.

Algorithm.] The Parts of single Arithmetick, as *Numeration*, *Addition*, *Substraction*, &c.

Arithmetic of Infinites.] The same as infinite Series, or converging Series.

Adfected Equations.] When in a Quadratic Equation, (which see *Sect.* 10. in this Chap.) the unknown Quantity is in one Term, and its Square in another.

Analogies.] The same as Proportions, as the Analogy of these four Quantities, $a . b :: c . d$, &c. and because the Rectangle of the two Extremes is equal to that of the means; therefore $ad = bc$, the Analogy being converted to an Equation.

Approximation.] See *Converging Series.*

Arithmetic of Negatives.] See *Negative*, under *N.*

Binomial Quantities or *Roots.*] Those consisting of 2 Names, Terms, or Members, connected together by the Sign $+$, as $u + d$, &c.

Coëfficients.] Those Numbers or known Quantities in a Quadratic, *&c.* Equation, which are (in the 2d Term commonly) multiplied in the Root of the first or unknown Quantity, as $uu + 2bu$, here $2b$ is the *Coëfficient*: And note, That where there is no Number nor Quantity multiplied in the Root in the 2d Term, then 1 or a Unit is the *Coëfficient*, as $uu + u$ is $= uu + 1u$. See *Compleating the Square.*

Compound Quantities.] Are such as are either the Product, Sum, or Difference of two or more single Quantities: Thus (up) is a Rectangle compounded of u and p, $u + d$, or $u - d$, the Sum or Difference of those two Quantities u and d.

Compleating a Square.] This is to square half the Coëfficient: As suppose two Members of a Square $uu + bu$ be given; to find what Quantity must be put to compleat the Square, is to square half the Coëfficient or $\frac{b}{2}$, which Square is $\frac{bb}{4}$, so that $uu + bu + \frac{bb}{4}$ is a compleat Square: For Proof of which you may multiply

$u+\frac{b}{2}$ (which is the Root of that Square) in itself. And if this were a Quadratic Equation, this Square of half the Coëfficient must be added to both Parts of the Equation, as admit $uu+bu=c$, half the Coëfficient being squared and added, will make it $uu+bu+\frac{bb}{4}=c+\frac{bb}{4}$.

Composing an Equation,] Is the way of proceeding (to resolve Problems) till we make some Quantities in the Question equal to some others; (see *Resolution of Equations*) as if I were asked, what Number that is which being multiplied by 7, and that Product divided by 3, the Quotient will make 1000, abating half the Number sought? Here, to compose this Equation, I put u for the unknown Number, or that sought; then 7 times that is $7u$; that divided by 3, makes $\frac{7u}{3}$. Then putting b to represent 1000, this Equation is composed from the Nature of the Question, *viz.* $\frac{7u}{3}=b-\frac{u}{2}$. (See this Equation resolved under the Word *Resolution.*) And where the Quantities in the Proposition or Problem can be reduced naturally into Analogy, then that Analogy is reduced, or an Equation composed from it: See *Analogy.*

Construction of an Equation,] Is to demonstrate Geometrically the Truth of the Equation, and Canon, from Lines or Geometrical Figures laid down for that purpose, as simple and quadratic from the Sides of the Triangles and Arcs of Circles; and cubical from the Parabola and Circle, *&c.*

Conversion of Equations,] Is properly the converting of an Equation into that which is not so, *i. e.* into Analogy, as if $ad=bc$; then it follows, that as the Quantity a is to c or b, so is the other of them to d, *viz.* $a.\ b :: c.\ d.$ See *Analogy*, of which Example this is the Converse, and they ought both to be remember'd. But some call this following, and the like, a Conversion, (but I call it Reduction by Multiplication, of which more under *Reduction of Equations,*) viz. $\frac{u}{r}+cd=p$, this by multiplying each Member by the Denominator r, gives $u+cdr=pr$, &c. of any other.

Converging Series.] Lines converge, when two right ones proceed nearer and nearer to each other, till they meet in a Point where they

they make an Angle: So in Numbers, it is the orderly approaching nearer and nearer to the Truth (as to the Root of a surd Number or Quantity) by the common Rules. Examples hereof, see *Section* the 13th following: *Note*, Sometimes this Method of Process is called also *Arithmetic of Infinites*, *Infinite Series*, *and Approximation.*

Contraction.] This in *Algebra* is to leave out all repeated or superfluous Terms, or single Quantities; as $4ab-3c+5c-3ab$, this contracted is $ab+2c$; for $4ab$, less $3ab$, leaves ab; and $5c$, less $3c$, leaveth $2c$. So also $7uu+3pu-du-3pu$ is $7u-d$; for first, u being in each Member $1u$, may be left quite out, and then 'tis $7u+3p-d-3p$; secondly, $+3p$ and $-3p$ destroy each other: So there rests but $7u-d$ of the Total, and the like may be observed in Contraction of other Quantities.

Calculus.] A Term in Fluxions: It is either *Calculus Integralis*, which is the Method of finding a flowing Quantity of any Fluxion given; or else *Calculus Differentialis*; which is the way of finding a Fluxion of any flowing Quantity.

Canon.] As it relates to *Trigonometry*, is the Rule of Proportion by which an Angle or sides of a Δ is found; the Tables of Artificial Numbers (Logarithms, Sines, and Tangents) are sometimes called *Canons*; but in *Algebra* it is a resolved Equation, being the Result of an Operation, and is the Value of some unknown Quantity in known Species, or Numbers, which being wrote down in Words, contains a plain general Rule for the Solution of any Question of like Nature; which is one of the great Excellencies of *Algebra*.

Cossick Powers.] Algebraick Powers of Number or Quantity; for the first, see *Chap.* I. *Sect.* 1. of this Treatise; whence it follows, that the Powers of any Quantity will stand thus, $u=$ a Root; $u^2=$ the Square or second Power; $u^3=$ the Cube or third Power; $u^4=$ the Biquadrat or fourth Power of u, &c. See Extraction of Roots in this Chapter, *Sect.* 6.

Cubic Equations.] Vide *Sect.* 12. of this Chapter.

Depression.] This Word is used in the Solution of Cubic &c Equations, where the Equation is depressed or reduced from one of the third Dimension to a Quadratic; that being one of the ways of resolving Cubical Equations: Or Depression is the reducing of an Equation by Division; as $puu+duu=cu-du$ by dividing by $p+d$, that Equation is reduced to $u=\frac{c-d}{p+d}$, first expunging or casting out u, because found in each Member or Term of the Equation: See this Chapter, *Sect.* 5.

Discrete Proportionals] Are Numbers or Quantities, where there is the same Proportion between the third and fourth, as between the first and second; but not the same between the second and third, as in *u. b* : : *b. k*, there is the same Analogy between *b* and *k*, as between *u* and *b*, but not the like between *b* and *b*; so that 'tis also called *Disjunct Proportion* in Number, 3. 7 : : 12. 28.

Dimensions.] In Equations, where the first Term is *uuuu*, that is one of 4 Dimensions; *uuu* is an Equation of 3 Dimensions; *uu* of 2, &c. *u* being unknown.

Equation.] An Equation in *Algebra* is, when one or several known or unknown Quantities are made equal to other Quantities upon a due and regular Process of *Composing*, (see that Word,) as if $u+c-d=rs+tu$, that is, *u* more *c* less *d* is equal to *r* multiplied in $s+tu$, which Equation is resolved as under *Resolution*; see also Simple, Quadratic, Cubical and Lateral Equations, in this part.

Evolution] Is marked *w*, (or the Extraction of a Root) and tells us, that the Number or Quantity before which it stands, is the Square Root of some other Step referred to in an Algebraical Operation.

Exponents] Are those small Figures placed almost over any Quantity, to shew (or expound) what Power of the said Quantity is expressed. See *Cossic Powers* above. And

Exponential Quantities] Is a Term in Fluxions, these being such Quantities, whose Exponents are variable or flowing Quantities.

Fluxions] Are defined to be the Doctrine, or Algorithm of the infinitely small Increment or Decrement of variable and indeterminate Quantities: For the best Notion we have of generating a Line, is by the Motion of a Point; of a Surface by the Motion of a Line; and of a Solid by the Motion of a Superficies: Now as this Increase of a Figure by Local Motion, is termed a *flowing Quantity*; so the Velocity of that Increase of the flowing Quantity is called *a Fluxion*; and is distinguish'd from the Character or Symbol of the flowing Quantity by a Point over it, &c. which is a kind of Artificial *Algebra*, but is wrought by Rules peculiar to itself, being a late Invention and a great Improvement.

Flowing Quantities.] See *Fluxions* above.

Generating.] In Geometry Points Lines and Superficies generate as under the word *Fluxion*; or in Arithmetic, what is produced by Multiplication or Involution is a *generated Number or Quantity* in *Algebra*.

Homogeneal] Is *of the same kind:* So such surd Roots as have the same radical Sign are *Homogeneal*, as $\sqrt{ab}$, $\sqrt{ac}$ ec; $\sqrt[3]{cd}$, $\sqrt[3]{hrke}$; $\sqrt[4]{lp}$, $\sqrt[4]{g}$, &c. See this Word in the *Introduction*, *Sect.* 1.

Homologous.]

Homologous.] In a Proportion of four Quantities. If $u . b :: c . d$. Here the two Antecedents u and c are *Homologous*, as are the two Consequents b and d, *&c.* and u is *Homologous* to c, as b to d.

Heterogeneal.] (*Of different kind*;) So Surds are *Heterogeneal* when their radical Sines are not the same, as $\sqrt[3]{bc}$; $\sqrt[4]{tu}$, $\sqrt[5]{rst}$, &c.

Infinite Series.] See *Converging Series* above, and *Chap.* 10. following, *Sect.* 13.

Involution. Is marked ⓐ, and shews that the Quantity before which it stands is the Square of some other which is referred to; the contrary to *Evolution.*

Ineffable Quantities.] Surds, whose Roots can only be expressed by the proper radical Signs.

Irrational Quantities.] The same as *Ineffable*, last above.

Lateral Equations.] Simple Equations which have but one Root, or in figurate Numbers they are those of the first Order, as 1, 2, 3, 4, *&c.* See *Sect.* 14 of this Chapter.

Like Signs.] Those Quantities in *Algebra* which have the same Sign, as $+b+c+d$, or $-b-c-d$, *&c.*

Like Quantities.] Such as have the same Symbols or Species, as $23bcd$, $14bcd$, $2bcd$, &c. there being the same Number of each Letter.

Maximis and Minimis] Is a Term used in *Fluxions.* It is a Method whereby Problems are resolved, which require the greatest or least Quantity attainable in that Case; it is a stable Quantity naturally: and therefore to determine to a *Maximum* or *Minimum* in any flowing Quantity, is to make it a permanent Quantity.

Members of an Equation.] Those Quantities contained between every Sign, as $ab+du-cp=r$; here ab, du, cp, and r, are the four Members or Terms of this Equation.

Moments.] A Term in Fluxions according to some; being such Parts of Quantity as are in a continual Flux, either decreasing or increasing: Or rather, Moments are the beginning (of the generating Principle) of Magnitude, and are themselves no Magnitude conceiveable, because infinitely small.

Multinomial.] See *Polynomial.*

Negative Signs.] See under the last Article (or Head) among the Characters used in *Algebra* for (—).

Negative, or the Arithmetic of Negatives,] Is that kind wherein Negative Numbers or Quantities are employed, which are known by this mark (—) before them.

The *Numeration* is no more than only adding the Words [lefs than nothing] to what is wrote down; as $-a$, or -7 is read *a* lefs than nothing, or 7 lefs than nothing, and $-.75\ ec$ is that Decimal lefs than nothing.

Addition] Shews the adding of Negative to Negative or Affirmative, as by the Rule of adding the Quantities of like Signs together, and taking the Difference for the Sum; as the Sum of $-7+-3$ is -10; $-7+3$ is -4; and $12+-13=-1$.

Substraction] Is done by changing the Sign of the Subtrahend, and then adding the Quantities; as from -7 take -3, refts -4; from -7 take -3, refts -10; and if from 12 you take -13, there refts 25, *&c.*

Multiplication] Is nothing difficult, for you only obferve, that if Quantities of like Signs be multiplied, the Sign of the Product is Affirmative or $+$; but if the Factors have unlike Signs, the Sign of the Product is Negative or $-$: Thus -7 by -3 is 21, and fo $+7$ by $+3$; but 7 by -3 is -21, and -12 by 13 is -156, *&c.* And the fame Rule ferveth for

Division of Negatives.] As to prove thofe in Multiplication, $21 \div -7$ quoteth -3; $-21 \div -3$ quoteth 7; and $-156 \div 13 = -12$. All which you'll find farther illuftrated, with variety of Examples, *Sect.* 3. of *Chap.* 7. alfo *Sections* 2, 3, 4, and 5 of this *Chap.* X. and the laft of my Examples in *Converging Series* near the end of the Book.

Nome.] Whence Binomial, Trinomial, and Multi- (or Poly-) nomial, *i.e.* Roots of 2, 3, or more Names; fee thofe Words.

Numeral Algebra.] When the Work is performed Algebraically with Figures or Numbers, and not Letters.

Periodical Degrees, or Parodical.] Of the Terms of a Cubical *ec* Equation, are when the Exponents of the Powers therein rife or fall gradually in an Arithmetical Ratio, as $u^4 - 59u^3 + 1154u^2 - 8080u = -12000$, *&c.* are regular Biquadratics, having all their Terms in a gradual Order.

Permanent Quantities.] In Fluxions fuch Quantities as are invariable without Motion, or at Reft, and confequently have no Fluxions to them.

Powers of Quantities.] See that of Numbers, *Chap.* I. *Sect.* 1. or Exponents here below; as *u* or *c*, *&c.* are Roots or firft Powers, *uu* or *cc* are Squares or fecond Powers, *uuu* or *ccc* the Cubes or third Powers.

Power

Power to raiſe the Powers of Numbers to that of any other.] As ſuppoſe I raiſe the Root b to the ſame Power as $cccc$ or c^4, this is to multiply b in itſelf *ec*, to bring it to the fourth Power or Biquadrat: Or ſuppoſe $\sqrt[3]{82}$, were to be multiplied by 2, here 2 muſt be raiſed up to the ſame Power or Cube with the 82, and it will ſtand either $2\sqrt[3]{82}$, or $\sqrt[3]{8} \times \sqrt[3]{82} = \sqrt[3]{656}$; this will be of Uſe to obſerve, when you come to the Doctrine of ſurd Quantities, *Sect.* 7. of this Chapter.

Polynomial Roots or Quantities.] Thoſe which conſiſt of above three Names or Numbers; as $u+s+t+n$, &c.

Poſitive Quantities,] The ſame with Affirmative; which are thoſe that are marked with or follow the Sign $+$, or elſe ſuch as have no Sign before them.

Quantity.] In *Algebra,* either that ſought for, or thoſe given in a Queſtion, or they are all thoſe Symbols which repreſent any Number. See *Simple, Compound, Surd,* and *Variable Quantity.*

Quadratic Equations] Are ſuch (or reduceable to ſuch) as have the Square of the unknown Quantity in the firſt Term, and the Root (or that multiplied in ſome known Quantity) in the ſecond Term, as $uu+bu+c=rs$. See two Sorts, *Sect.* the 10th of this Chapter.

Rational Quantities.] Either ſuch whoſe Roots can be extracted accurately; or ſuch as are already the Root of ſome Quantities, or which want no Extraction.

Raiſe the Powers of Number or Quantity.] See *Powers.*

Radical Signs] Are $\sqrt{}$ for the Square Root; $\sqrt[3]{}$ for the Cube; $\sqrt[4]{}$ for the Biquadrat Root, *&c.* ſee *Roots* and the *Powers* in the Article preceeding, of Algebraical Characters: And alſo

Roots of Quantities.] Are either (in *Algebra*) put actually down (if of Rational Quantities) or if of Surd Quantities the Root is expreſſed by putting the proper Radical Sign before the Quantity; thus in the firſt Caſe, the Square Root of bb is b, of cc is c, &c. the Cube Root of uuu is u, &c. and in the ſecond Caſe, the Square Root of $u+s$ is $\sqrt{\overline{u+s}}$; the Cube Root of $u+s$ is $\sqrt[3]{\overline{u+s}}$; the Biquadrat Root of it is $\sqrt[4]{\overline{u+s}}$, and the like of any other Surd Quantity. See *Binomial, Trinomal, Polynomial,* and *Reſidual. Note,* That the Surd $c\sqrt{d}$, ec, is c multiplied in the Square Root of d, ec.

Reſolution

Resolution of (or to solve) Equations.] After an *Algebraist* has done with the composing an Equation, as under that Word, he resolves it by several Rules requisite thereto, as by *Multiplication*, *Division*, *Addition*, *Substraction*, *Evolution*, &c. so the Equation composed under the word *Composing* as above, which is $\frac{7u}{3} = b - \frac{u}{2}$; being resolved by multiplying by the Denominators of the Fraction, and adding $3u$ to each Part of the Equation, dividing by 14 and 3 (or 17) gives the Solution of the Question, or the Canon $u = \frac{6b}{17}$. Now b we find is $= 1000$ in the Question, so u the unknown Quantity is easily found by dividing 6000 by 17. And the Equation under that Word, *viz.* $u + c - d = rs + tu$ being resolved, is $u = \frac{d - c + rs}{t - 1}$. Particular Rules for resolving Equations, see *Sect.* the 9th of this Chapter; where it appears, that an Equation is resolved when the unknown Quantity alone possesseth one Side of the Equation as here u doth.

Residual Roots or Quantities.] Such as have a Negative Sign between them, as $be - d$, &c. or $40 - 10 = 30$ the Residue.

Simple Quantities.] In a strict Sense are those single Letters which represent the several Numbers in a Problem, given or required: Or some take them in a larger Senfe, for such Quantities as are not connected by $+$ or $-$ to any other, though they be multiplied in other; of the first sort are b, or c, or d, &c. of the second are $4b$, bc, drs, &c.

Signs.] See the *Table of Characters*, Article IV. of this Section; also *Affirmative*, *Negative*, and *Radical*.

Simple Equations.] Such as are free from the Involution or Powers of the unknown Number or Quantity.

Simple Quadratic Equations.] Such as have (or are reduceable to have) the Square of the unknown Number or Quantity in only one of the Members or Terms, as $uu = pq$, that is, $u = \sqrt{pq}$, or $uuu + ub + udc$, which by the Seclusion of u in each, is $u = \sqrt{b + dc}$.

Series.] A Rank, Row, or Column of Numbers or Quantities orderly placed. See *Progression and Converging Series.*

Species.] In *Algebra*, are the Letters which represent the Numbers in a Question, they are also called *Symbols*; so specious Arithmetic is not Numeral, but Literal *Algebra*.

Substitution.]

Substitution] Is the putting one Symbol or Quantity into the Progress of Algebraical Operations, in the room of some other: As in the Solution of a Quadratic Equation, by changing the second Term, and as in Fluxions.

Surd Quantities.] Such as cannot have the Root expressed, and are therefore marked with their proper *Radical Sign*, which Term see above, and also *Roots of Quantities.*

Terms] *Of an Equation*, are commonly said to be the several Members thereof; (see that Word above) but if in a Quadratic, as $uu + ub + ud = k + c$, the Terms are called but three Terms, as uu the first; $ub + ud$ (because the known Quantities are multiplied in the same power of the unknown) the second; and $k + c$ the third Term.

Transposition] Is the changing the Places of the Members of an Equation, in order to make the unknown Quantities to possess one Side, and the known the other Side of the Equation; whereby the Knowledge is gained of what Value the Number sought in a Question, really is, by finding what it is equal to. See *Sect.* 9. of this Chapter, for Rules to perform the same.

Trinomial Roots or Quantities] Are such as have three Nomes or Names, as $u + rs + tu$, &c.

Vanish, or be Wanting.] A Quantity is so, when by Reduction or Contraction it is expunged or thrown out of an Equation: Or as in a Cubic, &c. When the highest and lowest Powers of the unknown Quantity are only expressed, as $x^3 + x$: here x^2 is wanting.

Variable Quantities] Are such as are supposed in continual Motion, whereby they generate Lines, Superficies, and Solids, the same as *Flowing Quantities.*

Unciæ] Are the Numbers prefixed to the Quantities, which are the Powers of some Binomial Roots; as in the Square of $a + b = aa + 2ab + bb$, the *Unciæ* is 1, 2, 1, for aa and bb are 1 aa and 1 bb; so in the Cube of $a + b$, viz. $a^3 + 3aab + 3abb + bbb$, the *Unciæ* is 1, 3, 3, 1, &c. for according to Sir *Isaac Newton*'s Rule, the *Unciæ* of any Powers are found (as suppose the third Power or Cube) by this Rule; $1 \times \frac{3-0}{1}\,(3 \times \frac{3-1}{2}\,(3 \times \frac{3-2}{3}\,(1$, which in Words is read thus: One multiplied in 3 the Power, less 0, divided by 1 is 3 = the second of the *Unciæ*; 3, the second, multiplied in 3 the Power, less 1, divided by 2, gives 3 = the third of the *Unciæ*; and 3, the third multiplied in the Power 3, less 2, divided by 3, gives 1 = the fourth of the *Unciæ.*

And

And by the same Rule, the *Unciæ* of the Surfolid is 1, 5, 10, 10, 5, 1, for every Quote and the first 1, are the *Unciæ* arising.

This $(1\times\frac{5-0}{1}\ (5\times\frac{5-1}{2}\ (10\times\frac{5-2}{3}\ (10\times\frac{5-3}{4}\ (5\times\frac{5-4}{5}\ (1$; see also *Sect.* 6.

Vinculum] Is a Compound Surd Quantity multiplied by a Fluxion; in which this is a Term.

Unfold.] To unfold an Algebraick Canon, expressed Literally, is to compare the known Quantities with the Numbers which they represent; and working with them, as the Canon directs, we discover the Value of the unknown Number, or that sought.

Wanting.] See *Vanish.*

Having in this Section laid a plain and copious Foundation; I shall be something briefer in the subsequent Rules, but no farther than is consistent with evident Demonstration.

SECT. II. *Addition of Algebra.*

Note $b = 70$

Example 1.

Literal	$9b = 630$	Numeral	Add
	$10b = 700$		
	$b = 70$		
Sum	$20b = 1400$	Sum or Proof.	

Note $c = 35$

Example 2.

Literal	$-12c = -420$	Numeral
	$-7c = -245$	
	$20c = 700$	
Sum	$= c = 35$	Sum or Proof.

Compound Quantities. *Note*, $d=4$; $g=16$; $b=6$.

Example 3.

$$\begin{array}{rcrr} 3ddb + bbg & = & 288 & +\ 576 \\ ddb - 2bbg & = & 96 & -\ 1152 \\ -5ddb + bbg & = & -480 & +\ 576 \\ \hline \text{Sum} - ddb\quad 0 & = & -96 & +\ 0 \end{array}$$

Sum for Proof.

The General Rule for *Addition.*

That the Quantities of like Signs are to be added together, and where there are different Signs, as in the second and third Examples, you must substract the Sum under one Sign from that under another, and put the Difference down marked with the same Sign as that Sum, wherein was the Excess: As I deduct $19c$ from $20c$ (in the second Example) and the Difference is c for the Sum required.

And in the third Example, (*Col.* 1.) the Excess lies in the Negative Sign, for taking $4ddb$ from $-5ddb$, the Remainer is $-ddb$ (or $-1ddb$.)

And

And in the second Series of Quantities in that third Example, the Signs making the Quantities to balance; I therefore put (o) down, all which is proved by the respective Numbers.

And whereas it may be thought strange, that a Sum Total should be ddb, or 96 less than nothing; it is no more than what falls out in many Cases: For if a Merchant draws on his Factor 480 *l.* (as the Number is in *Example* 3.) and that Factor has but 288 *l.* and 96 *l.* = *viz.* 384 *l.* to pay the Bills in his Hands; 'tis plain, when the Factor has paid what is drawn on him, the Merchant will have 96 *l.* less than 0 in the Factor's Hands, *&c.* And this is also plain from the Reason given for adding Affirmative and Negative Indices of Logarithms, *Chap.* IV. foregoing; which they who have considered, the Addition and Substraction of *Algebra* will to them appear evident and reasonable.

And when Numbers are to be added that have different Quantities annexed, there is nothing to do but to put them down in one Line with their Signs between them; as to add $5ab$ to $3ad$, is $= 5ab + 3ad$; and ubc to rst is $-rst + ubc$, &c. And the Quantities in the foregoing Examples are truly added by the same way, being afterward contracted as under the word *Contraction* in the last Section foregoing.

SECT. III. *Substraction of* Algebra.

The General Rule, &c. *see after the* Examples.

Example 1.		Example 2.	
Note, $b = 70$			
From $20b$	$= 1400$	From $c =$	35
Literal =	= Numeral	Take $-19c =$	-665
Take $11b$	$= 770$		
Rests $9b$	$= 630$	Rests $20c =$	700

Note (as in Addition of Compound Quantities) $d = 4$; $g = 16$, and $b = 6$.

Example 3.

	Literal	Numeral
From	$-ddb$	$= -96$
Take	$-5ddb - 2gbb$	$= -480 - 1152$
Rests	$4ddb + 2gbb$	$= 384 + 1152$

And as a fourth Example; if from $4ddb + gbb + bb$, I take $bb + gg - bb - d$. The Remainer will stand thus:

$$4ddb + gbb + bb; -bb - gg + bb + d.$$

For

For Proof of which. By the last Section it appears that $4ddb = 384$
From this Sum 5860 taking the Sum of the four Quantities in the Subtrahend (added according to the Rules of Addition,) which is 636, the Remainer is 5224 as by the Margin
And so it will be by this Rule.

$$\begin{array}{rr} 4ddb = & 384 \\ gbb = & 576 \\ bb = & 4900 \\ \hline \text{Sum} & 5860 \end{array}$$

$$\text{viz.}\ \left.\begin{array}{r} \left.\begin{array}{r} +bb=420 \\ +gg=256 \end{array}\right\} 676 \\ \left.\begin{array}{r} -bb=\ 36 \\ -\ d=\ \ 4 \end{array}\right\} -40 \end{array}\right\} \begin{array}{r} =636 \\ \hline 5224 \end{array}$$

The General Rule for Substraction, *i. e.*

Change the Sign of the Quantities to be substracted, and then proceed as in Addition.

So in the first Example, $20b - 11b = 9b$, for $1400 - 770 = 630$; which is $9b$, or 9 times 70.

And in the second Example, $c + 19c = 20c$, or $35 + 665 = 700$; which is 20 times 35.

In the third Example; I change the Sign of $2gbb$, (or 1152), and add the same to (0), makes $+2gbb = +1152$; and $-5ddb$, or -480 having the Sign changed and added to $-ddb$, and -96 gives $4ddb$, or $384 =$ the Remainer as you see. And

In the fourth Example; If the four Signs of the Subtrahend be changed and made as in the Remainer above, that Remainer will prove true and agreeable to the Deduction foregoing, altho' it be done differently and according to the General Rule for Substraction of Algebraical Integers, as by the Margin appears, where $5860 - 636 = 5224$.

Note, These Examples prove the Truth of those in Addition.

$$\begin{array}{rr} 4ddb = & 384 \\ gbb = & 576 \\ bb = & 4900 \\ \hline \text{Sum} = & 5860 \end{array}$$

$$\text{And}\ \left.\begin{array}{r} \left.\begin{array}{r} -bb=-420 \\ -gg=-256 \end{array}\right\} -676 \\ \left.\begin{array}{r} +bb=+\ 36 \\ +\ d=+\ \ 4 \end{array}\right\} +\ 40 \end{array}\right\} =-636$$

$$\text{Rests} = \ 5224$$

The Reason of the General Rule for Substraction.

1st, Either the Sign of the Subtrahend is $+$ or $-$, if $+$ as to take 2 from 6, or $\left.\begin{array}{r} +6 \\ +2 \end{array}\right\}$ to take 2 from 6 is (speaking Algebraically) to say $6 - 2$ (or 6 less 2) whereby the Sign of the 2 is of Course changed,

2dly, And

2*dly*, And if the Sign of 2 (ec) as given to be ſubſtracted were (—), then by what is ſaid of Subſtraction of Indices of Logarithms, as well as from the Reaſon of the Thing itſelf; if I take (0) from 6, there will remain 6; but if — 2 (2 leſs than 0) from 6, 'tis the ſame as to add 2 to 6, ſo there muſt remain 8; for by how much the leſs any Number is that is deducted, by ſo much the greater is the Remainer; and by how much ſoever the Number ſubſtracted is leſs than 0, by ſo much does it make the Remainer greater than the Sum ſubſtracted from.

SECT. IV. *Multiplication in* Algebra.

Note, $u=25$; $b=12$; $c=20$; $d=4$; $g=5$; $h=10$.

A General Rule, ſee after the Examples.

Prop. 1. To multiply ſimple Quantities by Simple.

Case 1.
Multiply $-u = 25$
by $-b = 12$
Product $bu = 300$

Case 2.
Multiply $d = 4$
by $g = 5$
Product $dg = 20$

Case 3.
Multiply $-c = -20$
by $h = 10$
Product $-ch = -200$

Case 4.
Multiply $u = 25$
by $-c = -20$
Product $-cu = -500$

Prop. 2. To multiply compound Quantities by Simple.

Caſe 1.
Multiply $2bu + d = 500 + 4$
by $c = 20$
Product $= 2cbu + cd = 10000 + 80 = 10080$

Caſe 2.
Multiply $-cd - h = -80 - 10$
by $-c = -20$
Product $+ccd + ch = +1600 + 200 = 1800$

Caſe 3.
Multiply $2bu + d = 500 + 4$
by $-c = -20$
Product $-2cbu - cd = -10000 - 80 = -10080$

Prop. 3. To multiply compound Quantities by Compound.

Case 1. Multiply $bb+g = 100+5 = 105$
by $gg+b = 25+10 = 35$

$$\begin{array}{ll} ggbb+ggg & 525 \\ \quad +bbb+gb & 315 \end{array}$$

Product $ggbb+ggg+bbb+gb = 3675$ Product.

Case 2. Multiply $-bb-g = -100-5 = -105$
by $-gg-b = -25-10 = -35$

$$\begin{array}{ll} ggbb+ggg & 525 \\ \quad +bbb+gb & 315 \end{array}$$

Product $ggbb+ggg+bbb+gb = 3675$ Product.

Case 3. Multiply $bb+g = 100+5 = 105$
by $-gg-b = -25-10 = -35$

$$\begin{array}{ll} -ggbb-ggg & -525 \\ \quad -bbb-gb & -315 \end{array}$$

Product $= -ggbb-ggg-bbb-gb = -3675$ Product.

To multiply *Cossick Powers*, see at the end of *Division* of *Algebra*.

The General Rule for Multiplication.

If the Signs of both the Factors are like, then the Sign of the Product is Affirmative: But if the Signs of the Factors are different, the Sign of the Product is Negative.

This Rule will appear to be observed in the nine Cases of *Multiplication* as above; which I have demonstrated by Numbers, which agree to the Quantities of the Factors and Products: For Instance, the Product of the first Case of *Prop.* 3. is as in the Margin, where the Value of the Product is equal to the Rectangle of those Numbers 105 and 35 (the Values of the Factors) being multiplied together.

Quantities.	Values.
$ggbb =$	2500
$ggg =$	125
$bbb =$	1000
$gb =$	50
Sum $=$	3675

The

The Reason of the General Rule for Multiplication.

1st, *Multiplication* being nothing but the Work of many Additions; it follows that $-ab$ being added to $-ab$, is $-2ab$, which is the same as multiplying $-ab$ by 2; and so $-gb$ $-2gb$ $-3gb$ added, is $= -6gb$, which is the same as $-gb \times 6$; which shews, that the Negative Quantity $-gb$ by the Affirmative 6, must give the Negative $-6gb$, and the like Reason holds for any other Negative by a positive Quantity.

2dly, And as in *Logic*, two Negatives make an Affirmative; so in *Algebra*, one Negative Number or Quantity multiplied by another Negative, produceth an Affirmative: For the Sign — being directly contrary to +, must make a Product of a direct contrary Nature when the Multiplier is —, then (as above) when it is +; as substracting a Negative, is adding really so much as we seem to substract, as -2 from 4 is 6.

SECT. V. *Division of* Algebra.

I shall make use of the same Letters and their Values as in the last Section, and the Examples shall be such as to prove both themselves and those in the last Section.

Prop. 1. To divide a simple Quantity by a Simple.

Case 1. Divide $bu = 300$	$-b)\ bu\ (-u$ = Quote = 25; for	$-12)\ 300\ (-25 = -u$
by $-b = -12$	bu	60
	0	0

Case 2 Divide $dg = 40$	$g)\ dg\ (d$ the Quote; for	$5)\ 40\ (8 = d$
by $g = 5$	dg	0
	0	

Case 3. Divide $-cb = -200$	$b)\ -cb\ (-c$ the Quote for	$10;)\ -200\ (-20 = -c$
by $b = 20$	$-cb$	00
	0	

Prop.

Prop. 2. To divide a Compound be a simple Quantity.

		Quote.
Case 1. Divide $2cbu + cd = 10000 + 80$ by $c = 20$	$c)\,2cbu + cd\,(2bu + d$ $2cbu$ $+cd$ cd o rests.	$20)\,10080\,(504$ $= 2bu + d$
Case 2. Divide $ccd + cb = 1600 + 200$ by $-c = -20$	$-c)\,ccd + cb\,(-cd - b$ ccd $+cb$ $+cb$ o	Quote. $-20)\,1800\,(-90$ $= -cd - b$
Case 3. Divide $-2cbu - cd = -10000 - 80$ by $-c = -20$	$-c)\,-2cbu - cd\,(2bu + d$ $-2cbu$ $-cd$ $-cd$ o	Quote. $-20)\,-10080\,(504$ $= 2cb + d$

Prop. 3. To divide compound Quantities by Compound.

Case 1. Divide $ggbb + ggg + bbb + gb = 3675$ by $gg + b = 35$

Quote. $gg + b)\,ggbb + ggg + bbb + gb\,(bb + g$ $ggbb + bbb$ Deduct $ggg - bbb + bbb + gb$ rest of the (Dividend. Or $= ggg + gb$, if Contracted. $ggg + gb$ Deduct. o rests.	Quote. $35)\,3675\,(105 = bb + g$ 175 o

Case 2. Divide $ggbb + ggg + bbb + gb = 3675$
by $-bb - g = -105$

$-bb - g)\,ggbb + ggg + bbb + gb\,(-gg - b$ = Quote. $ggbb + ggg$ $o + bbb + gb$ $+bbb + gb$ rests o	$-105)\,3675\,(-35$ = Quote. $= -gg - b$ 525 o

Case

Case 3. Divide $-gghh-ggg-hhh-gh$
by $-gg-h$

$-gg-h)-gghh-ggg-hhh-gh\,(hh+g=$ the Quote.	$-35)-3675(105$
$-gghh-hhh$	
$-ggg+hhh-hhh-gh=$ the rest of (the Dividend.	175
Or $-ggg-gh$ if Contracted.	0
$-ggg-gh$	
0 rests.	

General Rule as to the Signs.

It is easily seen by the 9 Cases above, that if you divide Quantities by those having like Signs with those divided, then the Signs of those put in the *Quote* are + or *Affirmative*; but if the Signs of the *Dividend* and *Divisor* are unlike, then the Sign of those Quantities put in the *Quote* are *Negative*.

General Rule as to the Quantities.

Proceed in the Method of common *Division*, putting those Quantities in the *Quote* which are not in the *Divisor*, as in the last Case; I ask how often $-gg$ in $-gghh$? the Answer therefore is hh, which I put in the *Quote*, and multiplying the *Divisor* by hh, is $=-gghh-hhh$ (in the last Case above), so the *Remainer* is $-ggg+hhh$ (because the Sign must be changed of hhh, to deduct it from $-ggg$); then because here is $+hhh-hhh$, they destroy one another, (as is shewn under the Word *Contraction*, in *Sect.* 1. of this *Chap.*) so there rests but $-ggg-gh$: and I find I can have $-gg$ of the *Divisor* in $-ggg$ of the *Dividual* $+g$, by which multiplying the *Divisor* $-gg-h$, produceth $-ggg-gh$, which deducted from the last compound Quantity contracted, leaves 0, or nothing for a *Remainer*.

But tho' this is pretty enough, and serves to shew the Coherence of *Division* in *Algebra*, with that in other kinds of *Arithmetic*; yet it is far from falling in use, so much as where the *Divisor* in Species will not divide any part of the *Dividend*; or especially where it wont divide it all, but some part; and the rest as a Fraction of the *Divisor* being put under the remaining part of the *Dividend*.

The first Kind of more common Cases in Division *are these.*

To divide gh by r, quoteth $\frac{gh}{r}$; so $pr+st-m$ by c, is $\frac{pr+st-m}{c}$

and

and also $cd-c+m$ by $pr+s-tu$, is $\frac{cd-c+m}{pr+s-tu}$, &c. which are done by only putting the Dividend over the Divisor.

The second kind of more common Cases in Division *are.*

Where the Divisor is found in some Quantities of the Dividend but not in all: As $su+stu+g$ being divided by $s+st$, will stand as in the Margin.

$$s+st)\ su+stu+gb\ \left(u+\frac{gb}{s+st}\right.=\text{the Quote}$$
$$\underline{su+stu}$$
$$+gb \text{ rests.}$$

So also to divide $msu+pru+gb-rs$ by $ms+pr$.

$$ms+pr)\ msu+pru+gb-rs\ \left(u+\frac{gb-rs}{ms+pr}\right. = \text{the Quote.}$$
$$\underline{msu+pru}$$
$$gb-rs \text{ rests.}$$

The Truth of these two Cases is easily demonstrated, by supposing the Species to represent any Numbers. But because I shall a little further have Occasion to shew how to solve Equations by Division, I shall refer the Numeral Work thereto.

A learned Friend put me in mind of giving this for my last Example in *Division.*

$$a+b)\ a^7+b^7\ (a^6-a^5b+a^4b^2-a^3b^3+a^2b^4-ab^5+b^6 = \text{the Quote,}$$

$$\begin{array}{l}
\underline{a^7+a^6b} \\
-a^6b+b^7 \\
\underline{-a^6b-a^5b^2} \\
+a^5b^2+b^7 \\
\underline{+a^5b^2+a^4b^3} \\
-a^4b^3+b^7 \\
\underline{-a^4b^3-a^3b^4} \\
+a^3b^4+b^7 \\
\underline{+a^3b^4+a^2b^5} \\
-a^2b^5+b^7 \\
\underline{-a^2b^5-ab^6} \\
+ab^6+b^7 \\
\underline{+ab^6+b^7} \\
0
\end{array}$$

$$\underline{a+b} \quad \text{(prov'd thus.}$$
$$a^7-a^6b+a^5b^2-a^4b^3+a^3b^4-a^2b^5+ab^6$$
$$\underline{+a^6b-a^5b^2+a^4b^3-a^3b^4+a^2b^5-ab^6+b^7}$$
$$\text{Sum}=a^7+b^7 = \text{the Prod. contracted.}$$
$$= \text{the Dividend.}$$

And

And *Note,* That the Differences of any two like Powers of two Quantities, will always be divided by the Difference of the Quantities without any Remainer, as $a^4 - b^4 \div a - b, = a^3 + a^2b + ab^2 + b^3$.

2. When the Exponents of the Powers are odd Numbers, the Sum of the Powers can be divided by the Sum of the Quantities without Remainer, as $a^3 + b^3 \div a + b, = a^2 - ab + bb$. And as in the above Example at large.

3. Alſo when the Powers are even, the Difference of the Powers can be divided by the Sum of the Quantities; as $a^6 - b^6 \div a + b, = a^5 - a^4b + a^3b^2 - a^2b^3 + ab^4 - b^5$.

SECT. VI. *Of raiſing the Powers of Numbers and Quantities and of Extracting the Roots.*

1. By the following Table you have the ſeveral Ways of expreſſing the Powers with their Exponents, both of Numbers and Species; what is ſaid of the Powers of 9 being expreſſed by their Exponents, being to be taken alſo in the other Roots 8, 7, 6, 5, 4, 3, 2, and 1, for as 9^{10} is to the 10th Power of 9, which by the Table is 3486784401: So is 8^{10} = the 10th Power of 8, which you ſee is 1073741824, which are the two various Ways of expreſſing the Powers of Numbers.

2. For the Powers of Quantities, whether Simple as thoſe of u, or compounded by Multiplication as thoſe of ub, viz. uu or u^2, and uuu or u^3, &c. for the ſecond and third Powers of u, or of ub, whoſe Square is $uubb$, or better u^2b^2, the Cube $= u^3b^3$; you have the beſt way of Notation in this Table.

A TABLE *shewing the several Ways of expressing the Powers, both of Numbers and Species.*

	Power.	Power.	Power.	Power.	Power.	Power.	Power.	Power.	Power.	Power.
Exponents.	1st	2d	3d	4th	5th	6th	7th	8th	9th	10th
	Root or —	Square or —	Cube or —	Biquadrat or —	Surfolid or —	Squared Cube or —	Second Surfolid or —	Squared Biquadrat or	Cubed Cube or —	Squared Surfolid or —
	1	1	1	1	1	1	1	1	1	1
	2	4	8	16	32	64	128	256	512	1024
	3	9	27	81	243	729	2187	6561	19683	59049
	4	16	64	256	1024	4096	16384	65536	262144	1048576
	5	25	125	625	3125	15625	78125	390625	1953125	9765625
	6	36	216	1296	7776	46656	279936	1679616	10077696	60466176
	7	49	343	2401	16807	117649	823543	5764801	40353607	282475249
	8	64	512	4096	32768	262144	2097152	16777216	134217728	1073741824
	9	81	729	6561	59049	531441	4782969	43046721	387420489	3486784401
Or more brief.	9 10	9^2 10^2	9^3 10^3	9^4 10^4	9^5 10^5	9^6 10^6	9^7 10^7	9^8 10^8	9^9 10^9	9^{10} 10^{10}
In Species thus:	u	uu	uuu	$uuuu$	$uuuuu$	$uuuuuu$	$uuuuuuu$	$uuuuuuuu$	$uuuuuuuuu$	$uuuuuuuuuu$
Or rather.	u	u^2	u^3	u^4	u^5	u^6	u^7	u^8	u^9	u^{10}
If compound thus:	ub	$uubb$	$uuubbb$	&c.	. . .	. . . .	. . . . .	. . . . . .	. . . .	. . . .
Or better thus:	ub	u^2b^2	u^3b^3	u^4b^4	u^5b^5	u^6b^6	u^7b^7	u^8b^8	u^9b^9	$u^{10}b^{10}$
	bcd	$b^2c^2d^2$	$b^3c^3d^3$	$b^4c^4d^4$	$b^5c^5d^5$	$b^6c^6d^6$	$b^7c^7d^7$	$b^8c^8d^8$	$b^9c^9d^9$	$b^{10}c^{10}d^{10}$

And the Notation of Powers with Fractional Exponents is thus:

Roots.	Squares	Cubes.	Biquadrat.	Surfolid.	Squar'd Cube.	Second Surfolid.	Squared Biquad.	Cubed Cube.	Squared Surfolid.
$a^{\frac{1}{2}}$	a	$a^{1\frac{1}{2}}$	a^2	$a^{2\frac{1}{2}}$	a^3	$a^{3\frac{1}{2}}$	a^4	$a^{4\frac{1}{2}}$	a^5
$a^{\frac{1}{3}}$	$a^{\frac{2}{3}}$	a	$a^{1\frac{1}{3}}$	$a^{1\frac{2}{3}}$	a^2	$a^{2\frac{1}{3}}$	$a^{2\frac{2}{3}}$	a^3	$a^{3\frac{1}{3}}$
$a^{\frac{1}{4}}$	$a^{\frac{1}{2}}$	$a^{\frac{3}{4}}$	a	$a^{1\frac{1}{4}}$	$a^{1\frac{1}{2}}$	$a^{1\frac{3}{4}}$	a^2	$a^{2\frac{1}{4}}$	$a^{2\frac{1}{2}}$, &c.
				Or with Literal Exponents, thus:					
$a^{\frac{m}{n}}$,	$a^{\frac{2m}{n}}$,	$a^{\frac{3m}{n}}$,	$a^{\frac{4m}{n}}$,	$a^{\frac{5m}{n}}$,	$a^{\frac{6m}{n}}$,	$a^{\frac{7m}{n}}$,	$a^{\frac{8m}{n}}$,	$a^{\frac{9m}{n}}$,	$a^{\frac{10m}{n}}$, &c.

Some

Some Observations *upon this Table.*

1. Any two Exponents added, give that of the Product of the respective Powers: As where the Root is 3, and the Exponents 4 and 5. I say, the Sum of 4 and $5 = 9$, which is equal to the Exponent of the Rectangle of the respective Powers $243 \times 81 = 19683$.

2. Double any Exponent, gives the Exponent of the Square of the respective Power, as in the 4th Power of 5, which is 625. I say, double the Exponent is 8, which is the Exponent of the Rectangle of 625×625, *viz.* of 390625.

3. I observe that 3, 4, *ec* times any Exponent, gives the Exponent of the 3d, 4th, *&c.* Powers: As three times the Exponent 2 is 6, so the Cube of the Powers 9, 16, *ec* under the Exponent 2, gives 729, 4096, *ec* the respective Powers under the Exponent 6.

4. If an 8th Power be to be divided by a 2d Power, the Exponent 2 taken from the Exponent 8, leaves the Exponent 6; so any of the Numbers under the Exponent 8, (as suppose 256,) divided by any of those respectively under the Exponent 2, (as for Instance 4,) the Quote will be respectively the Powers under the Exponent 6, (in this Example 64.)

5. The Cube *ec* Root of any Power, is the Power respectively of the Exponent of that Power given, divided by 3 *ec*; so the Cube Root of 262144 (its Exponent being 9, a third of which is 3,) is 64, found under the Exponent 3, and against the Power given. So likewise the Sursolid Root of 1024, (or any of the Numbers under the Exponent 10,) is 4, or the respective Numbers under the Exponent 2, the Quote of 10 divided by 5; for $4^5 = 1024$ the Proof. So much for raising the Powers of Quantities compounded by *Multiplication*; I proceed to shew,

III. *How to raise the Powers of Quantities compounded by* Addition *or* Subſtraction, *i. e. from a Binomial or Reſidual Root, and to extract Roots.*

$u+b=$ the Root or 1ſt Power.

$u+b$

$uu+2bu+bb=$ the Square or 2d Power of $u+b$

$u+b$

$uuu+2buu+bbu$

$+buu+2bbu+bbb$

$u^3+3buu+3bbu+bbb=$ the Cube or 3d Power of $u+b$.

$u+b$

$u^4+3bu^3+3bbuu+ub^3$

$+bu^3+3bbuu+3ub^3+b^4$

$u^4+4bu^3+6b^2u^2+4ub^3+b^4=$ the Biquadrat or 4th Power of $u+b$.

$u+b$

$u^5+4bu^4+6b^2u^3+4u^2b^3+ub^4$

$+bu^4+4b^2u^3+6u^2b^3+4ub^4+b^5$

$u^5+5bu^4+10b^2u^3+10u^2b^3+5ub^4+b^5=$ the Surſolid or 5th Power of ($u+b$.

$u+b$

$u^6+5bu^5+10b^2u^4+10u^3b^3+5u^2b^4+ub^5$

$+bu^5+5b^2u^4+10u^3b^3+10u^2b^4+5ub^5+b^6$

$u^6+6bu^5+15b^2u^4+20u^3b^3+15u^2b^4+6ub^5+b^6=$ the Squared Cube (of $u+b$.

$u+b$

$u^7+6bu^6+15b^2u^5+20u^4b^3+15u^3b^4+6u^2b^5+ub^6$

$+bu^6+6b^2u^5+15u^4b^3+20u^3b^4+15u^2b^5+6ub^6+b^7$

$u^7+7bu^6+21b^2u^5+35u^4b^3+35u^3b^4+21u^2b^5+7ub^6+b^7=$ the 2d (Surſolid of $u+b$.

$u^8+8bu^7+28b^2u^6+56u^5b^3+70u^4b^4+56u^3b^5+28u^2b^6+8ub^7+b^8$ ($=$ the 8th Power.

$u^9+9bu^8+36b^2u^7+84u^6b^3+126u^5b^4+126u^4b^5+84u^3b^6+36u^2b^7+$ ($9ub^8+b^9=$ the 9th Power.

$u^{10}+10bu^9+45b^2u^8+120u^7b^3+210u^6b^4+252u^5b^5+210u^4b^6+$ ($120u^3b^7+45u^2b^8+10ub^9+b^{10}=$ the 10 Power of $u+b$.

Obſervations

Observations from this last Table of Powers.

1. That the Sum of the Square of the Parts of any Number or Quantity more the double Rectangle is = the Square of the whole, as *per Euclid.*

2. That the foregoing Example of Powers may serve as a Table for any one to know any Power of any other Binomial Roots by, though of other Letters, *mutatis mutandis*; the Unciæ and Powers being the same as in this.

3. That the 10th Power is easily produced from the 9th or any subsequent Power from the preceding, by first finding the Unciæ as *per* the Rules foregoing, or as under the 5th Head following, and considering the Course of the Powers that precede that which you would find: By which Method I have produced the 8th, 9th, and 10th Powers without any Multiplication.

4. And that the Reader may not want a proper Way of expressing in Words any Power, I shall give an Example how the 10th Power is properly read; which is thus:

The squared Sursolid of *u*, more 10 times the Rectangle of *b* in the cubed Cube of *u*; more 45 times the Square of *b* in the squared Biquadrat of *u*; more 120 times the 2d Sursolid of *u* in the Cube of *b*; more 210 times the squared Cube of *u* in the Biquadrat of *b*; more 252 times the Sursolid of *u* in the Sursolid of *b*; more 210 times the Biquadrat of *u* in the squared Cube of *b*; more 120 times the Cube of *u* in the 2d Sursolid of *b*; more 45 times the Square of *u* in the squared Biquadrat of *b*; more 10 times *u* in the cubed Cube of *b*; more the squared Sursolid of *b*.

5. *A very brief Way of finding the Unciæ of any Power by that of the next preceding Power is this*; (which I have not seen given before.)

A second Way of finding the Unciæ.

Add the Unciæ of the 1st and 2d Members (for Example of the 9th Power to find the Unciæ of the 10th) together they make 10, which is that of the 2d Member of the 10th Power, 36 in the 3d, and 9 in the 2d Members of the 9th Power is the 3d Member in the 10th or 45; 36 and 84 in the 3d and 4th Members is 120 belonging to the 4th Member of the 10th Power, *&c.* of the rest.

And the Powers themselves, and their proper Exponents are found by the like Method, observing, That in which of the 2 Powers soever added,

added, is the greateſt Exponent, that Exponent muſt be put in the Sum.

Thus to make the 10th Power by theſe Rules from the 9th. The Unciæ of the 1ſt and 2d Members is 10 = the 2d in the 10th; and the Sum of the Powers is bu^9; (becauſe 9 is found the higheſt Exponent of u and b is only the 1ſt Power.) So $10bu^9$ is the 2d Member of the 10th Power. The 3d Member of the 10th is the 2d and 3d Members of the 9th, &c. done as in that laſt mentioned.

And this will be made plain by a Numeral Example of finding the Unciæ to the 15th Power of a Binomial, by an eaſy Addition of only two Numbers; which Unciæ, with thoſe of all the intermediate, are found much ſpeedier than that of the 15th Power alone can be by the Method before-mentioned; which requires Subſtraction, Multiplication, and Diviſion, and thoſe pretty tedious ones before thoſe of the 15th Power can be exhibited by this Method.

A new Table of the Unciæ to the 15th Power of a Binomial Root, made by Addition only.

add	add															
1	1	add														
1	2	1	add													
1	3	3	1	add												
1	4	6	4	1	add											
1	5	10	10	5	1	add										
1	6	15	20	15	6	1	add									
1	7	21	35	35	21	7	1	add								
1	8	28	56	70	56	28	8	1	add							
1	9	36	84	126	126	84	36	9	1	add						
1	10	45	120	210	252	210	120	45	10	1	add					
1	11	55	165	330	462	462	330	165	55	11	1	add				
1	12	66	220	495	792	924	792	495	220	66	12	1	add			
1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1	add		
1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	1	add	= the Unciæ of the 14 Power.
1	15	105	455	1365	3003	5005	6435	6435	5005	3003	1365	455	105	15		1 = the Unciæ of the 15 Power.

This Method, to me, is intirely New, and my own Contrivance: But ſee a Third Way a little farther.

6. It is obſervable, that the Unciæ encreaſe till you come to the middle, and then decreaſe; there being the ſame Unciæ to the right hand of the higheſt, as towards the left; ſo that when you

you have found half the Unciæ, you have all: Obſerving farther, That there are two higheſt Unciæ in the middle of the odd Powers, as the 3d, 5th, 7th, &c. Powers.

And that the Exponents of ſeveral Member's after you come to the higheſt, do change, as in the 10th Power in the former Table above; the Member toward the left hand of the higheſt is $210 u^6 b^4$, but toward the right of the higheſt, the Power is $210 u^4 b^6$, which is the ſame, only the Exponent's changed: So the 2d Member towards the left hand is $120 u^7 b^3$; but 'tis $120 u^3 b^7$.
The 3d $= 45 u^8 b^2$; but 'tis $45 u^2 b^8$.
The 4th $= 10 u^9 b$; but 'tis $10 u b^9$, &c. } toward the right.

7. You may ſee that four of the Unciæ, *viz.* the two firſt and two laſt are always known in all Powers, being one; and the Exponent of the Power. And by theſe Rules the Unciæ, the Powers, and Exponents of any Binomial Root may be found with much leſs Trouble than the common Way, or any I have ſeen before.

8. But the Unciæ of any Term, or Member of any of the Powers in the Table above, may be found without any of the previous Unciæ of Powers or their Terms, by eaſy Operations in Multiplication and Diviſion, as by theſe 6 Examples.

A third Way of finding the Unciæ.

Term or Member.	Power	Factors.	Products.	Quote or Unciæ required.
4 of the	4	1,2,3 =	6 the Diviſor	= 4
		4,3,2 =	24 the Dividend	
4	5	1,2,3 =	6 the Diviſor	= 10
		5,4,3 =	60 the Dividend	
5	10	1,2,3,4 =	24 the Diviſor	= 210
		10,9,8,7 =	5040 the Dividend	
7	10	1,2,3,4,5,6 =	720 the Diviſor	= 210
		10,9,8,7,6,5 =	151200 the Dividend	
5	15	1,2,3,4 =	24 the Diviſor	= 1365
		15,14,13,12 =	32760 the Dividend	
4	16	1,2,3 =	6 the Diviſor	= 560
		16,15,14 =	3360 the Dividend	

Here it may eaſily be ſeen that I tell the Figures from one to within one of the Term given, and multiply them together *for a Diviſor.*

And

And telling as many from the Power given backward; I multiply them one in another for a Dividend; whence the *Unciæ* of any Term of any Power ariseth by Division.

For suppose $p=$ the Exponent of any Power, and $t=$ the Term, and $u=$ the *Unciæ* sought, the Rule will be

$$\frac{p\times p-1\times p-2\times p-3,\ \&c.}{t-1\times t-2\times t-3\times t-4,\ \&c.}=u$$ (in this Example the *Unciæ* of the 5th Term or Member of the 10th Power $=210$.)

9. Any Number may be squared by the Canon exhibited in the Square, or 2d Power of the above Table of the Powers raised from a Binomial Root. For Example; What is the Square of 1234^9?

The Analytical Square or Canon directing us how to work, is $uu+2ub+bb$. Now suppose 1200 the 2 first Figures $=a$

And 34 the 2 next the right hand $=b$

It follows, That $u^2=1200^2=1440000$

$2ub$ or $u\times b\times 2$ is $=\ 81600=1200\times 34\times 2$

bb (or 34×34) is $=\ 1156$

Sum $=1522756=uu+2ub+bb-1234^2$.

10. And by the Cube or 3d Power we have a Rule how to cube any Number. Example: The Canon $=u^3+3u^2b+3ub^2+b^3$ What is the Cube of 2232?

I say $2000=u$

$200=b$

$8000000000=uuu$ or 2000^3

$2400000000=3uub=2000^2\times 200\times 3$

$240000000=3ubb=2000\times 200^2\times 3$

$8000000=bbb=200^3$.

$10648000000=$ Sum or 2200^3 or uuu in the 2d Operation; where

$2200=u$

and $30=b$

$435600000=3uub,=2200^2\times 30\times 3$

$5940000=3ubb=2200\times 900\times 3$

$27000=bbb=30^3$.

$11089567000=$ Sum, or 2230^3. So in the 3d Operation $2230=u$

and $2=b$

$29837400=3uub=2230^2\times 2\times 3$

$26760=3ubb=2230\times 4\times 3$

$8=bbb=2^3$.

$11119431168=2232^3=uuu+3uub+3ubb+bbb$.

This

This proves that the third Power in the foregoing Table is the the true Cube of $a+b$, as is here demonstrated by Numbers working with the Species exactly according to the Canon, without having regard to the way of placing Numbers in common *Multiplication.*

11. But that which renders the Powers of a Binomial Root more admirably useful is, that they shew plainly the Reason of all those mysterious Rules given in extracting the Root of any Power, as the Square, Cube, Biquadrat, &c. Roots in the former Part of this Book mentioned: For Example,

In Extracting the Square Root of 1522756.

Canon for the Square Root.
$=aa+2ab+bb.$

1522756(1000$=a$
1000000$=aa$ deduct.

$\div$ this Rest by $2a=2000$) 522756 rests, (200$=b=$ the Quote.

400000$=2ab.$
40000$=bb.$

440000$=2ab+bb,$ deduct from the (last Rest.

$\div$ by $2a=2400$) 82756 remains, and now 1000 + 200 (=1200$=a$ 30$=b$, the Quote.

72000$=2ab.$
900$=bb.$

72900$=2ab+bb,$ deduct from the last (Remainer.

$\div$ by $2a=2460$) 9856$=$rest; and now 1230$=$the 3 (Roots$=a$ 4$=b=$the Quote or last in the Root sought.

9840$=2ab.$
16$=bb.$

9856$=2ab+bb$, deduct from the (last Rest.

0 rests.

And the Reason of Extracting the Cube Root, will appear in the following Example, proceeding according to this Canon: $aaa + 3aab + 3ab^2 + b^3$.

Extract the

Cube Root of 11119431168 (2000 $= a$.

8000000000 $= aaa$, deduct.

12006000) 3119431168 resteth, which divide by $3a + 3aa$, (gives 200 $= b$.

(200 $= b$.

6000 $= 3a$, the Treble Root.

12000000 $= 3aa$, $=$ the Treble Square of the (Root a.

12006000 $= 3a + 3aa$, the Divisor.

8000000 $= bbb$ or 200^3.

240000000 $= 3abb = 6000 \times 40000$.

2400000000 $= 3aab = 4000000 \times 200 \times 3$.

2648000000 $= bbb + 3abb + 3aab =$ the subtrahend (to deduct from the last resolvend.

14526600) 471431168 resteth the second resolvend (30 $= b$, $=$ (the Quote.

6600 $=$ Treble Root 2200 $= 3a$.

14520000 $= 3aa = 2200^2 \times 3$.

14526600 $= 3a + 3aa =$ the Divisor for 2d Re- (solvend, gives 30 $= b$.

27000 $= bbb = 30^3$.

5940000 $= 3abb = 6600 \times 900$.

435600000 $= 3aab = 4840000 \times 30 \times 3$.

441567000 $= bbb + 3abb + 3aab =$ the subtrahend.

14925390) 29864168 $=$ third resolvend rests. 2 $= b$, the Quote.

6690 $= 3a$, the Treble Root $= 2230 \times 3$.

14918700 $= 3aa = 4972900 \times 3$.

14925390 $= 3a + 3aa$, the Divisor for the third (Resolvend.

8 $= bbb = 2^3$.

26760 $= 3abb = 6690 \times 4$.

29837400 $= 3aab = 14918700 \times 2$.

29864168 $= bbb + 3abb + 3aab$, the Sum or sub- (trahend, which taken from the

(0) 3d Resolvend there Rests (0)

So the Root is found 2000 + 200 + 30 + 2.

And

And the Process according to the Canon of the third Power of $a+b$, is repeated three times, *viz.*

1st, $2000=a$, and $200=b$	or	2000
2dly, $2200=a$, and $30=b$	+	200
3dly, $2230=a$, and $2=b$	+	30
	+	2

$= 2232$, the Root required.

These Examples, according to the 2d and 3d Powers of the Binomial Roots in the Table above, fully shew the Reason of the Method used in the Extraction of the Square and Cube Roots. And by the observing of the Canon given by the Powers, the Root of any other, even the squared Sursolid may be extracted: As I did that of the Biquadrat in *Sect.* 6. of *Chap.* I. Article the 2d; having never seen it done but only proceeding purely as the 4th Power directs.

IV. The Roots of Algebraic Quantities are either those of Rationals or Surds. 1. The Roots of Rational Quantities, whether those Quantities be compounded by Multiplication of Single Binomial, or Residual Roots are thus express'd.

1. How Single Roots of Powers are express'd in Species and Numbers.

	Powers. in Species.	in Numb.	Their Roots. Species.	Numb.
Square	$uu=$	4	$u=$	2
Cube	$bbb=$	27	$b=$	3
Biquadrat	$c^4=$	256	$c=$	4
Sursolid	$d^5=$	3125	$d=$	5
Squared Cube	$b^6=$	46656	$b=$	6
2d Sursolid	$k^7=$	823543	$k=$	7

2. How Binomials and Residual Roots are extracted.

Binomial Powers. Species.	Nº.	Their Roots. Spec.	Nº.	Residual Powers. Species.	Nº.	Their Roots. Spec.	Nº.
$uu+2ub+bb=$	25.	$u+b=$	5	$dd-2du+uu=$	9.	$d-u=$	3
$uu+2u+1=$	9.	$u+1=$	3	$bb-2b+1=$	4.	$b-1=$	2
$16bb+8bk+kk=$	961.	$4b+k=$	31	$16bb-8bk+kk=$	289.	$4b-k=$	17
$bbuu+2buu+uu=$	64.	$bu+u=$	8	$bbuu-2buu+uu=$	16.	$bu-u=$	4

 2*dly*, Surd

3dly, Surd Roots, or Roots of Surd Quantities, are expressed thus by their own Radical Signs.

Simple Surd Quantities.

	Powers. Roots.		Powers. Roots.
Square Root of	$u = \sqrt{u}$	or in N°.	$171 = \sqrt{171}$
Cube Root	$b = \sqrt[3]{b}$	———	$315 = \sqrt[3]{315}$
Biquadrat	$c = \sqrt[4]{c}$	———	$517 = \sqrt[4]{517}$
Surfolid	$d = \sqrt[5]{d}$	———	$735 = \sqrt[5]{735}$
Squared Cube	$g = \sqrt[6]{g}$	———	$379 = \sqrt[6]{379}$
2d Surfolid	$h = \sqrt[7]{h}$	———	$1231 = \sqrt[7]{1231}$
Squared Biquad.	$k = \sqrt[8]{k}$	———	$339 = \sqrt[8]{339}$
Cubed Cube	$l = \sqrt[9]{l}$	———	$1129 = \sqrt[9]{1129}$
Squared Surfol.	$m = \sqrt[10]{m}$	———	$1057 = \sqrt[10]{1057}$

Compound Surd Quantities.

Powers.	Roots.
$u+b$	$\sqrt{u+b}$
$u+b+c$	$\sqrt[3]{u+b+c}$
$u+b+c+d$	$\sqrt[4]{u+b+c+d}$
$u+b+c+d-g$	$\sqrt[5]{u+b+c+d-g}$
$u+b+c-d+g+h$	$\sqrt[6]{u+b+c-d+g+h}$
$u+b+c-d+g-h-k$	$\sqrt[7]{u+b+c-d+g-h-k}$
$u+b-c+d+g+h+k+l$	$\sqrt[8]{u+b-c+d+g+h+k+l}$
$u+b+c+d-g-h+k+l+m$	$\sqrt[9]{u+b+c+d-g+k+l+m}$
$17r+gb-bc+m$	$\sqrt[10]{17r+gb-bc+m}$

It must be observed, That the Sign connecting the Residual Root is always the same with that of the double Rectangle, *i. e.* Negative as under the 2d above.

Having fully shewn the Nature of raising the Powers of Quantities, and extracting the Roots, and illustrated the same by Numbers,

bers, it naturally leads me to *the Arithmetic of Surd Quantities*; which I shall also demonstrate by Numbers.

SECT. VII. *The Algorithm of Surd Quantities.*

In regard there is often Occasion for Working the Roots of Species or Numbers, as by Addition, Substraction, Multiplication, *&c.* of them; and that it would be tedious and less accurate to find near the real Roots of Numbers, and impossible to be done of all Algebraic Quantities; therefore the Business of this Section is to give Rules for such Operations in Powers without regarding the Roots farther than by the Radical Signs prefixed: So that the compleat Powers of Quantities, or Numbers may hereby be added, substracted, *&c.* as well, (and much sooner) than if the real Roots were known, and the Work done by them See Article 7. Sect. 1. of this Chapter.

I. *Reduction of Surd Quantities.*

Reduction is here taught first, because Quantities are required to be reduced to a common Radical Sign before those that have different Signs can be multiplied, divided, *&c.*

Example 1. Reduce $\sqrt{u}$ and $\sqrt[4]{c}$ to Surds having a common Radical Sign; or Surds Heterogeneal to Homogeneal: See the Operation.

4,	4	The common Radical Sign. (ponents.
2,	$1 =$	The lowest Terms of the given Radical Signs or Ex-
$\sqrt[2]{u}$,	$\sqrt[4]{c} =$	The Surds given to be reduced.
$\sqrt[4]{uu}$,	$\sqrt[4]{c} =$	The Answer or the Surds given in a common Radical (Sign.

By the Operation it is plain, that first I take half the Radical Sign 2 (given) and put it over the other given, or 4; and half the given Radical Sign 4, and place it over the other given or 2, which are the lowest Terms of the given Radical Sign.

2*dly*, I multiply the Radical Signs given by those standing over respectively, and they produce 4 = the common Radical Sign required.

3*dly*, I

3dly, I raiſe the Surd Roots given to the Powers of the loweſt Terms of the given Radical Signs which ſtand over the reſpective Root; as u raiſed to the 2d Power is uu, c in the 1ſt Power is $= c$, and placing the common Radical Sign before this uu and the c, I have the Anſwer $\sqrt[4]{uu}$ and $\sqrt[4]{c}$.

Example 2d ſhall be in Numbers, as to reduce $\sqrt[2]{9}$ and $\sqrt[4]{16}$ to a common Radical Sign.

See the Work by the ſame Rule as above; and in Multiplication and Addition it is ſhewn how theſe prove each other, or how the Totals agree, and conſequently prove the Truth of this Method of Reduction.

4,	4	= Common Radical Sign.
2,	1	= Leaſt Terms of given (Radical Signs, placed interchangeably.
$\sqrt[2]{9}$,	$\sqrt[4]{16}$	= Surds given.
$\sqrt[4]{81}$,	$\sqrt[4]{16}$	= Surds required, or thoſe (given reduced.

II. *Multiplication of Simple Surds.*

In this Article I ſhall ſhew,

1. How to multiply a Surd by a Rational Number or Quantity.
2. To multiply a Surd in others of the ſame kind, or the ſame Radical Sign.
3. To multiply Surds together having different Radical Signs.

1. *To multiply a Surd in a Rational Number or Quantity.*

This is expreſſed two Ways, for Example, to multiply $\sqrt[4]{c}$ by b, the Anſwer is very often expreſſed thus, $b\sqrt[4]{c}$ or b in the Biquadrat Root of c. But actually to multiply them is done by this *Rule.*] Raiſe b up to the Power of the Surd given, as here b in the fourth Power is $bbbb$ and putting the ſame Radical Sign before, is $\sqrt[4]{bbbb}$, which being multiplied by $\sqrt[4]{c}$, gives $\sqrt[4]{bbbbc}$.

Or in Numbers, ſuppoſe $c = 16$, and $b = 8$, then $b^4 = 4096$, and $\sqrt[4]{16}$ multiplied in $\sqrt[4]{4096} = \sqrt[4]{65536}$ the Product, and that Root being extracted is $= 16$, the Product in a Rational Number.

To

To prove both which Ways of working, by raising the Power of the Rational Number, to be genuine,

I say, c being $=16$, its $\sqrt[4]{\ }$, $=2$, which multiplied by $b=8$, gives 16 as before.

2. *To multiply a Surd Number by a Surd, which are Homogeneal, or when the Radical Signs are the same.*

Rule.] Multiply the one by the other, and the Product is the Answer, prefixing one of the Radical Signs given.

Example. Multiply $\sqrt[3]{dr}$ by $\sqrt[3]{u}$, and the Product is $=\sqrt[3]{dru}$, and putting $u=8$; $r=9$; and $d=3$. It is thus done in Numbers, $\sqrt[3]{27}$ by $\sqrt[3]{8}=\sqrt[3]{216}=6$ the Product.

Proved thus: By the real Roots without Surds. I say $\sqrt[3]{27}=3$, and $\sqrt[3]{8}=2$, and 2 times 3 is 6, as before in the Numerical Way last above, which is the same as the Literal.

3. *To multiply Heterogeneal or Surds, having different Radical Signs.*

Rule.] Reduce them to a common Radical Sign, and then work as in the last.

Example. Multiply $\sqrt[4]{c}$ by $\sqrt[2]{u}$. In a common Radical Sign, the Surds are $\sqrt[4]{uu}$ and $\sqrt[4]{c}$; whose Product is $\sqrt[4]{cuu}$.

Or in Numbers: $c=16$; and $u=9$; or $\sqrt[4]{16}$ to be multiplied by $\sqrt[2]{9}$, which in a common Radical Sign are $\sqrt[4]{16}$ and $\sqrt[4]{81}$, whose Product is $\sqrt[4]{1296}=6$.

Rule proved thus: By the real Roots, for $\sqrt[4]{16}=2$; and $\sqrt[4]{81}=3$; and $3\times2=6$.

III. Division *of Surd Quantities.*

There is nothing of Difficulty in this Rule; for if the Quantities of the Dividend can be divided by those of the Divisor, and the Radical Signs are the same, work as is taught in *Division of Algebra*, or of whole Numbers; as $\sqrt{ubc}\div$ by $\sqrt{bc}$ the Quote $=\sqrt{u}$, &c.

Or

Or in Numbers; $\sqrt{144} \div \sqrt{9} = \sqrt{16} = 4$, or $= 12 \div 3$. For Proof,

2. If the Radical Signs are not the same, they may be reduced to a common Radical Sign, and then divide the Power of the Dividend by the Divisor; or put it over that of the Divisor, as if the $\sqrt[4]{cuu}$ be divided by $\sqrt[2]{u}$. In a common Radical Sign they are $\sqrt[4]{cuu}$ for the Dividend, and $\sqrt[4]{uu}$ for the Divisor; so the Answer is $\sqrt[4]{\frac{cuu}{uu}} = \sqrt[4]{c}$. And this proves the last Example in the last Rule, and is farther proved by dividing the Numeral Rectangle there by one Factor, for the Quotient will be the other Factor.

IV. Addition of *Surd Quantities*.

This Rule is placed after *Multiplication*, because that is often used in this Rule of *Addition*. I shall shew

1. How to add when there are the same Quantities and Radical Signs.

2. When there are different Quantities of Species, but the same Radical Signs, which Quantities are supposed commensurable.

3. How to add Quantities, having the same Species, but different Radical Signs, or when both the Radical Signs are different, and the Quantities incommensurable.

1. To add $\sqrt[4]{u}$, $\sqrt[4]{u}$, and $\sqrt[4]{u}$.

Rule.] Multiply any one of the Surds by the whole Number of them, as here $\sqrt[4]{u} \times$ by $3 = \sqrt[4]{81u}$, (observing the first Rule of *Multiplication* last above,) which is the Answer.

Or in Numbers: Add $\sqrt[4]{16}$, $\sqrt[4]{16}$, and $\sqrt[4]{16}$. Here 3 the Number of Roots raised to the fourth Power is 81; and $\sqrt[4]{16} \times \sqrt[4]{81} = \sqrt[4]{1296} = 6$ proved thus; $\sqrt[4]{16} = 2$, and 3 times 2 is 6, or $2+2+2$, which proves the Rule.

2. When two unequal Surds are commensurable, and have the same Radical Signs, and when divided by their greatest common Measure, are reduced to such Quantities as are the compleat Powers of some Quantities or Numbers; they may be added thus, suppose $\sqrt[3]{40}$ to be added to $\sqrt[3]{135}$.

2d Rule.] Divide the Surds by their common Measure $\sqrt[3]{5}$, and they are reduced to $\sqrt[3]{8}$ and $\sqrt[3]{27}$, the Cube Roots of which are 2 and 3; the Sum of which is 5, which Sum of the Roots multiplied is $5\sqrt[3]{5}=\sqrt[3]{625}$ the Anſwer.

And for Proof, Extract the Cube Roots of the Surds given in the commom way of Extraction, and you'll find the Sum $=\sqrt[3]{625}$ $=8.55$.

How to add Quantities having the ſame Surds, but different Radical Signs; or when the Radical Signs are either different or the ſame, and the Surds are incommenſurable.

Rule.] There is no Occaſion for farther Operation, than to put the Radical Sign $+$ between the given Surd Roots: As

To add $\sqrt{cc}$ to the $\sqrt[3]{cc}$ and $\sqrt[4]{cc}$; the Sum is $\sqrt{cc}+\sqrt[3]{cc}+\sqrt[4]{cc}$.

Or the Sum of $\sqrt{5}$, the $\sqrt[3]{6}$ and $\sqrt[4]{13}$, is $=\sqrt{5}+\sqrt[3]{6}+\sqrt[4]{13}$.

Or $\sqrt{3}+\sqrt{7}+\sqrt{12}$, *&c.* is the Sum of ſuch Surds as $\sqrt{3}$, $\sqrt{7}$ and $\sqrt{12}$, *&c.*

There is a Rule which ſome pretend helps us to the Sum of theſe kind of Surds, when the Radical Sign is $\sqrt[2]{}$. The Rule is this; *To the ſquare Root* of the Sum of the Squares of the Surds given, add the double Rectangle of theſe Surds, and the Sum is the Anſwer.

Example. To $\sqrt{12}$, add $\sqrt{8}$; according to the Rule, the Sum of the Squares of the Surds is $=20$, its Root $\sqrt{20}$, and the double Rectangle of the Surds is $2\sqrt{96}=8\times12\times4=\sqrt{384}$, by the firſt Example of *Multiplication* of Surds. So by this Rule, the Sum of $\sqrt{12}+\sqrt{8}$ is reduced to $\sqrt{20}+\sqrt{384}$. I muſt confeſs I do not ſee the Uſe of the Rule, nor of the Alteration of the Surds, ſince to ſay the Anſwer is $\sqrt{12}+\sqrt{8}$ is more true and intelligible, than to call the Sum $\sqrt{20}+\sqrt{384}$.

V. Subſtraction *of Simple Surds.*

1. When the Surd Roots are commenſurable or reduceable to Rational Quantities, as in the ſecond Example in *Addition*; work

as is there directed, but when you come to the Roots, inſtead of taking their Sum, take their Difference, and multiply as in the ſame Example. To know the Difference between $\sqrt[3]{40}$ and $\sqrt[3]{135}$: Theſe $\div \sqrt[3]{5}$ reduces them to $\sqrt[3]{8}$ and $\sqrt[3]{27}$, whoſe Roots are 2 and 3, and the Difference of thoſe Roots is 1, which multiplied by the common Meaſure $\sqrt[3]{5}$, gives $\sqrt[3]{5}$ for the Remainer or Difference.

But when the Surds are incommenſurable, as in the laſt Example in *Addition*, you have nothing to do but place the Sign (—) between the Surds given; ſo the Difference of $\sqrt{b}$ above $\sqrt{g}$, is $\sqrt{b}-\sqrt{g}$.

VI. Addition *and* Subſtraction *of Compound Surds.*

1*ſt*, For commenſurable Surds, as thoſe under the ſecond Rule of *Addition* above, take this Example.

$$\begin{array}{ll}\text{To } \sqrt{45}+\sqrt{294} & \\ \text{Add } \sqrt{125}-\sqrt{96} & \text{Sum}=\sqrt{320}+\sqrt{54}.\end{array}$$

Here the Sum of $\sqrt{45}$ and $\sqrt{125}=\sqrt{320}$; and of $\sqrt{294}$ and $-\sqrt{96}=\sqrt{54}$ by the ſaid ſecond Rule.

2*dly*, For mixt Numbers, as Surds and Rational Numbers; ſee this Example.

$$\begin{array}{ll|ll}\text{To} & 331+\sqrt[4]{768}-\sqrt[4]{80} & \text{Or from} & 331+\sqrt[4]{768}+\sqrt[4]{80} \\ \text{Add} & 153+\sqrt[4]{243}+\sqrt[4]{1280} & \text{Take} & 153+\sqrt[4]{243}-\sqrt[4]{1280} \\ \hline \text{Sum}= & 484+\sqrt[4]{7203}+\sqrt[4]{80}. & \text{Refts}= & 178+\sqrt[4]{3}\quad+\sqrt[4]{1296}\end{array}$$

I need give no farther Examples, becauſe the *Addition* and *Subſtraction* of theſe Surds are done by the ſelf-ſame Rules as thoſe called *Simple Surds*, as by the Rule under the ſecond Example, and in *Subſtraction* of thoſe Surds.

The ſecond Example laſt foregoing is done thus: To add $\sqrt[4]{768}$ to $\sqrt[4]{243}$; I divide them by the common Meaſure $\sqrt[4]{3}$, which reduceth them to $\sqrt[4]{256}$ and $\sqrt[4]{81}$, whoſe Roots are 4 and 3, whoſe Sum

Sum is 7, so $7\sqrt[4]{3}$ is the Answer, or $7^4 \times \sqrt[4]{3} = \sqrt[4]{7203}$, and so of the Rest.

The Demonstration of the Rule for this last, as under Article IV. Rule 2.

This I shall do by this plain Instance of Numbers adapted on purpose, as in the Margin; where $\sqrt{64} + \sqrt{64}$, are given to have their Sum found.

$$\sqrt{4})\ \sqrt{64} + \sqrt{64} = \text{the supposed Surds given.}$$

$$\left.\begin{array}{l} \text{Quotes } \sqrt{16} = 4 \\ \text{Qt.} = +\sqrt{16} = 4 \end{array}\right. \text{Roots.}$$

$$8 = \text{Sum of the Roots.}$$

$$\text{and } 8\sqrt{4} = \sqrt{256} = 16 \text{ the Answer.}$$

$$= \sqrt{64} + \sqrt{64} = 8 + 8.$$

1. These divided by $\sqrt{4}$, are reduced to the Root $\sqrt{16} + \sqrt{16}$, whose real Roots are $4 + 4 = 8$. And $\sqrt{4} \times 8$, as by the first Rule of *Multiplication* above, gives the $\sqrt{256}$, whose Square Root being 16, is = the Sum of the Roots of the two Surds given.

VI. Multiplication *of Compound Surds.*

Three Examples or Varieties done in Species two Ways. — *The three Examples in Species proved by Numbers.*

Example 1. — Example 1.

To multiply $b\ \sqrt{ac}$ in $ac + \sqrt{b}$ } See the Operation, also in Numbers $\left\{\begin{array}{l} 4\ \sqrt{64} \\ 2 + \sqrt{16} \end{array}\right\}$ Multiply as the Species.

$$abc\sqrt{ac} + b\sqrt{abc} = \text{the Prod. or in N}^{\circ} = 8\sqrt{64} + 4\sqrt{64}\sqrt{6} = 192$$

Or reduced into all Surds. } $= \sqrt{aaabbccc} + \sqrt{abbbc}$

For $4 \times 8\ (= \sqrt{64}) = 32$
and that $32 \times 2 + 4 (= \sqrt{16}) = 6$ $= 192$

Example 2. — Example 2d proved thus:

To multiply $b\ \sqrt{ac}$ in $ac\ \sqrt{b}$ — Multiply by $\left.\begin{array}{l} 4\sqrt{64} \\ 2\sqrt{16} \end{array}\right\}$ Multiply as the Species are.

$$abc\sqrt{abc} = \text{the Prod, or in Numb.} = 8\sqrt{64}\sqrt{16} = 256$$

Or reduced to Surds. } $= \sqrt{aaabbbccc}$

For $4, \times 8\ (= \sqrt{64}) = 32$
And $32 \times 2 \times 4 (= 2\sqrt{16}) = 8$ $= 256$

Example 3.

To multiply $b+\sqrt{ac}$ in $ac+\sqrt{b}$

$$abc+ac\sqrt{ac}+b\sqrt{b}+\sqrt{abc}$$

Or more in Surds $\Big\}=abc+\sqrt{aaaccc}+\sqrt{bbb}+\sqrt{abc}$

Example 3.

Or in N°. thus: $\left.\begin{array}{r}4+\sqrt{64}\\ \times\text{ in }2+\sqrt{16}\end{array}\right\}$ By the 3d Examp.

$\left\{\begin{array}{l}=8+2\sqrt{64}+4\sqrt{16}+\sqrt{64}\sqrt{16}=72\\ \text{And so 'tis in reality for }4+\sqrt{64}=12\end{array}\right.$

And $2+\sqrt{16}=6$

And $12\times6=72$.

1. *Note*, That in multiplying b, multiplied in the Square Root of ac, by ac more the Square Root of b; I say $b\sqrt{ac}$ by $ac=abc\sqrt{ac}$, and stands so in the Example 1. More b (in the Multiplicand) $\sqrt{abc}$, &c.

2. *Note*, In like manner by the Numbers, Example 1. I say 2 times $4\sqrt{64}$ is $8\sqrt{64}$. 2*dly* $4\sqrt{64}$ by $\sqrt{16}$, is $=4\sqrt{64}\sqrt{16}$; so the Product is $=8\sqrt{64}+4\sqrt{64}\sqrt{16}$. Now the Square Root of 64 is 8, and of 16, 4; so that this Product is $8\times8=4\times8\times4=192$: And that this is right multiplied, will appear by the Numbers given, or $4\times$ in 8, (which 8 is the Square Root of 64,) gives 32 for the Value of the Multiplicand.

And 2, more 4, (which 4 is the Square Root of 16,) makes 6, which is the Value of the Multiplier; so 6 times 32 is $=192$ as before, which is a full Proof of the Rule by the first Example, and the rest are performed and proved much after the same manner, which it would be needless to insist farther upon.

Example 4. To multiply $\sqrt[3]{u}-cd$ by $\sqrt[3]{u}+cd$: See the Operation in the Margin.

$$\begin{array}{r}\sqrt[3]{u}-cd\\ \sqrt[3]{u}+cd\\ \hline \sqrt[3]{uu}-\sqrt[3]{cccdddu}\\ +\sqrt[3]{cccdddu}-ccdd\\ \hline\end{array}$$

$$\text{Prod.}=\sqrt[3]{uu}+\sqrt[3]{cccdddu}-\sqrt[3]{cccdddu}-ccdd$$

$$\text{Product contracted}=\sqrt[3]{uu}-ccdd$$

In Numbers demonstrated thus; $u=8$, $c=1$, $d=2$.

$$\begin{array}{r}\text{Then }\sqrt[3]{8}-2\\ \text{by }\sqrt[3]{8}+2\\ \hline \sqrt[3]{64}-\sqrt[3]{64}\\ +\sqrt[3]{64}-4\\ \hline \sqrt[3]{64}-4\end{array}$$

$\sqrt[3]{64}-4=$ the Sum or Product contracted.

Now the Cube Root of $64=4$, and $4-4=0$

So the $\sqrt[3]{8}=$ the Multiplicand $=2$, and $2-2=0\times2$ in the Multiplier ($=0$ the Proof.

5. Hence

5. Hence it appears, that if you multiply two Surd Numbers together, whose Radical Signs are $\sqrt{}$, the Square Root of the Product is the Answer in a Rational Number. But if the Root cannot be extracted without a Remainer, divide the Product by the greatest Square Number, that will do it without Remainer; so the Root of that Divisor in the Quote is the Answer, as in the first Case, $\sqrt{16}$ in $\sqrt{9} = \sqrt{144} = 12$ the Answer. And in the second Case, $\sqrt{72}$ in $\sqrt{8} = \sqrt{576}$, which $\div$ 144 quoteth 4, and the $\sqrt{144} = 12$; so that $12\sqrt{4}$ is the Answer.

VII. Division *of Compound Surds.*

This will appear very easy to those who understand common Division in Species, as before taught, and Multiplication of Surds.

Example, Divide $\sqrt{bu}+\sqrt{du}$ by $\sqrt{u}$; see the marginal Work.

$$\begin{array}{l} \qquad\qquad\qquad\qquad \text{Quote.} \\ \sqrt{u})\ \sqrt{bu}+\sqrt{du}\ (\sqrt{b}+\sqrt{d}. \\ \qquad \underline{\sqrt{bu}} \\ \qquad\qquad \sqrt{du} \\ \qquad\qquad \underline{\sqrt{du}} \\ \qquad\qquad 0 \text{ rests.} \end{array}$$

Example 2. To divide $abc+ac\sqrt{ac}+b\sqrt{b}+\sqrt{abc}$ by $b+\sqrt{ac}$, or in Numbers, $8+2\sqrt{64}+4\sqrt{16}+\sqrt{64}\sqrt{16}$ by $2+\sqrt{16}$.

Done Literally thus:

$$\begin{array}{l} b+\sqrt{ac})\ abc+ac\sqrt{ac}+b\sqrt{b}+\sqrt{abc}\ (ac+\sqrt{b} = \text{the Quote.} \\ \qquad\qquad \underline{abc+ac\sqrt{ac}} \\ \qquad\qquad\qquad\qquad b\sqrt{b}+\sqrt{abc} \\ \qquad\qquad\qquad\qquad \underline{b\sqrt{b}+\sqrt{abc}} \\ \qquad\qquad\qquad\qquad \text{rests} = 0 \end{array}$$

Done Numerally thus:

$$\begin{array}{l} 2+\sqrt{16})\ 8+2\sqrt{64}+4\sqrt{16}+\sqrt{64}\sqrt{16}\ (4\sqrt{64} = \text{Quote.} \\ \qquad\qquad \underline{8+4\sqrt{16}} \\ \qquad\qquad \underline{2\sqrt{64}-4\sqrt{16}+4\sqrt{16}} \\ \text{Or Contracted, and the rest brought down.} \Big\} = 2\sqrt{64}+\sqrt{64}\sqrt{16} \\ \qquad\qquad\qquad\qquad \underline{2\sqrt{64}+\sqrt{64}\sqrt{16}} \\ \qquad\qquad\qquad\qquad\qquad 0 \text{ remains.} \end{array}$$

There is nothing of Difficulty in these Operations, observing to change the Signs of what you substract: As in the Numeral Example, I am to take $4\sqrt{16}$ from $2\sqrt{64}$, therefore I say that $2\sqrt{64} - 4\sqrt{16}$ is the Remainer, to which if I bring down $4\sqrt{16}$, the $+$ and $-$ destroy the $4\sqrt{16}$: So that bringing down to the said Remainer $2\sqrt{64}$, the rest of the Dividend, which is $\sqrt{64}\sqrt{16}$, I find that $2+\sqrt{64}$ is contained in $2\sqrt{64}$; $\sqrt{64}$ times, which I put in the Quote, and multiply and deduct, so the Remainer is (0).

And the Truth is thus proved, 1*st*, In that it agrees with the Literal Example. 2*dly*, As it is the Product of Example the Third foregoing, divided by the Multiplier. And 3*dly*, and most plainly, thus:

The Value of the several Members of the Numeral Dividend are,

8 ———————— $= 8$
$2\sqrt{64}$, or 2×8 ———— $= 16$
$4\sqrt{16}$, or 4×4 ———— $= 16$
$\sqrt{64}\sqrt{16}$, or 8×4 ———— $= 32$

Value of the Divisor $2+\sqrt{16}=2+4=6$) $72=$ Sum ($12=$ Quote
$=4+8=4+\sqrt{64}$
for Proof.

SECT. VIII. *Algebraical Fractions.*

Although the Business of Fractions in Species are often used in the Solution of Analytical Questions: Yet since the Reader of this Part is supposed to have perfectly learnt *Reduction of Vulgar Fractions*, Chap. II. Sect. 1. and there being so very little Difference between the Method of working Fractions in Numbers and Species, therefore I shall not need to enlarge much on this Section.

1. *Note*, The fifth Case is done thus: Multiply the Quantity given by the proposed Denominator, and place the Product over the Denominator for the Answer.

2. And the sixth Case is only placing the Integral Quantity given as a Numerator, and a Unit for a Denominator.

3. And that the Quantities omitted, which are found in both the Terms, brings a Fraction into its lowest Terms.

The

The Cases of Reduction *of* Algebraical Fractions.

No. of Cases given.	To reduce	Examples, two to each Case.	The Answers.	Done by Rules *Ch.* II. *Sect* 1. and Cases.
1	Mixt Quantities to Fractions.	$ub+\frac{b}{c}$	$\frac{bcu+b}{c}$	1
		$u+\frac{b+c}{d}$	$\frac{du+b+c}{d}$	
2	An improper Fraction to a ~~whole or mixt~~ Quantity.	$\frac{bcu+b}{c}$	$bu+\frac{b}{c}$	2
		$\frac{du+b+c}{d}$	$u+\frac{b+c}{d}$	
3	A Fraction to its lowest Terms.	$\frac{ubbd}{ubc}$	$\frac{bd}{c}$	4
		$\frac{14udc+21dpu}{7dru}$	$\frac{2c+3p}{r}$	
4	Fractions of different to common Denominators.	$\frac{u}{c}, \frac{m}{r}, \frac{s}{t}$	$\frac{urt}{crt}, \frac{cmt}{crt}, \frac{crs}{crt}$	6
		$\frac{u+c}{m}, \frac{b+d}{gg}$	$\frac{ggu+cgg}{ggm}, \frac{bm+dm}{ggm}$	
5	A whole Quantity to a Fraction of the same Value, whose Denominator is given.	$b+c$ to that of Denominat. r	$\frac{br+cr}{r}$	
		$u+m$ to that whose Denominator $=r+s$	$\frac{ru+mr+su+ms}{r+s}$	
6	An Integeral Quantity to the Form of a Fraction.	bu	$\frac{bu}{1}$	
		$cu-pr+bb$	$\frac{cu-pr+bb}{1}$	
7	A Compound to a simple Fraction.	$\frac{u}{b}$ of $\frac{c}{d}$ of $\frac{r}{m}$	$\frac{cru}{bdm}$	3
		$\frac{3ub+cd}{m}$ of $\frac{r-5st}{b+n}$	$\frac{3bru+cbr-15bstu-5cdst}{bm+mn}$	

II. Addition, Substraction, Multiplication, *and* Division *of* Algebraical Fractions; *two* Examples *to each Rule.*

	Propositions or Examples.	Solutions or Answers.	Rules.
Addition	1. To $\frac{am}{u}$ add $\frac{2bb}{m}$.	Sum $= \frac{amm + 2bbu}{mu}$	Reduce the Fractions to a common Denominator and add the Numerators, as in Vulgar Fractions.
	2. To $\frac{u-r}{g}$ add $\frac{r}{g+u}$	$\frac{gu-gr+uu-ru+gr}{gg+gu}$	Note, $-gr$ and $+gr$ destroy each other.
Substraction	1. From $\frac{amm+2bbu}{mu}$ Take $\frac{2bb}{m}$	Rests $= \frac{am}{u}$	Reduce to a common Denominator and Work as in Vulgar, reducing the Remainers to their lowest Terms.
	2. From $\left.\frac{gu-gr+uu-ru+gr}{gg+gu}\right\}$ Take $\frac{r}{g+u}$	Rests $\frac{u-r}{g}$	
Multiplication	1. $\frac{2bu}{c}$ by $\frac{b}{n}$	Product $\frac{2bbu}{cn}$	Multiply the Numerators give $2bbu$, and the Denominator $= cn$ as in vulgar Fractions reducing $u + \frac{b}{r}$ into a Fraction, as by the first Reduction
	2. $u + \frac{b}{r}$ by $\frac{3mp}{s}$	Prod. $= \frac{3mpru + 3bmp}{rs}$	
Division	1. Divide $\frac{2bbu}{cn}$ by $\frac{b}{n}$	Quote $= \frac{2bu}{c}$	Work as in Vulgar, and reducing the Quotes to lowest Terms, shall be as you see.
	2. $\frac{3mpru+3bmp}{rs}$ by $\frac{3mp}{s}$	Quote $= u + \frac{b}{r}$	

These Examples above are very easy to such as have sufficiently acquainted themselves with *Vulgar Fractions*, and the two Examples in *Substraction* prove those in *Addition*, as those in *Division* do

do those in *Multiplication*; there is no need to illustrate these by Numbers, because that is done by comparing these with the like Examples in *Vulgar Fractions*. So that I proceed to

SECT. IX. *Rules to resolve Simple Equations.*

The Business of this is, (when unknown Quantities are intermixt with known,) to bring the unknown Quantity to possess solely one Side of the Equation (commonly that towards the left hand) now these complex Quantities being so either by the unknown Quantity, being added to, substracted from, multiplied in, or divided by known, or by most or all of these together; therefore Equations are resolved, or the Value of the unknown Quantity is discovered by *Addition*, *Substraction*, *Multiplication*, *Division*, and *Evolution*, &c. and that in a contrary Way to that by which the known and unknown Quantities are connected and placed together, *i. e.* those of *Substraction* solved by *Addition*; *Addition* by *Substraction*; *Multiplication* by *Division*; and those divided and standing Fraction-Ways are solved by *Multiplication*, as by these Examples.

Simple Equations Resolved.

1. *By Addition* - Note $\begin{cases} b=20 \\ c=12 \\ d=8 \end{cases}$ And that u is put for the unknown Quantity required.

Literally	Numerally
$u-d=b-c$	$u-8=20-12$
Add d $\quad$ d Add	Add 8 $\quad$ 8 Add
Sum $u=b-c+d$ the Answ.	Sum $u=20-12+8=16$

2. *By Substraction.*

$u+d=b+c$	$u+8=20+12$
Deduct d, $\quad$ d Deduct	Deduct 8, $\quad$ 8 Deduct
Rests $u=b+c-d$ the Answ.	$u=20+12-8=24$

3. *By Multiplication.*

$\frac{u}{4}=\frac{dc}{b}$	$\frac{u}{4}=\frac{96}{20}$
$u=\frac{4dc}{b}=$ the Answer.	Answer $u=\frac{96\times4}{20}=19\frac{1}{5}$

4. *By Division.* $du+cu=5b$ $u=\frac{5b}{d+c}$ the Anſwer.	$8u+12u=20\times5$ $u=\frac{100}{8+12}=5$
5. *By Addition, and Subſtraction, or Tranſpoſition* $u+c-d=b$ $u=b-c+d$ the Anſwer.	$u+12-8=20$ $u=20-12+8,=16$
6. *By Addition, Subſtraction, and Multiplication.* $\frac{u-d}{2}+c=b-d$ $u-d+2c=2b-2d$ $u=2b-d-2c$ the Anſwer.	$\frac{u-8}{2}+12=20-8$ $u-8+24=40-16$ $u=40-16+8-24$ Or $u=8$
7. *By Addition, Subſtraction, Multiplication, and Diviſion.* $\frac{cu+du}{b}+d-b=5c$ $cu+du+bd-bb=3bc$ $cu+du=3bc-bd+bb$ $u=\frac{3bc-bd+bb}{c+d}$ the Anſwer.	$\frac{12u+8u}{20}+8-20=3\times12$ or 36 $12u+8u+20\times8-20\times20=36\times20$ $12u+8u=720-160+400$ $u=\frac{720-160+400}{12+8}$ (or $u=48$)

Eight General Rules for reſolving Simple Equations, applied to the foregoing Examples.

1. *When one or more known Negative Quantities are connected with an unknown Affirmative, they are ſeparated, and the unknown made to poſſeſs ſolely one Side of the Equation (which is the ſame as to ſay that ſuch Equations are reſolved) by* Addition *only.* So in the Example of reſolving by *Addition*, I add the Quantity connected with u, as d to both Parts of the Equation, and the Sum is very plainly $u=b-c+d$.

The Reaſon of which is plain, and ſhews the Foundation of Tranſpoſition: (which Word ſee in the Alphabetical Account near the beginning of

of this Section) For adding the same Quantity or Number to both Parts of the Equation does preserve the Equality; but this Advantage is gained thereby, that $+d$ added to $-d$, destroys the Quantity $-d$, so that it vanishes and leaves u in Possession of one Side of the Equation, and that $+d$ being added to the other part of the Equation, makes $b-c+d$, to which u is therefore found equal. And the Reason appears still more plain in the Numerical Operation; for if $u-8$ be $=8$, (or $20-12$,) it must follow, that $u=8+8$ or 16, and this is no more than what is taught in *Substraction of intire natural Numbers.*

2. *When one or more known affirmative Quantities are connected with an unknown, that Equation is resolved by substracting such known Quantities from each part of the Equation.* Thus in the Example of the Resolving by *Substraction*, d taken from $+d$ (which is connected with u) leaves nothing but u on that Side of the Equation; and d deducted from the other part of the Equation, there rests $b+c-d$, to which therefore u is equal, and the Equation is resolved.

The Reason of this is evident from what was said of adding in the first Rule above; *for if equal Numbers or Quantities be either added to or taken from equal, the Sums or Remainers will be equal.* And the Reason plainly appears also in the Numerical Work; for if $u+8$ be $=32$, it follows that $u=32$, abating only the 8.

3. When in an Equation, there are several known Quantities on the same Side with the unknown, some Affirmative, and some Negative; it follows from the above Rules, that you may transpose such known Quantities to the other Side of the Equation, changing their Signs, as in the fifth Example: And this you may do both ways to the Right, or thence towards the left hand of the $=$, and that with either known or unknown Quantities, keeping close to the Rules above, your own Reason, and the Nature of the Question: I mean, you will not think it reasonable to make the unknown Quantity a Negative, by transplacing it, when the Reason and Nature of the Question does dictate, that what is on the other Side of the Equation, is so far from being equal to $-u$, that it ought to be at least equal to u, or to some absolute Number or Quantity which $-u$ is not.

4. When in an Equation, the unknown Quantity is divided by some Number or known Quantity, possessing one Side of the Equation; then every Member of such Equation must be multiplied by the Denominator of the Fraction, whereof the unknown Quantity is the Numerator, as in the third Example; I multi-

ply $\frac{u}{4}$ by 4, produceth u; and $\frac{dc}{b}$ by 4, produceth $\frac{4dc}{b}$; and so the Equation is resolved, the unknown Quantity being cleared and found $= \frac{4dc}{b}$, as the Numeral Work farther demonstrates; for equal Quantities multiplied by equal, produce equal, as well as adding and substracting.

The Reason is very evident: For if a fourth Part of u be $= \frac{dc}{b}$, it must follow, that the whole u must be equal to four times the $\frac{dc}{b}$, and the like holds for any other Divisor of the unknown Quantity, or Denominator as abovesaid.

5. Hence it appears, that in multiplying by the Denominator of such a Fraction, 1*st*, *That every Member of both Parts of an Equation must be multiplied.* And 2*dly*, *That the Numerator of that Fraction is multiplied by putting it down as a whole Number*, as $\frac{1}{2}$ × by 2, is $= 1$; $\frac{3}{4}$ by 4, is $= 3$; $\frac{5}{6}$ by 6, is $= 5$, &c.

6. *When Quantities known and unknown stand multiplied together in an Equation possessing one Side thereof, such Equations are solved by* Division; *in which Case the Quantities so multiplied in the unknown, are to be your Divisor*; as in the fourth Example or Division above. To separate u (in the two Members $du+cu$) from the known Quantity d and c; I divide both Parts of the Equation by $d+c$, and the Quotient is $u = \frac{5b}{d+c}$; *for equal Quantities divided by one and the same Quantity, must exhibit equal Quotients*; and at the time you free one Side of the Equation of complex Quantities, and shew the known, which the unknown Quantity is equal to. Thus in the Numeral Example, the Matter is plain, for where $8u+12u=100$; $20u$ is $= 100$; therefore u only must be equal to $\frac{100}{20}$, which is the same as in the Process, according to the Rule above; for $8u+12u \div 8+12=u$; and 20×5, or $100 \div 8+12=5$.

But Note, That when you have two Members, one mixt with a known Quantity, the other intirely unknown; in this Case, the known + or − 1 is your Divisor, as if $8u-u=49$, then $u = \frac{49}{8-1}$, and if $6u-u=25$, then $u=\frac{25}{6-1}=3$.

7. When

7. When (as in the seventh Example) 'tis found that unknown Quantities are so mixt with known, as to have Products, Quotients, Sums, and Remainers; you have nothing to do but to work as by these foregoing Rules, as in $\frac{cu+du}{b}+d-b=3c$: For if the whole is multiplied by b, it gives the next Step $cu+du+bd-bb=3bc$. And if $bd-bb$ be transposed, (as being all known Quantities,) the third Step stands thus; $cu+du=3bc-bd+bb$. And if the third Step be divided by $c+d$, (because those are the known Quantities that are multiplied in the unknown,) the fourth Step or Quote, which is the Answer, is $u=\frac{3bc-bd+bb}{c+d}$; each of which Steps appears still more plain in the Literal Work, which shews in this last Equation $u=48$.

8. There is one other Thing to be observed, but that is rather in composing or forming the Equation, than in the Resolution, *i. e.* If $u:c::b.d.$ what is the Value of u?

Here the second Step is found, as being the Rectangle of the two Extreams, which is equal to that of the means. And dividing by d, gives the Canon. See the Words *Analogy* in the beginning of this Chapter, and *Equation*.

$u.c::b.d$

$ud=bc$

$u=\frac{cb}{d}$ the Answer.

And if you would reduce an Equation into an Analogy, it is as u to c or b; so is the other to d in the two Examples above, or $\left\{\begin{array}{l} u.b::c.d \\ u.c::b.d, \text{ \&c.}\end{array}\right.$

SECT. X. *Resolution of Quadratic Equations.*

What simple and adfected Quadratics are, is shewed in the foregoing Definition of Terms near the beginning of this Chapter; where under the Words *Simple Quadratics*, I have said enough relating to that kind. As for adfected Quadratic Equations, (which Word see,) there are, by some Algebraists, reckoned three Sorts, but I make but two really distinct; to one of which, all other adfected Equations are reducible. I shall give Examples,

I. Of the two distinct Kinds of Square adfected Equations.

2*dly*, Of those reducible thereto, and how performed.

II. *Shew how these Equations are Resolved by compleating the Square.*

III. *How Resolved by Substitution; how done four different Ways, two of which (being the best) are new.*

1*st*, The

1st, The two distinct Forms of Quadratic Equations are these only.

In Species.	In Numbers.
1st, $uu+du=bc$ ——— ——— ———	$uu+6u=135$
2dly, $uu-du=\frac{bc}{5}$ ——— ——— ———	$uu-6u=27$

2dly, Equations reducible to one of the two Forms above, are these and such like.

Equations given.	Reduced to these Quadratics.
1st, $du-uu=m$	$uu-du=-m$, as the 2d Form.
2dly, $u+b=\frac{c-d+b}{2cu}$	$uu+bu=\frac{c-d+b}{2c}$, as the 1st Form.
3dly, $u=\frac{c+db}{3bu}+b$	$uu-bu=\frac{c+db}{3b}$, as the 2d Form.
4thly, $u=\frac{cu+tu}{3buu}-bb$	$uu+bbu=\frac{c+t}{3b}$, the 1st Form *ec.*

Notes upon these Reductions.

1st, That the first is performed by changing the Signs of all the Members, (as is directed in Rule the Third foregoing to *Simple Equations*) which makes $-m=-du+uu$, which is the same as to say $uu-du=-m$, as in the second Form; though some make this a distinct Sort, but with no more Reason than the three subsequent; for the Square is compleated, the Root extracted, and the Canon produced exactly as in one of the two Kinds abovementioned.

2dly, The second Equation above is reduced, as taught in the 4th and 6th Rules to the Examples of *Simple Equations*, multiplying by $2cu$, and dividing by $2c$ each part of the Equation.

3dly, The third Equation above is reduced by the second, fourth, and sixth Rules to the Examples of *Simple Equations*, transposing b, multiplying by $3bu$, and dividing by $3b$.

4thly, The fourth is reduced by multiplying by $3buu$, expunging u in each Member, and dividing the rest by $3b$.

II. *The*

II. *The Steps of resolving Quadratic Equations by compleating the Square.*

Equations propofed.	Squares compleated. 1*st*,	Roots extracted. 2*dly*,	Canons exhibited. 3*dly*,
$uu+du=bc$	$uu+du+\frac{dd}{4}=bc+\frac{dd}{4}$	$u+\frac{d}{2}=\sqrt{bc+\frac{dd}{4}}$	$u=\sqrt{bc+\frac{dd}{4}}-\frac{d}{2}$
$uu-du=\frac{bc}{5}$	$uu-du+\frac{dd}{4}=\frac{bc}{5}+\frac{dd}{4}$	$u-\frac{d}{2}=\sqrt{\frac{bc}{5}+\frac{dd}{4}}$	$u=\sqrt{\frac{bc}{5}+\frac{dd}{4}}+\frac{d}{2}$
$uu+2cu=3b$	$uu+2cu+cc=3b+cc$	$u+c=\sqrt{3b+cc}$	$u=\sqrt{3b+cc}-c$
$uu+u=2b$	$uu+u+\frac{1}{4}=2b+\frac{1}{4}$	$u+\frac{1}{2}=\sqrt{2b+\frac{1}{4}}$	$u=\sqrt{2b+\frac{1}{4}}-\frac{1}{2}$
$uu+6u=135$	$uu+6u+9=135+9$	$u+3=\sqrt{135+9}$	$u=\sqrt{135+9}-3$
$uu-6u=\frac{135}{5}$	$uu-6u+9=\frac{135}{5}+9$	$u-3=\sqrt{\frac{135}{5}+9}$	$u=\sqrt{\frac{135}{5}+9}+3$
$uu+u=90$	$uu+u+\frac{1}{4}=90+\frac{1}{4}$	$u+\frac{1}{2}=\sqrt{90\frac{1}{4}}$	$u=\sqrt{90\frac{1}{4}}-\frac{1}{2}$

Rules to be observed in this Method (which is the best) of the Resolution of Quadratic Equations, applied to the Examples above.

1*st*, *Whatsoever known Quantities are multiplied in the Root, or unknown, they are called the Co-efficients*; as d, $2c$, and 1 in the second Terms of the four first Examples: In the fourth of which there being no Co-efficient, therefore 1 must be taken for it, because a Unit is suppofed to stand before every Quantity, as u is $=1u$, $c=1c$, &c.

2*dly*, *To compleat any Square having only the two first Members given*, is done by adding the Square of half the Co-efficient: Thus in the first and second Examples, half the Co-efficients $\frac{d}{2}$, (the

(the Square of which is $\frac{dd}{4}$) added to the two other Members standing towards the left hand, the compleat Squares are $uu + du + \frac{dd}{4}$ and $uu - du + \frac{dd}{4}$. And in the third and fourth Examples $2c$ and 1 are the Co-efficients, the half of which are c and $\frac{1}{2}$; the Squares of which Halves are cc and $\frac{1}{4}$, the former of which compleats the Square of the two Members towards the left hand of the third Example, and the later (or $\frac{1}{4}$) added, doth compleat the Square of the fourth Example: See at the beginning of this Chapter.

3*dly*, And because this Addition of the Square of half the Co-efficients is made to one part of the Equations, where the unknown Quantities are; therefore it must be added also to the other part of the Equations, *That so the Equality may be continued*, (*as is said in the first and second Rules to the Examples of* Simple Equations,) and thus the second Column is made in all the seven Examples above.

4*thly*, The third Column from the left hand, are the Roots of those in the second.

1st, *For in all compleat Squares, the Square Roots are exactly those of the first and third Member.*

2dly, *The Sign connecting the Quantities of the Root, is always the same as that before the second Term of the compleat Square, or of the Equation given:* Thus the Roots of

Squares.		Roots.	
$uu + du + \frac{dd}{4}$	in the second Column, is	$u + \frac{d}{2}$	in the third Column.
$uu - du + \frac{dd}{4}$	is ——— ——— =	$u - \frac{d}{2}$	
$uu + 2cu + cc$	——— ——— =	$u + c$	
$uu + u + \frac{1}{4}$	——— ——— ——— =	$u + \frac{1}{2}$,	&c.

And for a Proof, any of these Roots being multiplied in itself, produces the respective Square above.

3*dly*, The

3dly, The Root of the 1ſt Part being extracted, in order to diſcover the Value of the ſingle unknown Quantity; therefore the Roots of the Quantities known, ſtanding on the other part of the Equation, (as is ſaid under the 3d of adding the Square of half the Coefficient) muſt have their Roots extracted; which is done by prefixing the Radical Sign, becauſe they are generally Surds.

4thly, To reduce the 3d Column to the 4th, or to the Canons or Anſwers, you have nothing to do but to tranſpoſe the Root of the 3d Member of the compleat Square, as $\frac{d}{2}$, c, and $\frac{1}{2}$ in the four firſt Examples changing the Sign as taught.

A farther Demonſtration or Proof of the Method for the Solution of Quadratics, by the Numeral Operations above.

In the Numeral Work theſe Numbers are equal to the Letters in the Specious, *viz.* $d=6$, $b=45$, $c=3$, and u is found equal to 9.

Here (proceeding by the Rules for Solution above) in the Equation $uu+6u=135$, u in the 4th Column is found $=\sqrt{135+9}-3$.

Now $135+9$ is $=144$, the Square Root of which is 12, whence take 3, and the reſt is $=9$, ſo $u=9$.

And u being found $=9$ by the Canon, grounded upon the Method of Proceſs aboveſaid; if ſuppoſing $u=9$, the 2 Parts of the Equations propoſed prove equal, it fully proves the Truth of the Method for Solution.

So $uu+6u=81+54$ in the 5th Equation given, is $=135=$ the other part of that Equation given.

$uu-6u=81-54$ in the 1ſt part of the 6th Equation is $=27=\frac{121}{3}$ the 2d part of that Equation.

And $uu+u=81+9$ in the 1ſt part of the 7th Equation is $=90$ in the 2d part of that Equation, &c.

Thus I have given the moſt plain and demonſtrative Rules and Examples for the underſtanding and proving the Method of ſolving Square Equations by the way of compleating the Square. I proceed to ſhew,

III. *The Method of Solving Quadratic Equations by Substitution.*

N°. of Steps.	*Steps towards the Answer.*	*How each Step is produced.*	*N°. of Steps.*	*Process towards an Answer.*	*Rules for each Step.*
1	$uu+cu=18d$ ——	$=$ *Equation given.*	1	$uu-cu=9d=$ ——	*the Equation given*
2	for u is put $a-\frac{c}{2}$ in this 1st Case.		2	for u is put $a+\frac{c}{2}$ in this 2d Case $u=a+\frac{c}{2}$	
3	$aa-ac+\frac{cc}{4}$ —— ——	$\left\{ \begin{array}{l} =\square\,\overline{a-\frac{c}{2}}=uu \\ \text{or } a-\frac{c}{2}\ \ominus \end{array} \right.$	3	$aa+ac+\frac{cc}{4}$ —— ——	$\left\{ \begin{array}{l} =\square\, a+\frac{c}{2}=uu \\ \text{or } \overline{a+\frac{c}{2}}\ @ \end{array} \right.$
4	$ac-\frac{cc}{2}$ —— ——	$=a-\frac{c}{2}\times c-cu$	4	$-ac-\frac{cc}{2}$ —— ——	$=a+\frac{c}{2}\times-c$
5	$aa-ac+\frac{cc}{4}+ac-\frac{cc}{2}=18d$	$=$ 3d Step $+$ 4th.	5	$aa+ac+\frac{cc}{4}-ac-\frac{cc}{2}=9d$	$=$ 3d $+$ 4th Steps.
6	$aa-\frac{cc}{4}=18d$ —— ——	$=$ 5th contracted.	6	$aa-\frac{cc}{4}=9d$ —— ——	$=$ 5th contracted.
7	$a=\sqrt{18d+\frac{cc}{4}}$ —— ——	$=$ 6th *ww*, and $\frac{cc}{4}+$	7	$a=\sqrt{9d+\frac{cc}{4}}$ —— ——	$=$ 6th *ww* and $\frac{cc}{4}+$
8	$u+\frac{c}{2}=\sqrt{18d+\frac{cc}{4}}$	$u+\frac{c}{2}$ put for a	8	$u-\frac{c}{2}\sqrt{9d+\frac{cc}{4}}$ ——	$\left\{ \begin{array}{l} u-\frac{c}{2} \text{ put for } a \\ \text{in the 7th Step.} \end{array} \right.$
9	$u=\sqrt{18d+\frac{cc}{4}}-\frac{c}{2}$	$\frac{c}{2}$ in the 8 $+$	9	$u=\sqrt{9d+\frac{cc}{4}}+\frac{c}{2}$ ——	$=-\frac{c}{2}+$ in the 8th Step.
	A 2d Example see towards the right hand.				

I have here inserted the Operations at large, to make them more plain and easy; which the 3d and 6th Columns from the right hand do so fully contribute to, that I have little to add, only,

1. To put the Reader in mind, that the Signs connecting the substituted Quantities are always contrary to that which connecteth the 2 Members of the Square given, as is plain by the 2 Examples above.

2. That any Letter as well as a (if it be not in the Equation given) $+$ or less half the Co-efficient, as above, may be substituted.

3. In the 8th Step I omit a, and substitute in place thereof $u+\frac{c}{2}$ in the 1st Example, because if $a-\frac{c}{2}=u$, then by transposing $-\frac{c}{2}$, a is $=u+\frac{c}{2}$, and in the 2d Example $a=u-\frac{c}{2}$.

A New and Better Way.

But tho' this is the only Way I know extant of solving these Equations by Substitution; *yet I am sure it is a better, to make the Vowel and half the Co-efficient in the Quantities given to be your Substitute:* and then it will fall right at the last, remembering that half the Co-efficient is to be added or deducted, and 'twill save some Steps of working. For Example, putting $u-\frac{d}{2}$ for u, in the Equation $uu+du=bc$.

$$uu-du+\frac{dd}{4}=\Box\, u-\frac{d}{2}$$

$$du-\frac{dd}{2}=u-\frac{d}{2}\times d.$$

$$uu-\frac{dd}{4}=bc;\ \left\{\textit{the Sum of the 2 last contracted.}\right.$$

$$u=\sqrt{bc+\frac{dd}{4}}-\frac{d}{2}=\textit{the Canon.}$$

Here it only needs to be noted, that when the Sign connecting the 2 first Terms of the Equation given is $+$, I make the half Co-efficient added to the Surd Root of the Answer $-$; and the contrary.

But in truth this way of solving Equations by Substitution, is at the best not so brief and easy a Method as that by compleating the Square; for if that be well considered, there arises from it,

IV. *These Examples and Rules for solving most Quadratic Equations by Inspection only.*

Examples. Note, $c=3$, and u found $=9$.	*Demonstrations of the Truth of the Solutions or Canons.*
1. Equations proposed. } $uu+10u=57c$	$\sqrt{57c+25}-5.=9$: so $u=9$ as *per* Canon 1.
Canon —— $u=\sqrt{57c+25}-5$	and $57c=171$: so is $uu+10u=171$ as *per* Equation 1.

2. Equations propofed.	$uu - 6u = 9c$	$\sqrt{9c+9}+3=9$: fo $u=9$ as *per* Canon 2.
Canon	$u=\sqrt{9c+9}+3$	$9c=27$: fo is $uu-6u=27$ as *per* Equation 2.

The Rules for Solving a Quadratic Equation by Inspection.

1. What I mean here, *by Inspection*, is the putting down only the Canon or Anfwer without any previous Operation: To do which, take the Root of the Square of the unknown Quantity, which here is u.

2. Make the Sign of Equation, and to the right hand of that, place the Root of the unknown Quantity or Quantities given, with the Sign $\sqrt{}$ before, as in Equation the 1ft, $u=\sqrt{57c}$.

3. Square half the Co-efficient, and add to the laft mentioned Quantity with the Sign $+$ always before it, and it will be $u = \sqrt{57c+25}$.

4. To the laft add half the Co-efficient, when the Sign is $-$, and fubftract it when the Sign before the Quantity where the Co-efficient is, is $+$; as in this Example I put -5, becaufe $10u$ is $+10u$: fo is the Canon $u=\sqrt{57c+25}-5$.

And thus you may give the Canon for any Quadratic Equation when reduced into either of the two Forms above.

Here is the proper place for giving the Geometrical Conftruction of Simple and Quadratic Equations; which I omit for thefe four Reafons: 1. Becaufe it is done by fo many other Authors. 2. For that this Treatife is defigned to be as eafy as poffible, in order to inftruct and encourage the Tyro, rather than to make a fhew of Learning. 3. It cannot be expected, where fo many things are contained in one fmall Volume. And, 4. It is fufficiently proved and demonftrated above, by comparing the Parts of the Canon and thofe of the Equation given, as grounded on the Solution, efpecially in the Numeral Operations.

SECT. XI. *Containing for Exercife, fome Queftions refolved by Simple and Quadratic Equations, and by various Pofitions.*

I. BY *Simple Equations.* I had the following propofed to me, which the Querift, my Friend, had a real Occafion to know, as falling in his Bufinefs.

Problem 1.] 365 Pounds is received for the Ufe of a publick Building, and the Receiver was to have 4 *d per* Pound for what he paid in: what was the Poundage, and what paid in?

Steps

Steps.	Done Literally thus.	The Column of Rules to both Ways	Numerally thus.	Steps.
1	u= the Poundage, and b is $=365$		$u=$ the Poundage.	1
2	$b-u=$ the Sum paid in.		$365-u=$ the Sum paid in.	2
3	$60\,u=b-u$		$60\,u=365-u$	3
4	$61\,u=b$	$-u$ m̃ 3d +	$61\,u=365$	4
5	$u=\dfrac{b}{61}$ the Canon.	the 4th ÷ 61	$u=\dfrac{365}{61}=$ the Canon.	5
	So $\dfrac{b}{61}=$ the Poundage. And $b-\dfrac{b}{61}=$ the Sum paid in.		$365\div 61=l.\ 5:19:8$ and $365-\dfrac{365}{61}=359:0:4$	

Problem 2.] *The whole Length of a May-Pole is 16 Yards, the major part of which is broke by the Wind, tho' not quite off, but so that the Top of the Pole extends from the Fracture to the Ground at 8 Yards distant from the Appearance of the Pole above the Surface of the Earth in which it was placed: how much is broken off, and how much is standing?*

Steps.	I put $16=k$ and $8=n$—			Steps.
1	$u=$ the Perpendicular or (Part standing.	*Rules*, viz.	$u=$ the part standing.	1
2	$k-u=$ Hypothenuse.		$16-u=$ the Hypothenuse	2
3	$nn+uu=kk-2ku+uu$	$n\odot+1\text{st}\odot+2\text{d}\odot$	$8^2+u^2=16^2-2\times16u+u^2$	3
4	$2ku=kk-nn$	$2ku+$, $nn+$	$2\times16\,u=16^2-8^2$	4
5	$u=\dfrac{kk-nn}{2k}=$ the Canon.	4th $\div\ 2k$	$u=\dfrac{16^2-8^2}{2\times16}=$ the Canon.	5
	Demonstration of both. $k^2=256-n^2=64=192\div 2k=32=u=b.$ Vid. *Euclid*'s Elem. *Lib.* 1. *Prop.* 47.		$16^2=256-8^2=64=192\div32=6=$ the part (standing.	

Problem 3.] *Numbers in Geometrical Progression being given, to find the Rule for the Aggregate of any Series. This is shewn in* Progression, *Chap.* II. *pag.* 82.

Problem 4.] *Supposing the Pendulum of a Clock which vibrateth Seconds (or 60 times in a Minute) be 39.2 Inches in length; how long must a Pendulum be to, vibrate as often in a Minute as it hath Inches in length?* This also was a Question proposed for me to answer, which I did thus:

Steps.

I put $k=60$ Seconds: and $n=39.2$ Inches.

Steps.		Rules for the Work.		Steps.
1	$n=$ the N° of Vibrat. requir'd.		$u=$ N° of Vibrat.	1
	The Analogy. $\square$ Vib. Le. Pen. $\square$ Vib. Le. Pen.			
2	$kk. \quad n. \quad uu. \quad \frac{kkn}{uu}$	Or $uu. kk :: n. \frac{kkn}{uu}$ in a direct Pro-(portion.	$60^2.39.2 : uu.39.2 \times 60^2 \div uu$	2
3	$u=\frac{kkn}{uu}$ —— ——	$uu\ uu = \frac{kkn}{uu}$ or $\sqrt{uu}=$ &c.	$u=\frac{60^2 \times 39.2}{uu}$	3
4	$uuu=kkn$	3d Reduced.	$uuu=60^2 \times 39.2$	4
5	$u=\sqrt[3]{kkn}$	$=\sqrt[3]{}$ of the 4th	$u=\sqrt[3]{60 \times 39.2}$ (*viz.* of 141120	5

$kk=3600, \times n(=39.2)=141120$; and $\sqrt[3]{141120}=52$ near. | $\sqrt[3]{141120}=52=$ the Vibrations in a Minute,

And Inches for the Length of such a Pendulum.

Problem 5.] *What Age is that Person when $\frac{2}{3}$ of the Time from his Birth is equal to $\frac{7}{8}$ of what he wants of 60?*

I put $60=k$, $\frac{2}{3}=n$, and $\frac{7}{8}=t$.

Steps.		Rules for the Steps.		Steps.
1	$u=$ the Person's supposed Age.		$u=$ his Age.	1
2	$k-u$ the time he wants of 60.	or $60-u$	$60-u$	2
3	$\frac{u}{n}=\frac{k-u}{t}$	$u \div n$, & $k-u \div t$	$u \div \frac{2}{3} = 60 - u \div \frac{7}{8}$	3
4	$u=\frac{kn-nu}{t}$	the 3d $\times$ by n	$u=\overline{60 \times \frac{2}{3} - u \times \frac{2}{3}} \div \frac{7}{8}$	4
5	$tu=kn-nu$	the 4th $\times$ by t	$\frac{7u}{8}=60 \times \frac{2}{3} - u \times \frac{2}{3}$	5
6	$tu+nu=kn$	nu in 5, +	$\frac{7u}{8}+\frac{2u}{3}=60 \times \frac{2}{3}$	6
7	$u=\frac{kn}{t+n}=$ the Canon.	6th $\div t+n$	$u=60 \times \frac{2}{3} \div \frac{7}{8} + \frac{2}{3}$	7

So that $\frac{kn}{t+n}=$ the Age; and $k-\frac{kn}{t+n}=$ it wants of 60.	$u=25\frac{35}{37}$ *Years*, the Age, and $60-25\frac{35}{37}=34\frac{2}{37}$ *Years*, the want of 60.
And $\frac{kn}{t+n} \div n = k - \frac{kn}{t+n} \div t$ for Proof.	And $25\frac{35}{37} \div \frac{2}{3} = 34\frac{2}{37}$ *Years* $\div \frac{7}{8}$ for a Proof.

Problem 6.] *Two Persons,* A *and* B, *have each a certain Number of Pounds, so that if* B *give* A 3 *of his, then they will have each a like Number: But if* A *give* B *three of his Pounds, then* B *will have* 5 *times as many as* A: *What Number of Pounds had each at first?*

I put $n=3$ and $k=5$.

1	$u=B$ hath at first.	u supposed	$u=B$ hath at first.	1
2	$u-2n=A$ hath at first.	1st$-\frac{2n}{2}$	$u-3\times2$	2
3	$u-n=B$ will have.		$u-3$	3
4	$u-n=A$ will have.	*ibid.* as 3d.	$u-3$	4
5	$u-3n=A$ will after have.	$=u-2n-n$	$u-9$	5
6	$u+n=B$ will then have.		$u+3$	6
7	$ku-3kn=u+n$	5th$\times k+$6th	$5u-3\times5\times3=u+3$	7
8	$ku-u=n+3kn$	$u+$ & $3ku+$	$5u-u=3+3\times5\times3$	8
9	$u=\frac{n+3kn}{k-1}$ the Answer.	8th $\div$ $k-1$	$u=\frac{3+3\times5\times3}{4}$ Answer.	9

$\frac{u+3kn}{k-1}=4n$, which B had. And A had $2n$: for $2n-n=n$: and $n+4n=5n$.	$\frac{3+3\times5\times3}{4}=12$, w^{ch} B at 1st had. And A had 6: for $6-3=3$; and $3+12=5\times3=15$.

Problem 7.] *A Hare, 50 of her Steps or Paces before a Greyhound, takes 4 Steps to the Dog's three; but then the Dog steps as much at twice as the Hare does at thrice: How many Steps must the Dog make before he catch the Hare?* This old Algebraic Question I answer thus:

I put $k=50$, $n=3$, $s=2$, and $b=4$.

1	$u=$ the supposed Answer (or Dog's Steps.		u	1
2	$n.b::u.\frac{bu}{n}$ all the Hare's (actual Steps.		$3.4::u.\frac{4u}{3}$	2
3	$n.s::\frac{bu}{n}+k.\frac{bsu}{nn}+\frac{ks}{n}$ (the Dog's Steps.		$3.2::\frac{4u}{3}+50.\frac{8u}{9}+\frac{100}{3}$	3
4	$u=\frac{bsu}{nn}+\frac{ks}{n}$	the 4th$\times nn$, &c.	$u=\frac{8u}{9}+\frac{100}{3}$	4
5	$nnu-bsu=kns$		$9u-8u=\frac{900}{3}$. *Ergo*	5
6	$u=\frac{kns}{nn-bs}$ the Canon for (Answer.	the 5th $\div$ $nn-bs$	$u=\frac{900}{3}=300$ Steps, the (Answer.	6

Problem 8.] A Ciſtern that hath four Vents of different Dimenſions containeth 72 Barrels of Liquid: If one Vent or Cock be opened, the Liquid will be ſpent in 6 Hours; if a ſecond be opened alone, it will run out in 8 Hours; if the third Cock be turned alone, the Liquid will be exhauſted in 9 Hours; and the fourth Cock or Vent is ſo ſmall, that if it alone be opened, it will be twelve Hours before the Ciſtern is empty: Now the Queſtion is, *in what Time the Ciſtern will be void of Liquid*, if all the four Vents be opened at the ſame Moment of Time. This Queſtion (which I have not as I know of, ſeen before anſwered by Algebra) I ſolve thus; tho' there is ſomewhat a ſhorter way: but I deſign it plain to the Learner, and if 72 or k be omitted and ſuppoſed 1, it will come to the ſame Canon, as you may make trial at your leiſure, for k is thrown out even in this Example.

For 72 I put k, $6=n$, $8=s$, $9=t$, $12=r$.

	$u=$ the unknown Time or ſuppoſed Anſwer.		u
1	$n.\ k :: u.\ \frac{ku}{n}$		$6.\ 72 :: u.\ \frac{72u}{6}$
2	$s.\ k :: u.\ \frac{ku}{s}$		$8.\ 72 :: u.\ \frac{72u}{8}$
3	$t.\ k :: u.\ \frac{ku}{t}$		$9.\ 72 :: u.\ \frac{72u}{9}$
4	$r.\ k :: u.\ \frac{ku}{r}$		$12.\ 72 :: u.\ \frac{72u}{12}$
5	So that $\frac{ku}{n}+\frac{ku}{s}+\frac{ku}{t}+\frac{ku}{r}=k$		$\frac{72u}{6}+\frac{72u}{8}+\frac{72u}{9}+\frac{72u}{12}=72$
6	$\frac{u}{n}+\frac{u}{s}+\frac{u}{t}+\frac{u}{r}=1$	5th Contract. or $\div$ by k	$\frac{u}{6}+\frac{u}{8}+\frac{u}{9}+\frac{u}{12}=1$
7	$u+\frac{nu}{s}+\frac{nu}{t}+\frac{nu}{r}=n$	the 6th $\times\ n$	$u+\frac{6u}{8}+\frac{6u}{9}+\frac{6u}{12}=6$
8	$su+nu+\frac{nsu}{t}+\frac{nsu}{r}=ns$	the 7th $\times\ s$	$8u+6u+\frac{6u\times 8}{9}+\frac{6u\times 8}{12}=6\times 8$
9	$stu+ntu+nsu+\frac{nstu}{r}=nst$	the 8th $\times\ t$	$8u\times 9+6u\times 9+6u\times 8+\frac{6u\times 8\times 9}{12}=6\times 8\times 9$
10	$rstu+nrtu+nrsu+nstu=nrst$	the 9th $\times\ r$	$8u\times 9\times 12+6u\times 9\times 12+6u\times 8\times 12+6u\times 8\times 9=6\times 8\times 9\times 12$
11	$u=\frac{nrst}{rst+nrt+nrs+nst}=$ Anſ.	the 10th $\div$ all that are $\times\ u$	$u=\frac{6\times 8\times 9\times 12}{8\times 9\times 12+6\times 9\times 12+6\times 8\times 12+6\times 8\times 9}$
	And $\frac{nrst}{rst+nrt+nrs+nst}=\frac{5184}{2520}$ Hours.		Numer. $=5184$; Denom. $=2520=$ ho. 2 : ′ 3 : ″ $25\frac{5}{7}$ which is the accurate Anſwer.

Problem 9.] How shall I find out a Canon for discovering the Discount of Money? See the Operation.

$d=$ Days to come. $c=100l.$
$p=$ Principal. $r=$ Rate of Int.
$y=$ a Year, or 365 Days.
$u=$ the Discount sought.

$$y.\ r::d.\ \frac{dr}{y}$$

And $c+\frac{dr}{y}.\ \frac{dr}{y}::p.\ u$

$$cu+\frac{dru}{y}=\frac{dpr}{y}$$

or $cuy+dru=drp$, and consequently $u=\dfrac{dpr}{cy+dr}$ the Canon for Discount.

Problem 10.] To find a Canon for discovering the present Worth of Money due at the end of any Days. Symbols as above.

$$y.\ r::d.\ \frac{dr}{y}$$

$$c+\frac{dr}{y}.\ c::p.\ u$$

$$cu+\frac{dru}{y}=cp$$

$cuy+dru=cpy$; and consequently $u=\dfrac{cpy}{cy+dr}=$ the Canon for present Worth.

These Canons apply'd under *Sect.* 8. *Chap.* 3. find the Discount of Money much sooner than by the common Way of the Rules of Proportion.

II. *To resolve Questions by Quadratic Equations, which they form.*

Problem 1.] There are three Numbers in Geometrical Proportion continued; the Mean $=42$, the Difference between the Extremes $=112$: *What are the two Extremes?* I put $k=112$; $n=42$.

1	$u=$ the lesser Extreme.		$u=$ the lesser Extreme.	1
2	$u+k$ the greater.		$u+112=$ the greater.	2
3	$uu+ku=nn$	2d × 1st $=n^2$	$uu+112u=42^2$	3
4	$u=\sqrt{nn+\frac{kk}{4}}-\frac{k}{2}$ *Answ.*	3d ω; and $\frac{k}{2}$ +	$u=\sqrt{42^2+56^2}-56$	4

$\sqrt{nn+\frac{kk}{4}}-\frac{k}{2}=14=$ the lesser. | $\sqrt{42^2+56^2}-56=14$ the lesser.

And $14+k=$ the greater Extreme. | And $14+112=126$ the greater.

And for Proof $14\times126=42^2=1764$.

Problem 2.] To divide a Line into extreme and mean Proportionals: or (which is one Use thereof) to find the side of the Dodecahedron

hedron by having that of the Octahedron given, whereof this is the greater proportional part.

A Line is thus divided, when the Square of the greater Part is = the Rectangle under the whole Line and the lesser Part.

I put $k=$ to the Side of the Octahedron ($=ab$) 120. a ———— c ——— b

1	$u=$ the greater part (or the side of the Dodecahed.)		$u=ac$ of the Line ab.	1
2	$k-u=$ the lesser part (as cb)	the 1st ⊖; & $k\times$ in 2d	$120-u=cb$ of the Line ab.	2
3	$uu=kk-ku$	$ku+$	$uu=120^2-120u$	3
4	$uu+ku=kk.$	the 3d uw and $\frac{k}{2}+$	$uu-120u=120^2$	4
5	$u=\sqrt{kk+\frac{kk}{4}}-\frac{k}{2}=ac$ (Answ.		$u=\sqrt{120^2+\frac{120^2}{4}}-\frac{120}{2}$	5

So that $u=ac$ of the Line $ab=$ the Side required.	And $\sqrt{120^2+\frac{120^2}{4}}-\frac{120}{2}=74.16$ (near.

Problem 3.] A Merchant sold a Ship for *l.* 2400, and gained after the Rate of what the Ship cost by laying out 10000 Pounds: What did the Vessel cost him?

I put $s=2400$, and $t=10000$.

1	$u=$ supposed Cost.		u	1
2	$s-u=$ the Gain.		$2400-u$	2
3	$u.s-u::t.\frac{st-tu}{u}$		$u.2400-u::10000.\frac{\overline{2400-u}\times10000}{u}$	3
4	$u=\frac{st-tu}{u}$	or the 1st = 4th Proportional.	$u=\frac{\overline{2400-u}\times10000}{u}$	4
5	$uu+tu=st$	4th $\times u$ and $tu+$	$uu+10000u=24000000$	5
6	$u=\sqrt{st+\frac{tt}{4}}-\frac{t}{2}$ $=2000$	5th Step uw, &c.	$u=\sqrt{24000000+25000000}-5000$ (=2000	6

And for Proof 2000. 400 :: 10000. 2000

Problem 4.]

Problem 4.] Admit *bin* (*Fig.* 9. *Plate* C.). to be the Curve of an Hyperbola, and that there are given

t ($=io$) the *Latus transversum* $=7.4$
l ($=ib$) the *Latus rectum* $=1.76$
o ($=zb$) the greatest Ordinat $=4.68$

What is the Length of the Abscissa, (iz) for which I put u because unknown?

1	$oo=lu+\frac{luu}{t}=$ the Property of the Curve.		$4.68^2=1.76\times u+\frac{1.76\times uu}{7.4}$	1
2	$ltu+luu=oot$	1st × by t	$1.76\times7.4\times u+1.76\times uu=4.68^2\times7.4$	2
3	$uu+tu=\frac{oot}{l}$	2d ÷ by l	$uu+7.4\times u=\frac{4.68^2\times7.4}{1.76}$	3
4	$u=\sqrt{\frac{oot}{l}+\frac{tt}{4}}-\frac{t}{2}$ $=6.7$	the 3d ɯ, & $\frac{t}{2}+$	$u=\sqrt{\frac{4.68^2\times7.4}{1.76}+\frac{7.4^2}{4}}-\frac{7.4}{2}$ $=6.7$	4

Problem 5.] What Number is that, whose Square multiplied by 10, being added to its Biquadrate, the Sum makes 30251?

I put $k=30251$, and $10=n$

1	u = the Answer unknown.		u	1
2	$u^4+nuu=k$	2d $+\frac{n^2}{2}$	$uuuu+10uu=30251$	2
3	$uuuu+nuu+\frac{nn}{4}=k+\frac{nn}{4}$	or $+\frac{nn}{4}$	$u^4+10uu+\frac{10^2}{4}=30251+\frac{10^2}{4}$	3
4	$uu+\frac{n}{2}=\sqrt{k+\frac{nn}{4}}$	3d ɯ	$uu+\frac{10}{2}=\sqrt{30251+\frac{10^2}{4}}$	4
5	$u=\sqrt{\sqrt{k+\frac{nn}{4}}-\frac{n}{2}}$	4th Step ɯ and $\frac{n}{2}+$	$u=\sqrt{\sqrt{30251+\frac{10^2}{4}}-\frac{10}{2}}=13$	5

And for Proof $13^2=169\times10=1690+13^4$ $(=28561)=30251$.

The last Canon for the Answer $=u$, is thus to be read: That if half n be deducted from the Square Root of $k+\frac{nn}{4}$, the Square Root of the Remainer is $=u=13$.

Problem 6.] A Farmer left his Son and a Nephew *l.* 400 to be so divided between them, that their Parts being alternately divided by each other, the Sum of the Quotients will be 30; but the greater part he devised to his Son: What did he leave to each?

Some make this Question, which I propose as above, to be solved by various Positions, others by the Doctrine of Surds: But I am sure it is most naturally reducible to the 2d of my 2 Cases of Quadratics before mentioned. And because I have not seen it done before, I shall insert the Solution at large as it is most easily understood, and that by the numeral Way; but shall wave the literal Method on purpose that the Reader may try his own Proficiency therein.

1	$u =$ the Son's Share.	
2	$400 - u =$ the Nephew's Share.	$400 =$ the 1st Step.
3	$\frac{u}{400-u} + \frac{400-u}{u} = 30$	1st ÷ 2d, and 2d ÷ by 1st
4	$u + \frac{160000 - 800u + uu}{u} = 12000 - 30u$	the 3d × by $\overline{400-u}$
5	$uu + 160000 - 800u + uu = 12000u + 30uu$	the 4th × by u
6	$160000 = 12800u - 32uu$	the 5th contracted, and $2uu - 800u +$
7	$5000 = 400u - uu$; and *Ergo*	the 6th ÷ by 32
8	$uu - 400u = -5000$ an Equation of the 2d sort.	5000 (in the 7th + and $400u - uu$ +
9	$uu - 400u + 40000 = 35000$	$\frac{400}{2}$ Q and added to each part of the Equation.
10	$u - 200 = \sqrt{35000}$	the 9th ω
11	$u = \sqrt{35000} + 200$	-200 in the 10th +

And

And $\sqrt{35000}+200=$ Son's Share $=l.\,387.083$; and $400-387$, &c. $=12.917$

And for Proof $387.083 \div 12.917=29.966$; and $12.917 \div 387.083=.034$; and $.034+29.966=30$: which answers in every part the Requisites.

Problem 7.] What Number is that by which if we divide 20, and to that Quote add 4 times that unknown Number the Sum is 24?

I put $b=4$, $t=20$, $c=24$.

$$\frac{t}{a}+ab=c$$
$$t+aab=ac$$
$$\left.\begin{array}{l} aab-ac=-t \\ aa-\dfrac{ac}{b}=-\dfrac{t}{b} \end{array}\right\}$$
$$aa-\frac{ac}{b}+\frac{cc}{4bb}=-\frac{t}{b}+\frac{cc}{4bb}$$
$$a-\frac{c}{2b}=\sqrt{-\frac{t}{b}+\frac{cc}{4bb}}$$
$$a=\sqrt{-\frac{t}{b}+\frac{cc}{4bb}}+\frac{c}{2b}=\text{ true}$$
$$(\text{Answer}=5=5=a$$

I have inserted this last Question that the Learner may see how to manage when the 2d Term (or the known Number multiplied in the Root) is a Fraction; as in the 4th Step foregoing.

III. *The Resolution of Questions which require various Positions.*

Having in *Sect.* 9, and 10. given Rules for solving Simple and Quadratic Equations; there was no need in *Sect.* 11. that I should repeat any thing farther than what I have done in the middle Column of Rules: But in this Head of various Positions having said nothing thereon before, I shall make it as plain as I can.

1. Then

1. Then by various Positions we mean Questions which have more Numbers than 1 sought for, and are discovered by assuming a Vowel for each Number sought.

2. And we proceed in the Work first with one unknown Number, not regarding the rest but as known: And when we have found the first unknown Quantity equal to some others, we omit that Quantity, and take instead thereof what it is equal to. Examples, and Rules thereupon, will make the matter plain.

Problem 1.] There is a Composition of 20 Integers, whose total Value is 82; but there are 2 different prized Ingredients in the 20, 1 of 4, the other of $4\frac{1}{4}$: How much of each Simple is there in the said Composition, and what the Value of each Ingredient in the whole of it, to make up the 82?

I put $q=20$, $4=k$, $4\frac{1}{4}=n$, and $82=t$.

Steps.		*Rules for the Operations.*		Steps.
1	$u=$ the Quantity at 4		u	1
2	$a=$ the Quantity at $4\frac{1}{4}$		a	2
3	$ku+an=t$	the 1st and 2d $\times k$ and n $an+$	$4u+4\frac{1}{4}=82$	3
4	$ku=t-an$		$4u=82-4\frac{1}{4}a$	4
5	$u=\frac{t-an}{k}$*	4th $\div$ by k	$u=\frac{82-4\frac{1}{4}a}{4}$	5
6	$\frac{t-an}{k}+a=q$	2d part of 5th $+a=q$	$\frac{82-4\frac{1}{4}a}{4}+a=20$	6
7	$t-an+ak=kq$	the 6th $\times$ by k	$82-4\frac{1}{4}a+4a=80$	7
8	$t-kq=an-ak$	$-an+ak+$ in 7th	$82-80=4\frac{1}{4}a-4a$	8
9	$a=\frac{t-kq}{n-k}=$ the Canon.	the 8th $\div$ $n-k$	$\frac{82-80}{4\frac{1}{4}-4}=a$	9

Hence appears the Canon to find the Quantity at $4\frac{1}{4}$, *viz.* $\frac{82-80}{4\frac{1}{4}-4}$ $=2\div\frac{1}{4}=8$. and $20-8=12=a$.

For Proof of which, 12 at $4=48$; and 8 at $4\frac{1}{4}=34$: Now $48+34$ $=82=$ the total Value.

Problem 2.] A Person dying, left by Will to 2 Sons each a Sum of Money; so that if to 5 times what he left the elder you add 3 times what

what he gave to the younger, the Sum will be 1675: And if from 7 times the Share of the elder Brother you deduct 8 times the Share of the younger, the Remainer will be 210: What Sum did the Father leave to each?

I put $s=1675$, $d=210$, $k=5$, $n=3$, $m=7$, and $t=8$.

1	$u=$ the Legacy (of the Elder.		u	1
2	$a=$ that of the Younger.		a	2
3	$ku+an=s$.	1st $\times k$; 2d $\times n=s$	$5u+3a=1675$	3
4	$u=\frac{s-an}{k}$ *	3d $\div k$; and $an+$	$u=\frac{1675-3a}{5}$ *	4
5	$\frac{ms-amn}{k}-at=d$	2d part, 4th $\times$ by $m-at=d$	$\frac{1675\times7-3a\times7}{5}-8a=210$	5
6	$ms-amn-akt=dk$	5th $\times$ by k	$1675\times7-3a\times7-8a\times5=210\times5$	6
7	$ms-dk=amn+akt$	$amn-akt+$ and $dk+$	$1675\times7-210\times5=3a\times7+8a\times5$	7
8	$\frac{ms-dk}{mn+kt}=a=175$. *Ergo*	7th $\div mn+kt$	$\frac{1675\times7-210\times5}{3\times7+8\times5}=a=175$, and	8
9	$\frac{s-175n}{k}=u=230$.	4th compared with 8th.	$\frac{1675-175\times3}{5}=u=230$	9

And for Proof $230\times5+175\times3=1675$. And $230\times7-175\times8=210$, as *per* the Question.

Problem 3.] A Surveyor having measured three Fields of Enclosure, hath lost the Contents: But remembers, that if 847 Acres be added to the 1st, the Sum will be equal to that of the other two Fields; if 847 Acres be added to the 2d Enclosure, the Sum will be double

double to the Content of the 1st and 3d; and if 847 Acres be added to the 3d Field, it will make treble the 1st and 2d: How many Acres is in each Field?

I put $a=$ 1st, $e=$ 2d, $u=$ 3d, and $k=847$.

1	$a+k=e+u$; so that $a=e+u-k$	$k+$ in 1st.	$a+847=e+u$ or $a=e+u-847$
2	$e+k=2a+2u$ $\quad e=2a+2u-k$	$k+$ in 2d.	$e+847=2a+2u$ $\quad e=2a+2u-847$
3	$u+k=3a+3e$ $\quad u=3a+3e-k$	$k+$ in the 3d.	$u+847=3a+3e$ $\quad u=3a+3e-847$
4	$e=2e+2u-2k+2u-k$	the 1st × in $2+2u$, $-k$ in 2d.	$e=2e+2u-\overline{847\times2},+2u-847$
5	$e=3k=4u$	the 4th contracted.	$e=\overline{847\times3},-4u$
6	$u=9k-12u+3u-3k+9k-12u-k$	the 5th $\times3+3u$ $-k+$, the 5th $\times3-k$	$u=\overline{847\times9},-12u+3u-\overline{847\times3},+847\times9-12u-847$
7	$u=14k-21u$	6th Step contracted.	$u=847\times14-21u$
8	$u=\frac{7k}{11}$*	$21u+$ in the 7th and $\div 22$, &c.	$u=\frac{847\times7}{11}=539=$ 3d Field. *Acres.*
9	$e=3k-\frac{28k}{11}$ or $\frac{5k}{11}$*	$=3k-4u$ as in the 5th, and u in the 8th is $=\frac{7k}{11}\times4$ $=$ 9th.	$e=\frac{847\times5}{11}=385=$ 2d Field.
10	$a=3k-\frac{21k}{11}-k$	$a=$ as *per* 1st compared with Steps 8, and 5.	$a=\overline{847\times3}-\frac{847\times21}{11}-847$
11	$a=2k-\frac{21k}{11}=\frac{k}{11}$*	the 10th Step contracted.	$a=\frac{847}{11}=77=$ the 1st Field.

Now $\frac{k}{11}=a$ the 1st Field $=77$ *Acres.*

$\frac{5k}{11}=e$ the 2d Field $=385$

$\frac{7k}{11}=u$ the 3d Field $=539$

And for Proof, $77+847=924=385+539$ = the 2d and 3d. *Acres.*

$385+847=1232=77+539\times2$ = 2d and 3×2.

$539+847=1386=77+385\times3$ = 1st and 2d×3.

Notes

Notes on the three Operations to the last three Problems above.

The middle Column has such plain Directions, as may suffice a docile Reader; but that I may render every thing plain, I shall observe that

In the first Operation, every Step is as easy as in common simple Equations, till you come to the sixth Step, which is formed thus:

Since u (an unknown Quantity in the 5th Step) is found $=\frac{t-an}{k}$, and since $u+a$ (the 2 Quantities of the different Ingredients in the 20) is equal to $q=20$; therefore that I may get rid of u, I take what it is equal to, and say $\frac{t-an}{k}+a=q$, and then proceeding by the common Rules laid down, I find $a=\frac{t-kq}{n-k}$, which are all known Quantities: and this Canon doth shew $a=8$.

Therefore u must be $20-8=12$. Or you may find it by a more artificial way, by having recourse to the 5th Step, where $u=\frac{t-an}{k}$ *ergo* $u=\frac{t-8n}{k}$, which you'll find $u=12$.

In the 2d Operation, I proceed as in common simple Equations to the 4th Step, and there, because $u=\frac{s-an}{k}$ I omit u, and take what it is equal to, $\frac{s-an}{k}$, which according to the Tenor of the Question being multiplied by m, and made less by at, the Remainer is $=d$; and so proceeding by the usual Rules, as before taught, I find $a=\frac{ms-dk}{mn-kt}=175$: And if you have recourse to the 4th Step, and instead of an there, put $175n$ (a being found $=175$) you will have $u=\frac{s-175n}{k}$, or $u=230$. And

In the 3d Operation, the three first Steps are composed according to the very Words of the Problem, and the 2d part of those Steps are the same, only having k transposed towards the right, there is shew'd what the 3 several unknown Quantities are simply equal to.

Now for the 4th Step I argue thus: Considering that in the 2d Step $e=2a+2u-k$, and that a is $=e+u-k$, therefore $e=2e+2u-2k$ (which is twice the first Step, $=2a$ in the 2d) $+2u-k$, which are the rest of the 2d Step.

Which 4th Step is in the 5th reduced to $3k-4u$.

Then I proceed to discover the Value of u by composing the sixth Step, thus: u being in the third Step towards the right hand $=3a+3e-k$; and the 1st Quantity in what a is $=$, being e; and e being in the 5th Step $=3k-4u$; therefore I say 3 a's is first 3 times $3k-4u$, *i.e.* $9k-12u$: and the Remainer of what a is $=$, being $u-k$, *ergo* that × by the 3, I say $3a=9k-12u+3u-3k$. Now the rest of what u is $=$ being $3e$, and e in the said 5th Step being $3k-4u$; therefore (to make the Process plain) I repeat $3e$ or $9k-12u$. And there being farther $-k$ to make up what u is $=$; therefore I say in the 6th Step $u=9k-12u+3u-3k+9k-12u-k$, which contracted and reduced, is easily found (as *per* the middle Column in the Example) $u=\frac{7k}{11}$. Therefore e in the 5th Column; I put $=3k$, -4 times what u is found equal to in the 8th Step.

Lastly, a being $=e+u-k$; I say it is $=e$ in the 9th Step, $+u$ (in the 8th Step) $-k$, or $a=2k-\frac{21k}{11}$, which is $=\frac{k}{11}$.

SECT. XII. *Concerning Cubical Equations.*

A Cubical Equation may be known by having commonly (for the first Term) the Cube of the unknown Quantity; the 2d Term, being the Square of that Quantity multiplied in a Co-efficient or some known Quantity; the third is the unknown Quantity sought in a known, as thus and the like; $uuu+cuu+du=k$, u being the Root or Quantity sought.

A Cubical Equation is formed from 1, 2 or 3 Binomials, &c. involved or multiplied in each other; representing either a compleat Cube or Parallelopiped, whose Base is either a Geometrical Square, or a Parallelogram: in which last, if $u+b$ be the greater Side, $u-b=$ the lesser Side of the Base, and $u+d=$ the Length; the Quantities multiplied produce this Cubical Equation, *viz.* $uuu+duu-bbu-bbd=k$ the Content. To find the Dimensions, which depend on the Value of u, it must be done by the Rules following.

2. In a Quadratic Equation, by the Methods of Resolution foregoing, you gain 1 Root; and by dividing the Equation given by that Root you find the other, if there be 2 different Roots.

Thus in the Equation $uu-16u=-63$, being resolved gives $u=9$: And if the $uu-16u+63$ (the 63 being transposed) be divided by $u-9$ (the 9 being transposed) the Quote will be $u-7$ or $u=7$. So are $u-9$ and $u-7$ found the 2 Roots or Dimensions of that Equation, *i.e.* 9 and 7.

I. To

3. I. *To resolve Cubic Equations by Tentation and Depression.*

Having by tryal, as in the following Example, found 1 Root, you must divide the Cubic Equation given by that Root, and that will depress the Equation into a Quadratic; which Quadratic you may find the Roots of, as is shewn last above.

Example 1.] What are the real Roots of this three Dimension (or Cubical) Equation $uuu - 57uu + 1040u = 6000$?

To answer this; I must first find by tryal a Root, which being multiply'd, &c. according to the given Equation, will be 6000.

To which end I suppose 10 *to be* 1 *Root* $= u$.

$$\left.\begin{array}{l}\left.\begin{array}{l}\text{Then } uuu \text{ (or } 10^3) \text{ is} = 1000 \\ \text{The 3d Term } 1040u = 10400\end{array}\right\}\text{Sum} = 11400 \\ \text{2d Term} - 57uu \text{ (or } 57\times10^2) \text{ deduct.} = 5700\end{array}\right\}\text{Rests } 5700$$

The Remainer should be 6000, but 'tis only 5700; therefore the 10 supposed is too little.

2*dly*, *I suppose therefore a Root to be* $12 = u$.

$$\left.\begin{array}{l}\left.\begin{array}{l}\text{Then } uuu \text{ (or } 12^3) \text{ is} = 1728 \\ +1040u \text{ ——— } = 12480\end{array}\right\}\text{Sum} = 14208 \\ -57uu \text{ (or } 57\times144) \text{ — — — — } = 8208\end{array}\right\}\text{Rests } 6000$$

So that I find $12 = u$ or $u - 12$ to be 1 Root.

By which Root having divided the Cubic Equation given, I depress it to the Quadratic Equation $uu - 45u = -500$. Which Equation being resolved, gives $u = 25$, or $u - 25$ for another Root. And dividing that Quadratic by this $u - 25$, the Quote or 3d Root is found $u - 20$, or $u = 20$. See the Operation.

$u - 12)\, uuu - 57uu + 1040u - 6000\,(= uu - 45u = 500 =$ *the Quote or Quadratic.*

$$\begin{array}{r} uuu - 12uu \\ \hline -45uu + 1040u \\ -45uu + 540u \\ \hline 500u - 6000 \\ 500u - 6000 \\ \hline \end{array}$$

or $u = \sqrt{6.25} + 22.5 = 25$ *Quadratic resolved, or a 2d Root.*

$u - 25)\, uu - 45u + 500\,(u - 20$ (= *a 3d Root.*

$$\begin{array}{r} uu - 25u \\ \hline -20u + 500 \\ -20u + 500 \\ \hline 0 \end{array}$$

So that the 3 Roots being found 12, 20 and 25; you may prove the Truth by putting u equal to each, transposing the Roots, and multiplying the Quantities one in another: for the 2d Rectangle will be the Cubical Equation given, transposing the Member next the right hand of the last Product.

1. *Note*, That in a Cubic Equation if the single Root or Square of the unknown Quantity be multiply'd in more than one known Quantity, with Signs between the Rectangles, as several Members of the Equation; where the said unknown Root is found squared in 2 &c. such Members, they make but 1 Term: and so do all those where the unknown Root is found single make another; as $u^3 - u^2b - u^2d + uc + ur + d^3 = o$. In this $-u^2b - u^2d$ is but 1 Term of the Equation, and $uc + ur$ is one other Term.

The Proof of the foregoing Work.

$$\begin{array}{r}
u - 25 \\
u - 20 \\
\hline
uu - 25u \\
-20u + 500 \\
\hline
uu - 45u + 500 \\
u - 12 \\
\hline
uuu - 45uu + 500u \\
-12uu + 540u - 6000 \\
\hline
uuu - 57uu + 1040u - 6000
\end{array}$$

or $uuu - 57uu + 1040u = 6000 =$ (*the Equation given, for Proof.*

2. *Note*, That in the Cubic suppose $u^3 = uc + ur + d^3 = o$; the 2d Term here is said to be wanting, and $uc + ur$ is the 3d Term.

Example 2.] Admit this Equation were to be resolved; $uuu - 30.5uu + 309u = 1039.5$: I suppose 1 Root $= 12$. and working as by the Equation, I find

$$\left.\begin{array}{rcl} u^3 = 12^3 & = & 1728 \\ -30.5u^2 & = & -4392 \\ +309u & = & 3708 \end{array}\right\}$$ The Sum as *per* Addition of Algebra is 1044, which is more than the 1039.5

2*dly*, I therefore suppose a lesser Number, *viz.* $11 =$ a Root, then

$$\left.\begin{array}{rcl} u^3 = 11^3 & = & 1331 \\ -30.5uu & = & -3690.5 \\ +309u & = & 3399 \end{array}\right\}$$ The Sum of which is $= 1039.5 =$ the Sum in the Equation given: so that 11 is 1 Root.

3*dly*, Dividing the Equation given (with the 1039.5 transposed) by $u - 11$, I depress it to the Quadratic $uu - 19.5u = -94.5$.

4*thly*, This Quadratic being resolved is $u = \sqrt{95.0625 - 94.5} + 9.75 = 10.5$ a 2d Root.

5*thly*, Dividing this Quadratic Equation by the 2d Root $= u$ or $u - 10.5$, the Quote or 3d Root is found 9. And for Proof, the 3 Roots 9, 10.5, and 11, (putting each $= u$, and transposing it) multiplied one in another, will give the Cubic Equation proposed to have its Root extracted, *viz.* $u^3 - 30.5u^2 + 309u = 1039.5$

From these 2 Examples it may be observed, That whether you happen to pitch upon the least Root (upon your Tryals) as in the first Example, or upon the greatest of the 3, as in the 2d Trial of the 2d Example; the Method of Process is the same: and so it would, if you happen first upon the middle Root. These

These 2 Examples are of Regular Cubics having all their Terms.

4. I shall give one Example of an Irregular Solid Equation: as suppose $uuuuu + .10uu = 20.61293$.

To resolve this, I can easily see that 2 is more than the Root; for 10 times the Square of 2 (alone, without the Sursolid) is 40. Therefore I take 1.5 for the Root, and (working with that according to the Equation) I find it too great; I therefore try 1.4, which is also a small matter too much: so that working according to the Equation with 1.3, I find it just $= 20.61293$. So that I discover the Root to be $1.3 = u$. for $1.3^5 + 10 \times \overline{1.3}^2 = 20.61293$.

5. *Biquadratic Equations* are resolved after the same manner as the Regular Cubics in the 2 first Examples: First, By supposing a Root till you pitch upon the right, and depressing the Biquadratic thereby into a Cubic; and then finding another Root by Tentation whereby to divide the Cubic to depress it to a Quadratic, *&c.* as in the said 2 first Examples above for all Regular Biquadratics (or such as have all their *Terms.*) Thus, *for Example*, to resolve or find the Roots of the Biquadratic $uuuu - 29uuu + 308uu - 1420u = 2400$.

(1.) By Tentation I find 8 to be 1 Root, and dividing the Equation given by $u - 8$, I depress it to the Cubical Equation $uuu - 21uu + 140u = 300$.

(2.) I make Tryal again, and find 10 to be another Root; so dividing the Cubic Equation by $u - 10$, I depress it to the Quadratic $uu - 11u = -30$, which being resolved, gives a 3d Root or $u = 6$.

(3.) Dividing the said Quadratic Equation by $u - 6$, I find the Quote to be $= u - 5$ or $u = 5$, the 4th Root: so are 8, 10, 6 and 5 the 4 Roots required.

There are 3 other Ways of resolving or finding the Roots of all Cubic and Biquadratic Equations, done more Mathematically: as,

II. By Substitution, Deduction, and Division.

III. By way of *Construction*, with the help of the *Parabola* and *Circle*.

IV. By Approximation or Converging Series.

II. *To resolve Cubic Equations by Substitution.*

It has been shewed how to resolve Quadratics by Substitution, and much after the same manner is a Canon found whereby to resolve Cubic, *&c.* Equations.

Example 1.] Admit the Value of u is required in this following Equation, *viz.* $uuu + duu - gu = k$, or $uuu + 120uu + 300u = 3714544$: Here 'tis plain that d is put for the given 120; g for 300; and k for 3714544.

The first Step toward the Answer is to substitute $b + e$ in the place of u in every Term of the Equation, in the Power that (u) is put.

Then

Then will this be the Canon for Resolution. $\left\{\begin{array}{l} bbb + 3bbc + 3bcc + ccc \text{ (instead of } uuu) \\ bbd + 2bcd + ccd \ldots\ldots \text{ (instead of } duu) \\ -bg - cg \text{ (in the place of } -gu) \end{array}\right.$

The whole Process and Rules are in the Margin, whereby u is found $= 124$.

1. Having your Canon before you, and also that $d = 120$, $g = 300$, proceed as the Canon directs you, first pointing over every third after the first Digit of the given Number; the 3 Points shewing the Root sought, will consist of 3 places, and therefore the 1 (= the first b) is 100.

2. You must compose a Number to deduct from that given, by taking those in the Canon which have not c found in them; as $b^3 + bbd - bg = 2170000$, which deducted reduceth the Number given to 1544544, which is your first Dividend, as by Step 7.

3. Take for a Divisor (in order to discover the Value of c) the Sum of the Numbers answering to the Sumbals mixed with or × by c, such are those in the 8th, 9th, 10th, 11th, and 12th Steps; which Sum is 54120

An Example of the Resolution of a Cubic.

Step.	
1	$3\dot{7}14\dot{5}44\dot{}$ (100 = b
2	1000000 = bbb } Add.
3	1200000 = bbd }
4	2200000 = $bbb + bbd$ = 1 + 2d Step.
5	30000 = $-bg$ deduct from the 4th.
6	2170000 rests, which deduct from the 1st Step.
7	1544544 ÷ = the 1st — the 6 Step ÷ by 13
8	30000 = $3bb$ } Add.
9	. . 300 = $3b$ }
10	24000 = $2bd$ }
11	. . 120 = d }
12	—300 = $-g$ }
13	÷ 54120 = the Sum, or Divisor.
	(20 = c
14	600000 = $3bbc$ } Add.
15	120000 = $3bcc$ }
16	. . 8000 = ccc }
17	480000 = $2bcd$ }
18	48000 = ccd }
19	—6000 = $-cg$ }
20	1250000 = Sum deduct from the 7th Step.
21	294544 ÷, rests of b, or 1st Step.
	Now 120 = b
22	43200 = $3bb$ } Add.
23	. . 360 = $3b$ }
24	28800 = $2bd$ }
25	. . 120 = d }
26	—300 = $-g$ }
27	÷ 72180 = Sum, by which to ÷ the 21 Step.
	(4 = c
28	172800 = $3bbc$ } Add.
29	. . 5760 = $3bcc$ }
30	. . . . 64 = ccc }
31	115200 = $2bcd$ }
32	. . 1920 = ccd }
33	—1200 = $-cg$ }
34	294544 = Sum, deduct from the 21 Step.
	0 rests.

54120 your Divisor, which may be had in the 6 Figures next the left hand of the Dividend 2 times, (which 2 must be 20, because it is the middle of the 3 that will be in the Root, as is observed above) this 20 is $=c$.

4. You must not make any farther Use of the Quantities in the 1st, 2d, and 4th Steps, but having found the Value of c, you proceed to find your 2d Subtrahend in the 20th Step with the several Quantities in the Canon; which you see in the 14th, 15, 16, 17, 18, and 19th Steps.

5. And having found your 2d Divisor by the very same Quantities that you had the 1st, (and observing that here $b=120$, and not to 100) you may easily finish the Operation by the Example, which is only a repetition of the Method above shewed, and you find the 3 several Roots put together is $124=u$ required: which I shall prove both from the Equation given, and by the Canon, to be the true Answer.

1. *By the Equation* $u^3=124^3=1906624$

$$\begin{array}{ll} +120uu & =1845120 \\ -300u & =-37200 \\ \hline & \text{Sum} =3714544=k \text{ in the Equation given.} \end{array}$$

2. *The Proof by the Canon* $bbb+3bbc+3bcc+ccc+bbd+2bcd+ccd-bg-cg$.

$$\left.\begin{array}{rl} bbb= & 1728000 \\ +3bbc= & 172800 \\ +3bcc= & 5760 \\ +ccc= & 64 \\ +bbd= & 1728000 \\ +2bcd= & 115200 \\ +ccd= & 1920 \end{array}\right\} \begin{array}{l} \text{Sum} = \text{———} 3751744 \\ \left.\begin{array}{l} -bg=36000 \\ -cg=\ \ 1200 \end{array}\right\} -37200 \\ \hline 3714544 \text{ rests} =k \end{array}$$

Note, $b=120$; $c=4$
$d=120$; $g=300$

Biquadratics are resolved after the same Method, raising the Power of $b+c$ (which is substituted) accordingly in making the Canon; and proceeding by that, as has been in the last Example observed.

Example 2.] Of an irregular Cubic Equation, as suppose $uuu+32334u=823975=k$, or $u^3+fu=k$.

1. Make your Canon as before, substituting $b+c$ in the 2 Members of the Equation (as u is in power) in the place of u.

2. Where the Equation has not all its Terms, and the Co-efficient is large, you must put such a Number first in the Root, as being

ing multiply'd by the Co-efficient, and added to its own Cube, the Sum will be as near as possible to, and less than that given on the right side of the Equation; as in the Example finding $50=b$ much too great (as in the 4th Step) I try $30=b$, which I still find too much, as in the 7th Step. Then putting $20=b$, I find its Cube as in the 8th Step, and the *Co-efficient* 32334 by 20, the Sum is deducible as in the 10th Step from the first, and then you proceed with the Remainer as in the last Example, and the Value of $u=25$.

Step.	The Canon is $bbb+3bbc+3bcc+ccc+bf+cf=k$.
1	$\dot{8}2\dot{3}97\dot{5}$ $(50=b$ $(30=b$ $(20=b$
2	$125000=bbb$
3	$1616700=bf$
4	$1741700=bbb+bf>$ 1st Step, and cannot be deducted.
5	$27000=30^3=b^3$
6	$970020=bf$
7	$997020=b^3+bf>$ 1st Step.
8	$8000=b=20^3$
9	$646680=bf$
10	654680 deduct from the 1st Step.
11	$169295\div$ rests of the 1st Step.
12	$1200=3bb$ } Add.
13	$60=3b$
14	$32334=f$
15	$\div 33594=$ the 3 last Steps a Divisor. $(5=c$
16	$6000=3bbc$ } Add.
17	$1500=3bcc$
18	$125=ccc$
19	$161670=cf$
20	169295 Sum, deduct from the 11th Step.
	0 rests.

3. It sometimes happens where the Co-efficient is very large, and the Root of the highest Power (or Value of u) is small, that you must make that Figure 1st put in the Root one place (or more) less than the Points over the Number given (as in the 1st Step above) denote. See the following Example.

Example 3.] Admit $u^3+82334u=2073975$, what is the Value of u? 82334 being $=f$, and $b+c$ substituted in the room of u.

Here

Here 'tis plain the Numb. given admits of 3 Cubical Points, but the Root u (or $b+c$) is but 25.

1. For if I should put 1 as the Root of 2 next the left hand; that 1 (there being 3 Points) must be 100. Now if I multiply the Co-efficient by 100, *i. e.* if I suppose two Cyphers put towards the right hand of it, it will be 8233400, w^{ch} (without the Cube of the 100, which must, as *per* the Examples above, be added) is much too great to be taken from the 2073975 in the first Step.

Step.	*The Canon* } $= b^3 + 3b^2c + 3bcc + c^3 + bf + cf$ *as before.*
1	$\dot{2}07\dot{3}97\dot{5}$ ($20 = b$
2	$8000 = b^3$
3	$1646680 = bf$
4	$1654680 =$ Sum, from the 1st Step.
5	419295 rests of Step 1st.
6	$1200 = 3bb$
7	$60 = 3b$
8	$82334 = f$
9	$83594 =$ Sum or Divisor. ($5 = c$
10	$6000 = 3bbc$
11	$1500 = 3bcc$
12	$125 = ccc$
13	$411670 = cf$
14	$419295 =$ Sum, from the 5th Step.
	0 rests.

2. Seeing a lesser Digit than 1 cannot be put first, or in hundreds place in the Root; I must therefore conclude, that the Root will have but two places.

3. I therefore suppose 10 in the Root, by putting 1 Cypher towards the right hand of the Co-efficient (to represent bf) which would be 823340, which (by doubling the 8 next the left hand I find 16) may easily be taken from 20, (next the same left hand of the Number given) I therefore put the double of 10 (or 20) in the Root and so proceed; all the difficulty in this case being to put the first true Figure in the Root: to which end I have not seen any Rules that contribute to do so easily and certainly as these above.

Example 4.] *Of Resolving Cubics by Substitution*, &c.

Admit $uuu + 350u^2 = 1225125$. Or that $uuu + duu = k$; and that $b + c$ is put $= u$.

This Example is of an irregular Cubic composed of 2 Members different from the former; where tho' there be 3 Cubical Points (or Ternaries) yet the Root has but 2 places, *viz.* 55.

For if we suppose $b=$ but 1 in the 3d place (or 100) you will find $bbb+bbd$ to be much more than can be taken from the Numb. pointed; and $10=b$ is much too little: therefore I try $50=b$, and find that to be the Root, that produces (by its Cube added to bbd) the next Number to and less than 1225125 given, and therefore I proceed as by the Canon, as directed in Example the 1st, last foregoing.

Step.	
	The Canon is $bbb+3bbc+3bcc+ccc=u^3$
	$bbd+2bcd+ccd=duu.$
1	1225125 $(50=b$
2	$125000=bbb$
3	$875000=bbd$
4	1000000 = Sum
5	225125 = 1st — 4th Step, a Dividend.
6	$7500=3bb$
7	$150=3b$
8	$35000=2bd$
9	$350=d$
10	$\div$ 43000 = Sum, a Divisor.
	$(5=c$
11	$37500=3bbc$
12	$3750=3bcc$
13	$125=ccc$
14	$175000=2bcd$
15	$8750=ccd$
16	225125 = 11 + 12 + 13 + 14 + 15th Steps
	rests 0 5th — 16th Step.

III. *To resolve Cubic Equations by way of Construction.*
Vid. *Plate* A. *Fig.* 27.

This being done by the Parabola and Circle, it is necessary to shew the making of the Parabola, *&c.*

A Parabola is one of the 5 Sections of a Cone, being made by cutting a Cone by a Plane parallel to the Side which is opposite to that wherein you cut; *and it is Geometrically formed thus:*

1. Draw the Line ($tvF\&dy$) the Axis of the Section. Then cross the same at right Angles with a Line at pleasure, as (ap) and according as you would have the same (or if the ap, which is called the

the *Latus rectum*, or Parameter, be given) suppose 1 Inch, set half an Inch from F to a, and to p.

Then take half of one of those Halfs (or $\frac{1}{4}$ of the Parameter, or of 1 Inch) and set it from F to v, and from v to t, so is $F =$ the Focus, and v the Vertex of the Parabola.

Then draw as many Lines as you please parallel to ap (as ool, *&c.*) with a Ruler on purpose for drawing parallel Lines; and the Distance t to o in the Axis extends from F the Focus, to l in the same Parallel forming the Curve ($vplll$, *&c.*) which you may draw through the Points (l, l, *&c.*). And having found the Curve on one side of the Axis, as vl, set it off on the same Parallel, on the other side of the Axis, as vo, vo, *&c.* and draw the Curve (oov) so is the Section finished, and aFp is the *latus rectum*, ool an Ordinat Applicate (or whole Ordinat) oo the Semi-Ordinat, (which is commonly called the Ordinat) any vo in the right Line is called an Abscissa.

$$\left.\begin{array}{l}\text{Now suppose a Cu-}\\ \text{bic Equation were}\end{array}\right\} \left.\begin{array}{l} uuu + buu - pu - q = 0 \\ \text{Or } uuu + 3uu - 2u - 2 = 0\end{array}\right\} \text{ so } \left\{\begin{array}{l} b = 3 \\ p = 2 \\ q = 2\end{array}\right.$$

1*st*, To find the Value of u. From the same Scale by which you set off the *Latus rectum* of your Parabola, take 3 (the Co-efficient multiply'd in the Square of the Root sought) and applying it at right Angles to the Axis (vy) towards the right hand, because $+3$ it will cut b in the Curve.

2*dly*, Draw a Line at pleasure cutting a supposed Line (vb) in the middle at right Angles, as qq.

3*dly*, Where that Line intersects the Axis, as at ⊕ set half p downward, because $-p$, *viz.* 1, from the same Scale extends from ⊕ to n.

4*thly*, On n erect a perpendicular Line nm which will cut the infinite Line qq in r.

5*thly*, From r set half $q = 1$ towards the right hand because $-q$, and it extends (by the same Scale) from r to s, which is the Center of the Circle sought.

6*thly*, Extend the Compasses from s to b, and describe the Arch λeb, which cutting the Curve at e the perpendicular Line ee measured on the same Scale, gives $u = 1$, which is $+1$ or affirmative, because the Section e is in the right hand part of the Curve. See *Philosophical Transactions*, N°. 188 and 190; Dr *Halley*'s Method.

SECT. XIII. *Approximation or Converging Series.*

What is meant hereby, is ſhewn under the words *Converging Series* in the foregoing Alphabetical Account at the beginning of this Chapter, and will be farther explained by the Rules and Examples following: as $\overline{u}^{1} - \overline{u}^{2}b + \overline{u}^{3}b^2 - \overline{u}^{4}b^3 + \overline{u}^{5}b^4 - \overline{u}^{6}b^5$, &c, is a Series; which I multiply by $u^2 - b^2$ as followeth.

$$\overline{u}^{1} - \overline{u}^{2}b + \overline{u}^{3}b^2 - \overline{u}^{4}b^3 + \overline{u}^{5}b^4 - \overline{u}^{6}b^5$$
$$u^2 - b^2$$

$$u - b + \overline{u}^{1}b^2 - \overline{u}^{2}b^3 + \overline{u}^{3}b^4 - \overline{u}^{4}b^5 + \overline{u}^{5}b^6, \text{ \&c.}$$
$$- \overline{u}^{1}b^2 + \overline{u}^{2}b^3 - \overline{u}^{3}b^4 + \overline{u}^{4}b^5 - \overline{u}^{5}b^6, \text{ \&c.}$$

$u - b =$ the Product.

Here it is plain, that if the Series was continued never ſo far, the Negatives would deſtroy the Affirmatives: But my intent in this Section is not to ſhew the way of repreſenting or expreſſing by a Series, a Product, Root, &c. for though that be a pretty Speculation, yet 'tis not ſo eaſy to diſcover the Roots, &c. therefrom. But I chuſe rather here to ſhew the great Uſe of Converging, in finding with much eaſe and expedition the real Roots themſelves.

II. *To find the Square Root*, as ſuppoſe of 13.

Here I will put s for the Square or Surd Number given; $r =$ the 1ſt two Figures in the Root; $a =$ the 2d Approach to the true Root; and $e =$ the 3d Approach; the ſeveral Approaches being the coming nearer and nearer to the Truth of the Root by ſo many Diviſions, by 1 of which ſeveral places in the Root are gained, as is ſhewed under the fifth Head following.

If in the Example above it be conſidered what Number ſquared will be next to and leſs than 13, or what 2 Digits will be next to 13.00 and put r for thoſe 2 places it will then be $s - rr \div 2r = a$ the 2d Approach.

$s - \overline{r + a}^2 \div \overline{r + a} \times 2 = e$ the 3d Approach, &c.

Or in Numbers $13 - 3.6^2 \div 3.6 \times 2 = .00555 = a$ the 2d Approach, $36. = r$.

13—

$13 - 3.60555^2 \div 3.60555 \times 2 = .00000127546 = e$, and $r + a + e$. Or $3.6 + .00555 + .00000127546$ which is $=$ $3.605551275464 - =$ the Square Root required to 13 places (but see more under General Head the 5th following). Or e will be $= \frac{s - rr - 2ra - aa}{2r + 2a}$ which facilitateth the Operation as to squaring 3.60555, &c.

Or instead of 3.6 at the 1st Approach, you may, as in the common Way of Extraction, put 3, whose square is next to and less than that of any other Digit deducible from 13.

Prop. 2. *To find the Square Root of a larger Number*, as of $371.00071 = s$, &c. as before.

$$\begin{array}{l} 371.00071 = s. \quad 19 = r \text{ the 1st Approach.} \\ -361 = rr \\ \hline \end{array}$$

$10.00071 \div 2r\ (= 31) = .26 = a$ the 2d Approach, and $r + a = 19.26$.

Again $371.00071 - 19.26^2 = .05311 \div 19.26 \times 2 = 0013787 = e$ the 3d Approach.

And $r + a + e = 19 + .26 + .0013787 + = 19.2613787 =$ the Root sought.

So by the same Rules of Converging Series the Square Root of 4712345 is found $2170.7936 +$

III. *Approximation, or the Extraction of the Cube Root by the Method of Converging Series.*

I shall also proceed in this with as much Plainness as I can, obviating what only is to the purpose.

1. Suppose then I would know the Cube Root of $13 = s$, for the first in the Root which is known here I put r. Then the Process, as in common Extraction, (which is the best way in this Case, as being most accurate and certain for the 2d in the Root) will be

$$s - r^3 \div 3rr = R = .3 \text{ the 2d in the Root.}$$

Or supposing 2.3 put in the Root together (for I can easily see that 2.3 is the Root of the next Cube Number to and less than $13.\dot{0}0\dot{0}$) it will be as before to find the 2d Approach, $2.3 = r =$ the 1st Approach.

Then $s - r^3 \div 3rr = .05 =$ the 2d Approach, and now $2.35 = R$.

And $s - R^3 \div 3RR = .001333 + s =$ the 3d Approach. So is $2.351333 + =$ the Root.

Or

Or in Numbers more plainly thus;

$13 - 2.3^3 (=12.167) = .833, \div 2.3^2 \times 3 \; (=15.87) = .05$ the 2d Approach.

And $13 - 2.35^3 (=12.977875) = .022125 \div 2.35^2 \times 3 \; (=16.5675) = 001333 + =$ the 3d Approach.

And the 1ſt Approach + the 2d + the 3d = 2.351333+, which is the Cube Root of 13, done with near 200 Figures fewer than in the common Way of Extraction.——But ſee *Head* 8. following.

2. And if we put $r =$ the known part of the Root, and a for each new Approach, you have

$rr + 2ra + aa =$ the Square of $r + a$

$r^3 + 3r^2a + 3ra^2 + a^3 =$ the Cube of $r + a$

So likewiſe $r^4 + 4r^3a + 6r^2a^2 + 4a^3r + a^4 =$ the Biquadrate of $r + a$

And $r^5 + 5r^4a + 10r^3a^2 + 10r^2a^3 + 5ra^4 + a^5 =$ the Surſolid of $r + a$

And as in the Square and Cube Roots it appears to be (putting s for the Number given to have its Root extracted)

$s - rr \div 2r = a$ the ſeveral Approaches in the Square Root.

$s - r^3 \div 3rr = a =$ the Approaches in the Cube Root.

So for the Biquadrate and Surſolid Roots it will be

$s - r^4 \div 4r^3 = a =$ the Approaches to the Biquadrate Root.

And $s - r^5 \div 5r^4 = a =$ the Approaches to the Surſolid Root.

$$\text{Approaches to the} \begin{cases} \text{Squared Cube} = 8 - r6 \div 6r5 = a \\ \text{2d Surſolid} = 8 - r7 \div 7r6 = a \\ \text{Squ. Biquad.} = 8 - r8 \div 8r7 = a \\ \text{Cubed Cube} = 8 - r9 \div 9r8 = a \end{cases}$$

Hence it appears, That the known part of the 2d Quantity next the left hand in the abovementioned Powers of $r + a$, is your Diviſor. I come next,

IV. *To illuſtrate by Numeral Examples the Extraction of the Biquadrate and Surſolid Roots by way of Converging Series.*

1. *To find the Biquadrate Root* of 314. The Proceſs is (the 1ſt r being = 4.)

1ſt, $314 - r^4 (=256) = 58 \div 4r^3 (=256) = .2 = a$. So is 4.2 a new r.

2dly, $314 - r^4 (=311.1696) = 2.8304 \div 4r^3 (=296) = .009$; ſo is $4.209 =$ a new r.

And by repeating the ſame Method with that laſt r, you have 4.2099+ for the Biquadrate Root of 314.

2. *To find the Surfolid Root of* 314 (the firft $r=3$) the Procefs will be

1ft, $314-3^5\ (=243)=71\div5r^4\ (=405)=.1=a$. So is 3.1 a new r.

2dly, $314-3.1^5\ (=286.29)=27.7185\div.461.75\ (=5r^4)=.05=a$, and $3.15=$ a new r.

3dly, $314-3.15^5\ (=310.1364)=3.8636\div5r^4\ (=492.28)=.007=a$. So is $3.157=$ a new r.

And proceeding in the fame Method with that new r, the Surfolid Root of 314 is found $3.1578+$

But there is yet

V. *Another Way of Converging (for Example) to the Square Root.*

For as in the 1ft Example in the Square Root in this Section the Canon was (putting $s=$ the Square Number or Surd given, and r for each Approach to the true Root) $\frac{s-rr}{2r}=r$ the 2d, *&c.* (or here to a) fo in this Method I am going to fhew, the Canon is $\frac{s+rr}{2r}=r$ the 2d, *&c.*

Now fuppofe $s=13$, it follows that the firft r (or Figure in the Root) is 3, fo that $13+3^2\div6=3.66=r$ the 2d.

$13+3.66^2\div7.32=3.605955=r$ 3d.

And $13+3605955^2\div7.21191=3.60555129=r$ 4th.

Note, *The Divifors are always the double of the* r *next before.*

From the laft Example you may note,

1. That though the preceeding Number be falfe, yet by every Repetition of Procefs according to the Canon, it convergeth or comes nearer to the true Root or Value of r.

2. That the Places of the Digits in the feveral Approaches are true, according to the Progreffion whofe Ratio is 2, and the 1ft Term $=1$. So

	Places.		
In the 1ft Approach	1	*viz.* 3	Are true, the reft not fo
2	2	3.6	
3	4	3.605	
4	8	3.6055512+	

$=r=$ the Square Root of 13 *propè.*

And if the Operation were repeated, as $3.60555129^2+13\div3.60555129\times2$, the Quote will give 16 places true in the Root, for

for every Approach affords double to that preceeding. But the Reader may chuſe which of the two ways he likes as the beſt, this, or the firſt way, which I take to be ſomewhat eaſier and ſhorter in the Operations of Diviſion.

Example 2.] *Of this Way of Converging; to extract the Square Root of* 29507710.4132345.

29507710.4132345 (5 = r the 1ſt.
25
$2r = 10$) 545 —— (54 = r the 2d.
45
2950
+ 2916 = 54^2
$2r = 108$) 5866 —— (5432 = r the 3d.
466
347
237
29507710 = of the Data.
+ 29506624 = 5432^2
$2r = 10864$) = 59014334 —— (5432.1000 = r the 4th.
46943
34873
22814
10864
29507710.41323450
+ 29507710.41000000
$2r = 10864.2$) 59015420.82323450 (5432.100000297721 — = r the 5th Anſwer.
469442
348740
228148
108642
0323450
1061660
838820
783260
227660
103760

VI. *But*

VI. *But to perform the last Operation by the Method first taught making* $a =$ *each Approach, and* $t =$ *the Total of the Approaches; it will stand thus:*

$\frac{s-tt}{2t} = a$ 29507710.4132345 $= s$ (5 $= t$ and $t + a = t$ 2d $= 54$
10)45 — — — — — 04 $= a$

2950
2916 $= tt$ and t the 2d $+ a$ the 2d $= t$ the 3d $= 5432$

$2t = 108$) 347 — — — (0032. $= a$ the 2d.
237 t 3d $+ a$ the 3d $= t$ 4th.
and t the 4th $= 5432.1000$

29507710
29506624 $= tt$

$2t = 10864$) 10864 — — — (0000.1000 $= a$ 3d.
0132

29507710.41323450
29507710.41000000 $= tt$

$2t = 10864.2000$) $=$ .00323450000 (0000.000000297721 $= a$ the 4th.
1061660
838820 and a the 4th $+ t$ the 4th.
783260
227660 $= 5432.100000297721$—
103760 $=$ the Squ. Root sought.

This Method is shorter than the last (as I said before) because the Figures in the Approaches are not repeated by dividing: or the Sum of the 1st t and the a's is $=$ the said Root.

Note, The Approaches are added as Decimals, or put so many Cyphers towards the right hand of each Approach as there are significant Places in the next subsequent Approach, and so add.

And as in the last Example the Operations in Division are shorter than in that next before; so the Canon for the Approaches (next after the 2 first, when the Squaring of the Approaches grows operose) does shorten the Work of Multiplication; or that Prop. of *Euclid* (I think 1, 2.) which says, That of two Lines (or Num-

 bers)

bers) if one be divided into any Number of Parts, the Sum of the Rectangles of one by the ſeveral Parts of the other, is = the Rectangle of one whole by the other.

But it being my Deſign here, only to ſhew how the large Multiplications for producing the 4th, &c. Approaches may be contracted; I ſhall explain the ſaid Canon, which is grounded on another *Prop.* of *Euclid*, *i. e.* any Line (or Number) divided into two Parts; the Sum of the Squares of the Parts, more the double Rectangle of thoſe Parts is = the Square of the whole.

So in the laſt Example putting t the 3d $= 5432$
a the 3d $= .1000$
Ergo $aa + 2ta + tt = \overline{t + a}^2$

There ariſes agreeable to the Canon under the 2d Head of Converging toward the Square Root, this Equation.

Now whereas it would be a little tedious to ſquare the whole of t the 4th $= 5432.1000$ (eſpecially if the Cyphers were ſignificant Figures) I therefore always divide it into 2 parts juſt at the meeting of the Approaches, as t the 3d, and a the 3d, *viz.* into 5432, and .1000, then as the Canon directs.

$.10 = aa$
$1086.4 = 2ta$
$29506624 = tt$ in the laſt Approach before, (ready done.)

$29507710.41 = \overline{t + a}^2 =$ the Sum $= 5432.1^2 = tt$ in the laſt Example.

In this Operation (this way performed) you ſave the trouble of working and inſerting 12 Figures in 29.

And this later Method being uſed, after the 3d, &c. Approaches, does much facilitate the Operations. And thus I think I have made this matter ſhorter, more certain in the Approaches, and plainer than has been done before.

VII. *Another*

VII. *Another Example of Approximation to the Cube Root* shall be to extract the Cube Root of 33148648517294436663656267 $= c$

$\frac{c - s^3}{3s^2} = a =$ the Theorem or Canon.	Root sought $= r$ Each Approach $= a$ And the Sum of the Approaches $\} = s$

1. *Note*, You always bring down so much of c, as will admit of s^3 to be deducted.

2. If any thing had remained upon deducting the last s^3, you must have put Cyphers, and divided again by $3s^2$. See the Operation, and the Probation thereof.

33148648517294436663656267 (3 $= a$ the 1st. $= s$

$3s^2 = 27$) 61 (——————————— 02 $= a$ the 2d
7 $+ a$ the 1st $= 32 = s$.

33148 $=$ part of c.
32768 $= s^3$

$3s^2 = 3072$) 3806 (——————————— 00123 $= a$ the 3d.
7344 $+ 32 = 32123 = s$
12008
2792

33148648517294 $=$ part of c.
33147310244867 $= s^3$

$3s^2 = 3095661387$) 13382724274——————000004323 $= a$ the 4th.
10000787263 $+ s = 32123$
7136031026 $= 321234323$
9447082526 $= s = r$.

33148648517294436663656267 $= c$
33148648517294436663656267 $= s^3$

o rests, *ergo* 321234323 $=$ Root required.

 This

This might be proved by the common Way of Extracting, tho' with very many more Figures: I shall chuse therefore rather to prove it by Involution of the Root, multiplying by 2 Digits at once, as taught *Compend.* 6. *Chap.* III.

$$\begin{array}{r}
321234323 = r = \text{the Root} \\
\times\ 321234323 \\
\hline
7388389429 \quad\quad\quad\quad \\
13813075889 \quad\quad\quad\quad\quad\quad \\
7388389429 \quad\quad\quad\quad\quad\quad\quad\quad \\
6745920783 \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \\
963702969 \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \\
\hline
103191490273268329 = r^2 \\
321234323 = r \\
\hline
2373404276285171567 \\
4437234081750538147 \quad\quad \\
2373404276285171567 \quad\quad\quad\quad \\
2167021295738634909 \quad\quad\quad\quad\quad\quad \\
309574470819804987 \quad\quad\quad\quad\quad\quad\quad\quad \\
\hline
33148648517294436663656267 = c = \text{the Proof.}
\end{array}$$

3. *Note*, That tho' you put a Figure or 2 too many or too few, or too little or too great, next the right hand of your Quotients, it will be rectify'd in the following Operations, if you proceed as *per* the Theorm. But the safest way is not to proceed farther at one Division than in the Example.

VIII. To illustrate what is said in *Note* 3: I shall shew how the first Example of Approximation to the Cube Root foregoing in this 13th Section, is done by different Approaches and Figures to what it is there: And how to manage when Negative Numbers fall in, as when the Cube of *a* or *s* exceed the Cube given from which they are to be substracted.

Thus

Thus to extract the Cube Root of $13=c$ Approaches $=a$
Sum of the Approaches $=s$
Root $=r$

The Theorem $\frac{c-s^3}{3s^2}=a$ the 1st, 2d, &c.

13.0 ———— $(2=a=s.$

$3s^2=12)$ 50 $(.4=a$ 2d. $+a$ 1st. $=s=2.4=s$

13.000 $=c$
13.824 $=s^3$

$3s^2=17.28)$ —.824000 rests $(-.0476=a$ the 3d. $+a$ the 2d. $=2.3524=s.$
13280
11840

13.
13.01767621824 $=s^3$

$3s^2=16.60135718)$—.017676218240000 $(-.0010647=a$ the 4th.
10748610600 $+2.3524=2.3513353=s.$
7877962920
12374200480

13.
13.0000101554648092379 77 $=s^3$

$3s^2=16586333079078 27)$—.00001015546480923797770000 $(-.00000061 2278=a$ the 5th.
20366496179101 50 $+2.3513353$
3780163100023230 $=2.3513346877 22$ $(=s.$
4628964842075760 $=r$ the Root of the
13116982262601060 $(c$ given *propè*.
15045491072462710

1. It may be obſerved from the laſt Operation, That there are different Digits in every Approach, from thoſe in the Approaches under the firſt Example of the Cube Root, in this *Sect.* foregoing. But 2*dly*, That tho' the Digits towards the right hand in theſe Approaches be wrong, yet thoſe *a*'s ariſing from the ſubſequent Operations proceed in each nearer and nearer to the Truth.

2. Although

2. Although the Cube of s does all along exceed c the Cube Number given, and cannot therefore in the common way be subſtracted; yet by the Rules foregoing in Logarithms and Algebra, and under the word *Negative* (Arithmetic) in the Alphabetical Explanation at the beginning of Algebra, it may; (for Example) 13.824 may be taken from 13: but then there will always remain ſo much leſs than nothing to be mark'd with the Sign — as the Subtrahend (or Cube of s) exceeds (c) *i. e.* in this Example of deducting a^3 (the firſt) from c (or 13.824 from 13) there muſt remain —.824 to be divided by the laſt Approach ſquared and multiply'd by 3. And therefore that Quote muſt be negative, *i. e.* —.0476, which added to $2.4 = a$ the firſt (as *per* the Doctrine of Addition of Algebra) the Sum is $= 2.3524$. And ſo forward of the Management of the other Negative Numbers or Quantities; as is taught in the Arithmetic of Negatives under the word *Negative*, at the beginning of Algebra, *Chap.* X. and alſo *Sect.* 2, 3, 4, 5. of that Chapter; and alſo *Sect.* 3. of *Chap.* VII.

Theſe are the moſt brief and accurate Ways that I know of Converging Series, tho' I am not ignorant of others which ſome Authors have been fond of exhibiting; but they are neither ſo ſhort nor accurate as the common way of Extraction. I ſhall give the Inſtance which they have exemplify'd in finding the Square Root of 133225.

Their Way by Converging Series.

$$133225 = c$$

$$r = 3)\ 66612.5 = \frac{c}{2}$$

$$-4.5 = \frac{rr}{2}$$

$$\textit{Diviſor } r = 3)\ 216\ (6 = s$$

$$18 = rs$$

$$+18 = \frac{ss}{2}$$

$$= 198 = \text{Sum.}$$

$$r = 36)\ 1812.5\ (5 = s$$

$$180 = rs$$

$$12.5 = \frac{ss}{2}$$

$$1812.5 \text{ Sum.}$$

$$\textit{Root} = 365 \qquad 0$$

The common Way of Extraction.

$$133225\ (365 = \textit{Root.}$$

$$6)\ 432$$

$$72)\ 3625$$

$$0$$

Done by putting down 20 Figures: Whereas their Way by Converging takes 48, beſides the Signs and Symbols, and is therefore not worthy of Conſideration.

Their

Their other Inſtance of Converging, in the Example of 2, takes 25 Digits more in the Operation than the common Way of Extraction, and is not true in the 3d Decimal Place: Wonderful new Invention!

SECT. XIV. *Concerning Figurate Numbers, with univerſal Series's to exhibit and give all the Figurate Numbers of any Order.*

I have added what follows becauſe the Series's will be found very uſeful in a great many Caſes, as how the Unciæ of Powers of a Binome are found; they are uſeful likewiſe in determining the Numbers of Combinations, and the Laws of Chances, &c.

The Table of Figurate Numbers.

Monads — — — — —	1	1	1	1	1	1	1
Laterals — — — — —	1	2	3	4	5	6	7
Triangulars — — —	1	3	6	10	15	21	28
Pyramidals — — —	1	4	10	20	35	56	84
Triangular Pyramidals	1	5	15	35	70	126	210
Pyramid Pyramidals —	1	6	21	56	126	252	462

The Conſtruction and Uſe of the Table.

The Monads, or Units, being added give the lateral Range, which are called Numbers of the 1ſt Order, as 1 and 1 is 2, and 1 is 3, and 1 is 4, &c. or 1, 2, 3, 4, 5, 6, 7.

The Laterals being added give the Triangulars: as 1 and 2 is 3, and 3 is 6, and 4 is 10, &c. ſo that theſe Triangulars 1, 3, 6, 10, 15, 21, 28, are called Numbers of the 2d Order.

The Triangular range being added give the Pyramidals or Numbers of the 3d Order: as 1 and 3 is 4, and 6 is 10, &c. giving 1, 4, 10, 20, 35, 56, 84.

The Pyramidals being added as 1 and 4 is 5, and 10 is 15, give 1, 5, 15, 35, &c. called triangular Pyramidals or Numbers of the 4th Order; and

The Triangular Pyramidals added as before, give the range called Pyramid Pyramidals which are Numbers of the 5th Order, &c.

The Names of theſe Numbers are analogous to the Coſſic Numbers or Powers: For if in the ſide of an Equilateral Triangle there be

be 2 Points ∴, the number of Points in the Triangle are $3 = 1 + 2$ = the triangular Number of the lateral 2; if there be 3 points in the Side as ∴· the Number of Points in the Triangle are 6, = the Triangular of 3; if 4 Points are in the Side as ∴∴, the Number of all the Points are 10 = the Triangular of 4, and ſo in the reſt. In the ſame manner, the Numbers in the Pyramidal Range will make the number of Points of which an equilateral triangular Pyramid may be ſuppoſed to conſiſt, if it be imagined to be compoſed of Triangulars that are parallel to the Baſe, and that the Sum of all the Triangulars make up the Pyramid. That Number in the Lateral Range, which is the perpendicular Column over any Figurate Number, is called the Side; thus 5 is the Side of the Triangular Number 15, and 7 is the Side of the Triangular 28, and of the Pyramidal 84.

Problem 1. The Laterals given, to find the reſpective Triangulars? *Note*, a Period, (in theſe Caſes) between 2 Numbers, *&c.* is the ſame as $\times$, = multiplyed by.

The Triangular of the Lateral 2 is $2+1=3=\frac{2.3}{2}$ Anſwer.

Of 3, it is $\frac{2.3}{2}+3=\frac{2.3+2.3}{2}=\frac{3.\overline{2+2}}{2}=\frac{3.4}{2}$ Anſwer.

Of 4, it is $\frac{3.4}{2}+4=\frac{3.4+2.4}{2}=\frac{4.\overline{3+2}}{2}=\frac{4.5}{2}$ Anſwer.

Of 5, it is $\frac{4.5}{2}+5=\frac{4.5+2.5}{2}=\frac{5.\overline{4+2}}{2}=\frac{5.6}{2}$ Anſwer.

Univerſally therefore if the Side be l, the Triangular Number will be $\frac{l.\overline{l+1}}{2}$, or if you ſuppoſe $l=n-1$, the Triangular of $n-1$ will be $n.\frac{n-1}{2}$.

Prob.

Problem 2. The Laterals given to find the correſpondent Pyramidals.

The Pyramidal of the Lateral 2 is $= 4 = \frac{2.3.4}{2.3}$

Of 3 is $= \frac{2.3.4}{2.3} + \frac{3.4}{2} = \frac{2.3.4}{2.3} + \frac{3.3.4}{2.3} = \frac{3.4.\overline{2+3}}{2.3} = \frac{3.4.5}{2.3}$

Of 4, is $= \frac{3.4.5}{2.3} + \frac{4.5}{2} = \frac{3.4.5}{2.3} + \frac{3.4.5}{2.3} = \frac{4.5.\overline{3+3}}{2.3} = \frac{4.5.6}{2.3}$.

Of 5, is $= \frac{4.5.6}{2.3} + \frac{5.6}{2} = \frac{4.5.6}{2.3} + \frac{3.5.6}{2.3} = \frac{5.6.\overline{4+3}}{2.3} = \frac{5.6.7}{2.3}$

And univerſally if the Side be l. the Pyramidal of the Side will be $\frac{l}{1}. \ \frac{l+1}{2}. \ \frac{l+2}{3}.$

Or making $n-2=l$; it is $\frac{n}{1}. \ \frac{n-1}{2}. \ \frac{n-2}{3}.$

Problem 3. The Lateral Numbers given to find the Triangular Pyramidals.

The Triangular Pyramidals of 2 is $5 = \frac{2.3.4.5}{2.3.4}$

Of 3, it is $\frac{2.3.4.5}{2.3.4} + \frac{3.4.5}{2.3} = \frac{2.3.4.5}{2.3.4} + \frac{3.4.4.5}{2.3.4} = \frac{3.4.5.\overline{2+4}}{2.3.4}$

$= \frac{3.4.5.6}{.3.4}$

Of 4, $\frac{3.4.5.6}{2.3.4} + \frac{4.5.6}{2.3} = \frac{3.4.5.6}{2.3.4} + \frac{4.4.5.6}{2.3.4} = \frac{4.5.6.\overline{3+4}}{2.3.4}$

$= \frac{4.5.6.7}{2.3.4}$

Of 5, $\frac{4.5.6.7}{2.3.4} + \frac{5.6.7}{2.3} = \frac{4.5.6.7}{2.3.4} + \frac{4.5.6.7}{2.3.4} = \frac{5.6.7.\overline{4+4}}{2.3.4}$

$= \frac{5.6.7.8}{2.3.4}$

And universally if the Side be l, the Figurate Number of the 4th Order is $\frac{l}{1}.\ \frac{l+1}{2}.\ \frac{l+2}{3}.\ \frac{l+3}{4}.$ And if we make $n-3=l$, the Figurate of $n-3$ will be $\frac{n}{1}.\ \frac{n-1}{2}.\ \frac{n-2}{3}.\ \frac{n-3}{4}.$

In the same manner we shall find, That in the Figurate Numbers of the 5th Order; if the Side be $=l$, the Figurate Number answering is $\frac{l}{1}.\ \frac{l+1}{2}.\ \frac{l+2}{3}.\ \frac{l+3}{4}.\ \frac{l+4}{5}.$ Or supposing $l=n-4$, the Series will be $\frac{n}{1}.\ \frac{n-1}{2}.\ \frac{n-2}{3}.\ \frac{n-3}{4}.\ \frac{n-4}{5}.$

Hence the following Series will give us all the Figurate Numbers of any Order whatsoever: *i. e.* $\frac{l}{1}.\ \frac{l+1}{2}.\ \frac{l+2}{3}.\ \frac{l+3}{4}.\ \frac{l+4}{5}.\ \frac{l+5}{6}.\ \frac{l+6}{7}.\ \frac{l+7}{8},$ &c. Or this Series. $\frac{n}{1}.\ \frac{n-1}{2}.\ \frac{n-2}{3}.\ \frac{n-3}{4}.\ \frac{n-4}{5}.\ \frac{n-5}{6}.\ \frac{n-6}{7}.\ \frac{n-7}{8},$ &c.

AN APPENDIX

To an intire *Syſtem of* ARITHMETIC.

By *EDWARD HATTON*, Gent.

Of the Menſuration of Superficies and Solids.

MY Intent is to be as brief, and yet as copious as poſſible; not doubting but the Variety of Figures here meaſured, more than any other Treatiſe of Meaſuring contains, that I have ſeen or believe to be publiſhed, will meet with the Approbation of the Studious in this Geometrical Science.

The Dimenſions I will ſuppoſe taken in Feet and Decimal Parts, as beſt anſwering that Accuracy and Brevity which will be expected from me: But at the Cloſe of this Appendix I ſhall give my Advice what Inſtruments are moſt proper for the ſeveral kinds of Workmen; and alſo by what Denomination the ſeveral kinds of Work is valued.

CHAP. I.

To measure any Superficies.

A *Superficies* is properly *defined* to consist only of Length and Breadth or Girt, without taking any Notice of Thickness.

SECT. I. *The Geometrical Square and Parallelogram, Rhombus and Rhomboides.*

Proposition 1. *To measure the Geometrical Square.*

Definition. This is a Figure consisting of 4 equal Sides, and as many right Angles: as *Fig.* I. *Plate* C.

Rule. Take the Dimensions of one Side, as *a b*, and multiply it by itself, gives the Content.

Example. The Side of any Square being 11.28, the Content is found 127.238, as *per* Marg.

```
        11.28
        11.28
   ----------
        31584
       12408
   ----------
Answ.=127.2384
```

Prop. 2. *To find the Area of the Parallelogram.*

Defin. This is a Figure consisting of, or bounded by, 2 longer equal parallel Sides, and 2 shorter equal and parallel Sides, having 4 right Angles: as *Fig.* II. *Plate* C.

Rule. Multiply the Length by the Breadth, and the Product is the Area.

Example. The Length $ab = 23.53$, Breadth $bc = 2.15$. See the Work.

```
        23.53
         2.15
   ----------
Answ.=50.5895
```

Prop. 3. *To find the Area of a Rhombus.*

Defin. This Figure is bounded by 4 equal Sides, and hath 4 Angles, the 2 opposite ones being equal, and the Sides parallel: as *Fig.* III. *Plate* C.

Rule. The distance between 2 opposite Angles, as *b d*, multiplied by the Line *e c*, gives the Answer.

Example.

Example. The Diagonal $bd = 1.34$; $ec = .85$; the Answer is 1.139

$$\begin{array}{r} 1.34 \\ .85 \\ \hline 1.1390 \end{array}$$

Prop. 4. *To find the Area of the Rhomboides.*

Defin. This Figure is bounded with 4 right Lines, the opposite Sides equal and parallel, and the opposite Angles equal: as *Fig.* 4. *Plate* C.

Rule. Multiply the Diagonal ab by the Perpendicular cn or mr, and the Rectangle answers.

Example. The Diagonal $ab = 12.47$, the Perpendicular $cn = 4.127$, and the Answer is therefore, as *per* Margin, $= 51.46369$.

$$\begin{array}{r} 12.47 \\ 4.127 \\ \hline 33669 \\ 51127 \\ \hline \end{array}$$

Answ. $= 51.46369$

SECT. II. *To measure the Conic Sections.*

Prop. 1. *To measure a Triangle.*

Defin. Right-lined Triangles are of 3 kinds (besides the equilateral, whose Sides are equal) *viz.* Right angled, as arn or xrn right angled at r in *Fig.* 5. 2*dly*, Scalenous Triangles, as oro, the Sides of which are all unequal. And, 3*dly*, Isoceles Triangles as anx appears; 2 of whose Sides are equal. This last is one of the Sections of a Cone, made by cutting the Cone ($ewagxna$) down the Axis, as op, *Fig.* 5. *Plate* C.

Rule. Multiply the Base (as xa) by half the Perpendicular (as nr) or half the Base by the Perpendicular, or the Perpendicular by the Base; and taking half the Product, will give the Area of any Triangle as above mentioned: for all Triangles are half a Square of the same Base and Perpendicular.

Example. The Base ax being $= 9.35$, the Perpendicular $nr = 12.42$, the Area is 58.0635, as *per* Margin.

$$\begin{array}{r} 9.35 \\ 6.21 \\ \hline 19635 \\ 5610 \\ \hline \end{array}$$

Answ. $= 58.0635$

Prop. 2. *To measure the Circle.*

Defin. This is formed by cutting the Cone ($xwagxna$) parallel to the Base ($xwagx$) as mc, *Fig.* 5. *Plate* C.

Rule. There are several ways of finding the Area of a Circle. As

1*st*,

1*st*, As 7 to 22, so the Diameter (as *r s*, *Fig.* 6) to the Circumference.

2*dly*, As 1 to 3.1416, so the Diameter to the Circumference.

Then multiply the Circumference by a 4th of the Diameter, or a 4th of the Circumference by the Diameter, gives the Answer.

Or the Area of a Circle is found without the Circumference thus:

As 1 is to .7854 : : so is the Square of any Diameter to the Area.

Example. Admit the Diameter *r s*, *Fig.* 6. be = 14.32, what is the Area?

By the last Method

1. .7854 : : 205.0624. 161.056 = Answ.

14.32
14.32
45824
20048
□ = 205.0624

Prop. 3. *To measure the Ellipsis, or Oval.*

Defin. This is a Section of a Cone, made by cutting it through both its Sides, but not parallel to the Base; as the Section *l e* *Fig.* 5. *Plate* C. which produceth *Fig.* 7. *Plate* C.

Rule. Multiply the *Latus transversum* (or transverse Ax) *l t*, by the Diameter Conjugate *c d* and the Product is = the Square of the Diameter of a Circle equal to the Oval: then work as in the last.

Example. Admit *l t* = 15.34, and *c d* = 12.28 the Answer is found 147.95 *fere:* for

1. .7854 : : 188.3752. 147.9499

l t = 15.34
c d = 12.28
42952
18408
< = 188.3752

Prop. 4. *To find the Area of the Parabola.*

Defin. This Section is formed by cutting the Cone parallel to the opposite Side, as in the Line *t s*, *Fig.* 5. which is parallel to the Side *a n* and produceth a Parabola; as *Fig.* 8. *Plate* C.

Rule. This Figure being 2 Thirds of a Parallelogram made of the whole Ordinat *p a* and the Abscissa *s s* therefore multiplying those 2 Lines together, 2 Thirds of the Rectangle is the Area.

Example. Admit *a p* = 3.84, and *s s* = 2.54; 2 Thirds of the Rectangle is = the Answer = 6.5024.

3.84
2.54
20736
768
Rectangle = 9.7536
$\frac{1}{3}$ = 3.2512
$\frac{2}{3}$ *or Answer* = 6.5024

Prop.

Prop. 5. *To find the Area of the Hyperbola (or Hyperbolic Space).*

Defin. This Section of a Cone is formed by cutting it (not parallel to the Side, as in the Parabola, but) so that if the Axis of the Section were continued upward, it would intersect the opposite Side of the Cone produced: the Section's Axis is represented by the Line *y z*, *Fig.* 5. and produceth an Hyperbola, as *Fig.* 9. *Plate* C.

To measure this Figure, there is no general Rule that can be given for Practice: But the Investigation is performed by the help of the Asymptotes, an Infinite Series, and partly by the Method of Fluxions; which 'tis not proper here to insert.

SECT. III. *To measure any of the Regular Polygons.*

Defin. These are Figures consisting of above 4 equal Sides, and as many equal Angles. As,

The Pentagon having 5 equal Sides, and as many Angles.

Hexagon	6
Heptagon	7
Octogon	8
Nonogon	9
Decagon	10
Undecagon	11
Dodecagon	12

Rule. To measure any of these, draw 2 Lines from any 2 Angles that include a Side to the Center: then find the Content of that Triangle, as taught *Prop.* 1. *Sect.* 2. above. Lastly, Multiply the Area of that Triangle by the Number of Sides, and the Product is the Content of the whole Polygon. Or multiply half the Sum of the Sides by the nearest Distance from the Center to one of the Sides, gives the Answer.

Example. The Side of the Pentagon (*s d*, &c.) *Fig.* 10. is = 15.34, the Perpendicular (or Radius of a Circle inscribed) is *r c* = 10.5: So that 15.34 multiplied by 5, the Number of Sides, gives = 76.7 = Sum of the Sides; half of that is = 38.35, which multiplied by *r c* = 10.5, gives the Area = 402.675.

$$\begin{array}{r} 38.35 \\ 10.5 \\ \hline \end{array}$$

Answer = 402.675

SECT.

SECT. IV *To measure the Trapezium, the Parallelogramic, the Polygram, (or Multangle, or irregular Polygon) the Area or Superficies of the Cylinder, Cone, and Sphere.*

Prop. 1. *To measure the Trapezium.*

Defin. This is a Figure of 4 Sides and 4 Angles, uncertain whether parallel or equal, but is commonly neither; as *Fig.* 11. *Plate* C.

Rule. Multiply the Diagonal $(a\,n)$ by half the Sum of the Perpendiculars $m\,p$ and $c\,p$; or the Sum of the Perpendiculars by half the Diagonal, and either of the Products is the Answer.

Example. The Diagonal $a\,n = 17.42$; the Perpendicular $m\,p = 4$, and $c\,p = 2.6r$, half the Diagonal is $= 8.71$, and $m\,p$ more $c\,p = 6.6r$, the Product of which 2 last is 58.06666, *&c.*

Or half the Product of the Diagonal in the Perpendiculars gives the Answer.

$$\begin{array}{r} 8.71 \\ 6.6r \\ \hline 5226 \\ \hline \textit{Answ.}{=}58.06r1 \end{array}$$

Prop. 2. *To find the Area of the Parallelogramic, or Parallelopleuron.*

Defin. This Figure hath 4 Sides, 2 of which opposite are parallel, and of 4 Angles the 2 at each end being equal: as *Fig.* 12. *Plate* C.

Rule. Multiply half the Sum of the 2 Sides $(v\,r)$ and $(s\,t)$ by the Length $(t\,w)$ gives the Answer. Or a Diagonal $(r\,t)$ multiplied by half the Sum of 2 Perpendiculars let fall on that Diagonal from s and v.

Example. The Ends $r\,u = 0.79$; $s\,t = 0.7$; Length $w\,t = .8888$, *&c.* half the Sum of .79 and .7 multiplied in $.8r = .662r1$.

$$\begin{array}{r} \text{Sum } r\,u + s\,t = 1.49 \\ \hline \tfrac{1}{2} = .745 \\ .8\,r \\ \hline 5960 \\ \hline .662r1 \end{array}$$

Prop. 3. *To find the Area of the Polygram, (sometimes called an irregular Polygon, or Multangle.)*

Defin. This is any Geometrical Figure having above 4 Sides, which are uncertain, and generally unequal; as are also its Angles: as *Fig.* 13. *Plate* C.

Rule. First divide your Figure into Trapeziums and Triangles, and then measure the same as under *Prop.* 1. *Sect.* 2. and *Sect.* 4.

Example.

Example. Figure 13 is divided into,

1. The Trapeziums (*u t m q v*)
2. — — — — and *q m p r q*
3. And the Triangle *q r a q*.

In the 1st, $qt = 18.26$, the $\frac{1}{2} = 9.13$
$ux = 5.17$, more $bm = 3.5 = 8.67$

61171
7304
Prod. $= 79.1571$

In 2d Trapez. $qp = 15.73$, the half $= 7.865$ } *Prod.* $= 53.56065$
$mn = 4.5$, more $rs = 2.31 = 6.81$ — — }

In 3d or the △, $qa = 10.72$; the half $= 5.36$ } *Prod.* $= 17.5272$
$ro =$ the Perpend. $= 3.27$ — — — }

The Sum = the Area of the whole *Fig.* 13. $= 150.24495$

Prop. 4 *To find the Area of a Cylinder.*

Defin. This Figure is defined under *Prop.* 1. *Chap.* 2.

Rule. Multiply the Ambit or Circumference of the Cylinder by its Length, and to that Rectangle add the double Area of the Base or End, and the Sum is the whole Area.

Area of the curved part of the Cylinder is therefore — } 3.1416
Area of the Base $= .7854$
Ditto $= .7854$
Whole Area (or Sum) $= 4.7124$

Example. Ambit of the Cylinder $= 3.1416$; Altitude as also the Diameter of the Base $= 1$.

But there are 2 other ways of doing this, where the Diameter and Altitude are equal.

As, 2*dly*, Multiply the Area of the Sphere having the same Diameter by 3, and divide by 2.

Or, 3*dly*, 6 times the Area of the Base, or End, is the Answer. Thus 6 times .7854 is $= 4.7124$, as above.

Prop. 5. *To find the Convex Superficial Content of a Sphere.*

Defin. This Body is defined where its Solidity is measured, *Prop.* 4. *Chap.* II. and 'tis represented *Fig.* 6. *Plate* C.

Rule. The Area is 4 times that of a great Circle; therefore having by the Diameter found the Area of a great Circle of any Sphere, multiply it by 4, and you have the Answer.

Exam. The Diameter of a Sphere being 12, the Area of the same is 452.39, as *per* Margin.

1. .7854 : : 144.
144
Prod. $= 113.0976$ *Area Circle.*
4
452.3904 $=$ *Area of (the Sphere.*

Prop. 6. *To find the Superficies (or Area) of a Pyramid.*

Defin. This is a solid Body, having (generally) a Square or some regular Polygon for its Base, being tapering like the Spire of a Steeple, &c.

Rule. Multiply the Circumference of the Base (or Sum of the Sides), by half the Altitude, and it gives the Superficial Content.

Example. The Side of the Base of a Pyramid (being an Octogon) is $=5.65$, that multiplied by $8=$ the Number of Sides, and that Product by half the Altitude $=30.7$, the last Product is the Area sought.

	5.65
	8
The Total Sides	$= 45.20$
Half the Altitude	$= 30.7$
Area	$= 1387.64$

Prop 7. *To find the Area of a Cone.*

Defin. The Cone is a Solid which hath a Circle for its Base, from whence it is gradually tapering upward till it terminate in a Point directly over the Center of its Base; as *Fig.* 5. *Plate* C.

Rule. As the Radius of the Base: Is to the Side of the Cone :: So is the Area of the Base to the Area of the Cone.

Example. Diameter of the Base $= 6$; Side of the Cone $= 15$: So the Area of the Base of the Cone is found $=$ 28.2744; and the Area of the Cone, as by the 2d Analogy $= 141.37$.

1. .7854 :: 36
36

28.2744 = *Area of the Base.*

Rad.	*Bas. Side.*	*Ar. Base*	*Ar. Cone*
3.	15 ::	28.2744.	141.37

Or *Note* farther, That the Area of a Cone is $=$ that of a Triangle, whose Perpendicular is the Altitude of the Cone, and the Base $=$ the Circumference of the Cone's Base: For in Example, 1. 3.1416 :: 6. 18.8496 $=$ the Base or Circumference; half of which multiplied by the 15, gives the Answer as before $=141.37$.

SECT. V. *To find the Area of the Parts of a Circle; as the Semicircle, Quadrant, Sector, Segment, Lune or Crescent, and the Trochoid or Cycloidal Space.*

Prop. 1. *To measure the Semicircle, Quadrant, and Sectors of a Circle, also the Segment.*

Defin. The Semi- or Half-Circle is the Space between *d m b d.* *Fig.* 14. *Plate* C; the Quadrant or Quarter of the Circle is *d m c d*, the Sector is *c b r e c* or the like, less or more than a Quadrant.

Rule

Rule. Multiply 1 fourth of the Ark, or Curve Line, by the Diameter of the Circle, and the Product is the Area required.

Example. The Circumference is = 3.1416, the Diameter of that Circle = 1.

The Ark of the Half-Circle *d c b m d* as *b m d* = 1.5708
of the Quadrant *c m d c* as *m m d* = .7854
of the Sector *c b r e c* as *e r b* = 1.05

So that 1 fourth of 3.1416 = .7854 × by 1, gives .7854 the Area of the Circle.
1 fourth of 1.5708 = .3927 × by 1, gives .3927 the Area of the Half-Circle.
1 fourth of .7854 = .1963 × by 1, gives .19635 the Area of the Quadrant.
1 fourth of 1.05 = .26271 × by 1, gives .26271 the Area of the Sector.

And the Area of the Sector *c b r e c*, less the Area of the Triangle *c b n e c*, gives the Area of the Segment *e n b r e*; *Fig.* 14. *Plate* C.

Prop. 2. *To find the Area of the Lune.*

Defin. This is a Figure like a Crescent or increasing Moon; as *a r n o a*, *Fig.* 15.

Rule. Find first the Area of the Semicirle *n c a r n*, and from that deduct that of the Segment *n c a o n*, and the Remainer is the Area of the Lune.

Example. The Diameter *n a* = 1, and consequently the Ark *a r n* = 1.5708, and Area = .3927.
The Area of the Sector *e a o n e* is = .256, from which deducting the Area of the Triangle *n e a n* = .135, the rest is the Area of the Segment = *n o a c n* ..e, there = .121; which taken from the Area of the Sem..
resteth the Answer = .2717.

.3927
.121
———
..

..ual Space.

Prop. 3. *To find the Area of the Trocboid, or* ..erated by the Circle

Defin. This Figure (as 16, *Plate* C.) is ..Line *a r c*, the Point at *b* (*l b o r*) which turneth round on .. the Line again at *c*, the Cy-
beginning at *a*, and when it to.. ..d the Space from that to the Line
cloidal Curve *a b c* is finishe.. ..d generating Circle.
a r c contains 3 times the ..und the Area of a Circle, whose Diameter

Rule. Therefore having .. ..y 3, and it produceth the Area of the Cy-
is (*r b*) multiply tha..
cloidal Space.

Example. Th.. ..ine *r b* = 1; Area of the Circle = .7854, and of the Cy..oidal Space = 2.356.

CHAP. II.

The Mensuration of Solids.

SECT. I. *To find the solid Content of a Cylinder, Parallelopiped, Prism, Cone, Pyramid, and Sphere.*

PROP. 1. *To measure the Cylinder.*

Defin. A Cylinder is a solid Body having a Circle for its Base, and is of equal Circumference from one end to the other, like a Rolling-Stone for Walks, &c. as the *Figure* 17. *Plate* C.

Rule. Multiply the Area of the Base (found as under *Prop.* 2. of *Sect.* 2. *Chap.* 1.) by the Length, and the Product is the Answer.

Example. The Diameter of the Base, as (*a b*) or (*c d*)=14.32, the Area of that Circle will be found 161.056; which multiplied by the Length (*b c*) =14.32, or (*a d*) the solid Content, is = 2306.32192, as *per* Margin.

161.056
14.32
5153792
2254784
2306.32192

Prop. 2. *To find the solid Content of the Parallelopipedon and of the Prism.*

This Solid is bounded by 4 Parallelograms, or Par[illegible] parallel, for the Sides; and 2 Squares [illegible]grams, equal and parallel, for the Ends: as *Fig.* 18. *Plate* C.

Rule. Multiply the A[illegible] of the Base or End by the Length, and the Rectangle is the A[illegible]

Example. The End is a Square whose Side is 3.72, and the Length = 31.44. See the Operation, where (according to the Method of working *Case* 5. of *Sect.* 5. of *Ch.* 3.) the Solidity is found 435.0793.

at the end 138384	3.72
th inverted 44.13	3.72
415152.	26784
138384	1116
55354	
[illegible]	13.8384=*Area*
435.0793=*Solid Content*	

Note,

Note, That a *Priſm* is meaſured alſo by multiplying the Area of the Baſe (which is either a Triangle or a Polygon) by the Length of the Priſm. This or the laſt Figure are three times a Pyramid of the ſame Baſe and Altitude.

Prop. 3. *To find the ſolid Content of the Cone: And 2dly, of a Pyramid.*

Defin. This Figure hath a Circle for its Baſe, and is defined *Prop.* 7 *Sect.* 4. of *Chap.* 1. of this Appendix.

Rule. Multiply the Area of the Baſe by one third of the perpendicular Altitude of the Cone, and that Rectangle is the Anſwer.

But Note, That to find the Height (it being a ſolid Body) muſt be done by this Rule, as *Euclid* 47. 1.

From the Square of the ſlant Height (*m l Fig.* 19.) deduct the Square of the Semidiameter of the Baſe as (*m a*), and the Square Root of the Remainer is the perpendicular Altitude (*x l*).

Example. The Diameter *m d* is $=6$, the ſlant Height *m l* $=5$; and conſequently the true Altitude (*a l*) $=4$: for 5 ſquared leſs $\frac{6}{2}=$ *m a* ſquared, $=16=$ the Square of *a l*; that is, 5 ſquared leſs 3 ſquared, is $=16$; whoſe Root is $4=$ *a l*. So that as $1.\ .7854 :: 36.\ 28.274 =$ the Area of the Baſe: Which multiplied by 1 third of $4=1.3r$, the Product is $=37.6986r1$.

28,274
1.3r1
————
84822
————
94246
28274
————
37.6986r1 = *Anſ.*

To meaſure the Pyramid, you muſt alſo multiply the Area of the Baſe by a Third of the perpendicular Altitude: for as a Cone is 1 Third of a Cylinder of the ſame Baſe and Altitude, ſo is a Pyramid 1 Third of a Priſm or Parallelopiped. 'Tis defined *Prop.* 6. *Sect.* 4. *Chap.* 1. of this Appendix.

Prop. 4 *To find the ſolid Content of a Sphere.*

Defin. This is a ſolid Body, every way perfectly round, ſo that all Lines drawn from the Convex Area to the Center are equal. It is 2 Thirds of a Cylinder of the ſame Diameter of the Baſe and Altitude with the Diameter of the Sphere, or of a Cylinder circumſcribed.

Rule. Find the Area of a great Circle, as *Prop.* 5. of *Sect.* 4. *Chap.* 1. which multiply by the Diameter; and 2 Thirds of the Rectangle is the ſolid Content required.

Example. The Diameter of a Sphere being 12, the Area of a great Circle (as *Fig.* 6. *Plate* C.) is $=113.0976$; which multiplied by the

the Diameter (or Length of a Cylinder) 12, the Rectangle is = 1357.171, the Solidity of the Cylinder; 2 Thirds of which is the solid Content of the Sphere = 904.7808.

SECT. II. *To find the Solidity of the five Platonic, or Regular Bodies, viz. the Tetrahedron, Hexahedron, Octahedron Dodecahedron, and Icosihedron.*

Prop. 1. *To measure the Tetrahedron.*

Defin. This is a solid Body bounded with 4 equal equilateral Triangles; and being measured as a Pyramid, it needs no other Rule than as under *Prop.* 3. of *Sect.* 1, *Chap.* 2. See *Fig.* 20. *Plate* C.

Prop. 2. *To find the Solidity of the Hexahedron or Cube.*

Defin. This Solid is bounded with 6 equal Geometrical Squares.

Rule. Multiply a Side in itself, and that Rectangle by the Side gives the Answer. See *Fig.* 21. *Plate* C.

Prop. 3. *To find the solid Content of the Octahedron.*

Defin. This Figure is bounded with 8 equal equilateral Triangles, which are the Bases of as many Pyramids, which meet in the Center of a Sphere, (out of which this Body is cut) so that it being 8 Pyramids. (See *Fig.* 22. *Plate* C.)

Rule. First find the Area of one of the Triangles, then take with Callipers, *&c.* the Distance between the Center of that Triangle and that of the opposite; by one sixth part of which multiply the Area of any one Triangle; and that Product multiply'd by 8, this last Rectangle gives the Solidity required.

Prop. 4. *To find the solid Content of the Dodecahedron.*

Defin. This is a Solid bounded by 12 equal Pentagons, which are the Bases of so many Pyramids that meet in the Center of this Solid, or of a Sphere out of which this Body is cut. See *Fig.* 23. *Plate* C.

Rule. Therefore take with a Pair of Callipers the Distance from the Center of one Pentagon to that of its opposite, and by one sixth of that multiplying the Area of any of the Pentagons; 12 times that Rectangle gives the Solidity. *Or twice the Distance of the opposite Pentagons multiplied by the Area of* 1 *Pentagon*, gives the Answer.

Prop. 5. *To find the Solidity of the Icosihedron.*

Defin. This is the last of the 5 Platonic or Regular Bodies: it is bounded with 20 equal equilateral Triangles, which are Bases to so many Pyramids, whose small Ends terminate in a Point in the Center of a Sphere which circumscribeth this Solid and out of which it is cut. (See *Fig.* 24. *Plate* C.)

Rule.

Rule. Multiply the Area of one Triangle by 1 sixth of the Distance of the Centers of 2 opposite Triangles (taken with Callipers) and that Rectangle multiplied by 20 (the Number of Pyramids in the Body) gives the Solidity sought.

I have not thought it necessary to give Examples to these 5 Rules because there is not much more therein besides the repeated Mensuration of a Pyramid, which is taught above. But I have chiefly inserted them for the sake of the Definitions, which I do not remember to be in any Tract of Measuring, no more than many other things contained in other Propositions of this Appendix.

SECT. III. *To find the solid Content of the Parabolic Conoid, the Parabolic Spindle, the Cylindroid, the Sphereoid, and the Hyperbolic Conoid.*

Prop. 1. *To find the solid Content of the Parabolic Conoid.*

Defin. This Solid is generated by the Rotation of a Semi-Parabola, as (*pss Fig.* 8. *Plate* C.) round its Axis *ss*; and is = half of the circumscribing Cylinder *pqsxasp* (or 1 and half of the Cone inscribed). Therefore

Rule. Multiply the Area of the Base by the Axis or greatest Absciss, and half the Rectangle is the Answer.

Example. The Diameters of the Figure (as *ap*) = 14.32, the Area of the Base will be found 161.056; which multiplied by the Absciss (*ss*) = 14.32, the Rectangle is = 2306.32192; the half of which is = 1153.16096 the Answer.

Prop. 2. *To find the Solidity of the Parabolic Spindle.*

Defin. This Figure is generated by the Rotation of the Semi-parabola round its Ordinat, and is $\frac{8}{15}$ of the circumscribing Cylinder. Therefore

Rule. Multiply the Content found as a Cylinder by 8, and divide by 15, the Quote is the Answer, which needs no Example.

Prop. 3. *To find the solid Content of the Cylindroid.*

Defin. This Figure only differs from that under *Prop.* 1. *Sect.* 1. of *Chap.* 2. inasmuch as that hath a Circle, this hath an Ellipsis for its Base: so that to measure it,

Rule. Multiply the Base found as *per Prop.* 3 *Sect.* 2. *Chap.* 1. by the Length, and that Product is the Answer.

Example. Admit the *Latus transversum* (as *lt*, *Fig.* 7. *Plate* C.) by *cd* = the Diameter Conjugate, (*i. e.* suppose *lt* = 15.34, and

cd =

$cd = 12.28$) the Area is $= 147.95$, which multiply'd by the Length $= 1.8r1$, the Anſwer is $= 279.45$.

Prop. 4. *To find the ſolid Content of the Sphereoid.*

Defin. This Figure is generated by the Rotation of the Semi-Ellipſis round its Axis, or *Latus tranſverſum*; and 'tis by ſome called the Problate Sphereoid, to diſtinguiſh it from that which is generated by the Rotation of a Semi-Ellipſis, which turns round its Diameter Conjugate, and is called and Oblate Sphereoid, (of which Figure, 'tis aſſerted by the moſt Learned, our Earth is, the Diameter at the Equator being greater than between the Poles.) This Problate Sphereoid is 2 Thirds of a Cylinder, whoſe Baſe's Diameter is = the greateſt Diameter of the Sphereoid, and its Altitude = the *Latus tranſverſum* of the Ellipſis.

Rule. Multiply the Area of a Circle, whoſe Baſe is the Conjugate Diameter, (or here the greateſt Diameter of the Sphereoid) by the Length, and 2 Thirds of the Product is the Content of the Sphereoid.

Example. The Diameter in the Middle $= 14.32$, as cd *Fig.* 7. the Area of that Circle 161.056, which multiply'd in the Length $lt = 14$, gives 200.48; 2 Thirds of which $= 133.65$, the Anſwer.

Prop. 5. *To find the Solidity of the Hyperbolic Conoid.*

Defin. This Figure is generated by the Rotation of the Semi-Hyperbola ($zbri$) *Fig.* 9. round the Abſciſſa zi; which formeth the Hyperbolic Conoid $brinzb$.

Rule. The *Latus tranſverſum* $= io$, multiply'd by 6; more the Axis or the Abſciſſa iz multiply'd by 6 is = the Diviſor.

2*dly*, Multiply $baisnzb$ = the Content of the Cylinder (whoſe Diameter is equal to the whole Ordinat bn, and its Length = the Abſciſſa zi) in 3 times the *Latus tranſverſum*, more 2 times the Abſciſſa, and that Rectangle is = a Dividend; which divided, there ariſeth the Quotient, which is the Solidity required.

Example. The whole Ordinat (or Diameter) $bn = 3.51$, the Abſciſs or Axis $iz = ba = 2.43$, and the *Latus tranſverſum* $io = 2.16$.

The Content of the Cylinder $basnzb$, is therefore = 23.513 } mult.
The *Latus tranſverſum* mult. in 3 = 6.48 }
The Axis or Abſciſſa mult. in 2 = 4.82 } Sum = 11.3
265.6969 = *Prod.*
The *Latus tranſverſum* mult. in 6 = 12.96
The Axis multiplied in 6 is = 14.46
= Sum Diviſor = 27.42
18916 (9.69 =
24649 (*the Anſ.*

SECT. IV.

SECT. IV. *To find the Solidity of the Frustums of a Cone, Sphere, and Sphereoid.*

Prop. 1. *To find the solid Content of the Frustum of a Cone or Pyramid.*

Defin. This may be done several ways: as to find the Content of that part of the Cone (*d r r m a d*) *Fig.* 19. *Plate* C.

Rule. 1. Find the Content of the whole Cone (*d l m d*) then deducting the Content of the upper part (*r l r*) the rest is the Content of the Frustum (*d r r m d.*)

All the difficulty (more than as *Prop.* 3. *Sect.* 1. of this *Chap.*) is to find the Height (*a l*) which is done by this Rule, That the homogeneal Sides of similar Triangles are in proportion: so that as *m n*, *m r* : : *m a*, *m l*; whence a Perpendicular let fall to *a*, is the true Altitude of the Cone.

Rule 2. *But more general for a Cone, Pyramid,* &c. *without the upper part*: Multiply the Areas of the greater and lesser Ends together, extract the Square Root of the Product; then multiply the Sum of the said Root, and 2 Areas of the Ends by one third of the Altitude of the Frustum, and the Product is the Content.

Example. In *Fig.* 19. *Plate* C. the Area of the greater Base (*m o d x m*) is = 36.72; of the lesser End (as *r s r s r*) = 15.5; the Product of which is = 569.16, whose Square Root is 23.857; which added to the 2 Areas, gives = 76.077; which multiply'd by 1 third of the Altitude *n r* = 6.6*r*, the Product is the solid Content of the Frustum = 507.179*r* 1.

Prop. 2. *To find the solid Content of the Frustum of Sphere or Globe.*

Defin. By a Segment is meant a part of the Sphere cut off perpendicular by a Plain: as *b n e r b*, *Fig.* 14. *Plate* C.

Rule. Find the Area of the Segment's Base, which multiply by the Altitude, reserving the Product. Then

To half the Altitude of the greater Frustum add 1 sixth of the Altitude of the lesser, and multiply the Sum by the reserv'd Rectangle, and divide this last by the greater Frustum's Altitude, and the Quote is the lesser Frustum's Solidity required.

Example. Diameter of the Frustum = 15.832 = *b e*, Area = 179.46

Its Altitude = *n r* . . . . . = 5

Altit. of the greater Frustum = *no* = 7 | Reserv'd Rectan. = 897.3

Half that Altitude is — — = 3.5
1 Sixth of the Altit. of the lesser = .83*r* 1 } Sum multipl. = 4.3*r* 1

Product = 3885.309

 1 Seventh

1 Seventh of which last Product is = 555.044 = the Solidity of Frustum required.

Note, *That if you divide the Square of half the Diameter of the Frustum's Base by the Altitude of either Frustum, the Quote is the other Frustum's Altitude.*

Prop. 3. *To find the solid Content of the middle Frustum of a Sphereoid.*

Defin. This is represented by *raiitr*, *Fig.* 7. like a Cask;

Rule. The easiest Rule, and what is near the Truth, is this: Multiply the Difference of the Bung and Head Diameters by .7, and add the Product to the Head Diameter; to that Sum find the Area of a Circle, and multiply it by the Length of the Cask, and that Product is the Answer.

Example. The Bung Diameter 20 = *rd*, Head 15 = *ra* or *ii*, Length *ab* = 12. The Difference between the Bung and Head = 5, which multiply'd by .7 is 3.5; which added to the Head 15, gives 18.5 for a mean Diameter, whose Area is = 268.803; which multiplied by the Length = 12, the Solidity is = 3225.638 = the Content of the said Frustum (*rudioir*).

Note, That if the Sphereoid be cut in the middle of (*tm*) by a Plain (*ra*) that part (*rstuor*) is 5 Ninths of a Cylinder, whose Base is (*roa*) and the Altitude (*ot*.)

I believe I have, in the Pages preceeding of this Appendix, shewn the best Ways of Measuring a greater Variety of Superficies and Solids than any Book wrote purely on that Subject contains: and to close this Chapter of Solid Measure, I may farther add, There is no Body, tho' never so irregular, but what may have its Content discovered by the help of Water For Example, I would know the solid Inches in a Faggot.

First, I put Water into a Vessel (suppose 36 Inch. by 20, and is a Parallelopipedon; and taking the Depth, I find it 30 Inches deep) into which I immerse the Faggot, so as to cover it with Water; and taking the Depth again, I find it 33 Inches deep, or 3 more than before the Faggot was was in: so the said 36 by 20 = 720 multiply'd in this 3, produceth 2160, the Inches solid contained in the Faggot.

Brief

Brief and Useful Observations on the most Proper Instruments to be used in the Mensuration of several things, &c.

BRicklayers *Work* is best measured by the Foot and 100 Parts: for the Divisor being 272 and a quater, (which is the Feet in one Rod, or the Square of 16 Foot and half) to reduce Feet into Rods, by which Denomination they value their Work: it is necessary that the Superficies of the Walls being the Dividends, should be Feet and Decimal Parts, the Divisor or 1 Rod, Brick and half thick is——— = 272.25 Feet

Or without the Rod, the shortest Way is to multiply any Number of Feet by the Value of 1 Foot at the Rate agreed on *per* Rod, as at *l.* 5 : 10 : 0, the Foot is in Value *l.* .020202, *&c.*

	.02*r*2
	5.4450
	5.499*r*1

Here the Multiplier being .020202, *&c.* or .02*r*2 I only multiply the Feet on the flat Wall by 2, and add every other Figure of the Product till there are but 2 places left, which put down as in

Example 2. What is the Value of 1361.25 Foot, Brick and half thick, at *l.* .0202020202, *&c.* or .02*r*2 *l.* *per* Foot, or *l.* 5 : 10 *per* Rod. See the Operation.

Foot 1361.25
Mult. =02*r*

As *per* Rule, add every other of = 27.2250
Sum or Answer = 27.4999, &c. or *l.* 27.5

Note, *That all Walls must be reduced to Brick and half thick by Multipling the Superficial Feet by the half Bricks which the Wall is thick, and Dividing by* 3.

Carpenters Work. The common two Foot serves as well as any other Instrument to measure Flooring, Partitioning, and Roofing; they being each valued by the Square of 100 Foot: so that Feet multiply'd by Feet, and 2 Figures cut off towards the right hand, give the Squares towards the left hand.

Painters and Plaisterers giving in and valuing their Work by the Yard, I think the best Instrument for them to take their Dimensions with, to be a Yard divided into 100 equal Parts.

Glasiers

Glaisiers Work and *Glass* being valued by the Foot, and Crown-Glass being to be done accurately, ought to be measured by the Foot, and Decimal, or 100 Parts of a Foot.

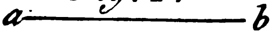
Joiners Work (as Wainscoat, especially Oak) being valued by the Yard, ought to be measured by the Foot divided into 100 Parts, (as being of considerable Value) and the Sum of the Feet divided by 9. (the Square Feet in a Yard.)

Land. The best Instrument to measure it with, is a 4 Pole Chain, divided into 100 Parts or Links: for 40 single Pole long and 4 broad making 160 square Perches or 1 Acre; if the Chain be 4 Pole long, then 10 long and 1 broad make an Acre. So that whatever the Dimensions be in square Chains and Links (or hundred Parts) instead of dividing them by 160, if they were measured by a one Pole Chain, here you have nothing to do but to cut off 5 places from any Product to give the Acres. As suppose a Field be 40 Chains and 30 Links long, and the Breadth 30.40 Chains; the Content, by dividing the Product by 10, (or cutting off 1 place besides the 4 which are Decimals) is 122.512 Acres.

Chains.
40.30
30.40
———
122.51200 *Acres*

FINIS.

Fig. 1.



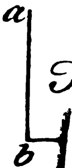


Fig. 2.

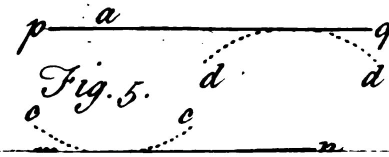
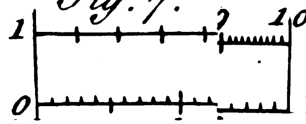


Fig. 5.

Fig. 7.




The DIAGONAL
70
75
80

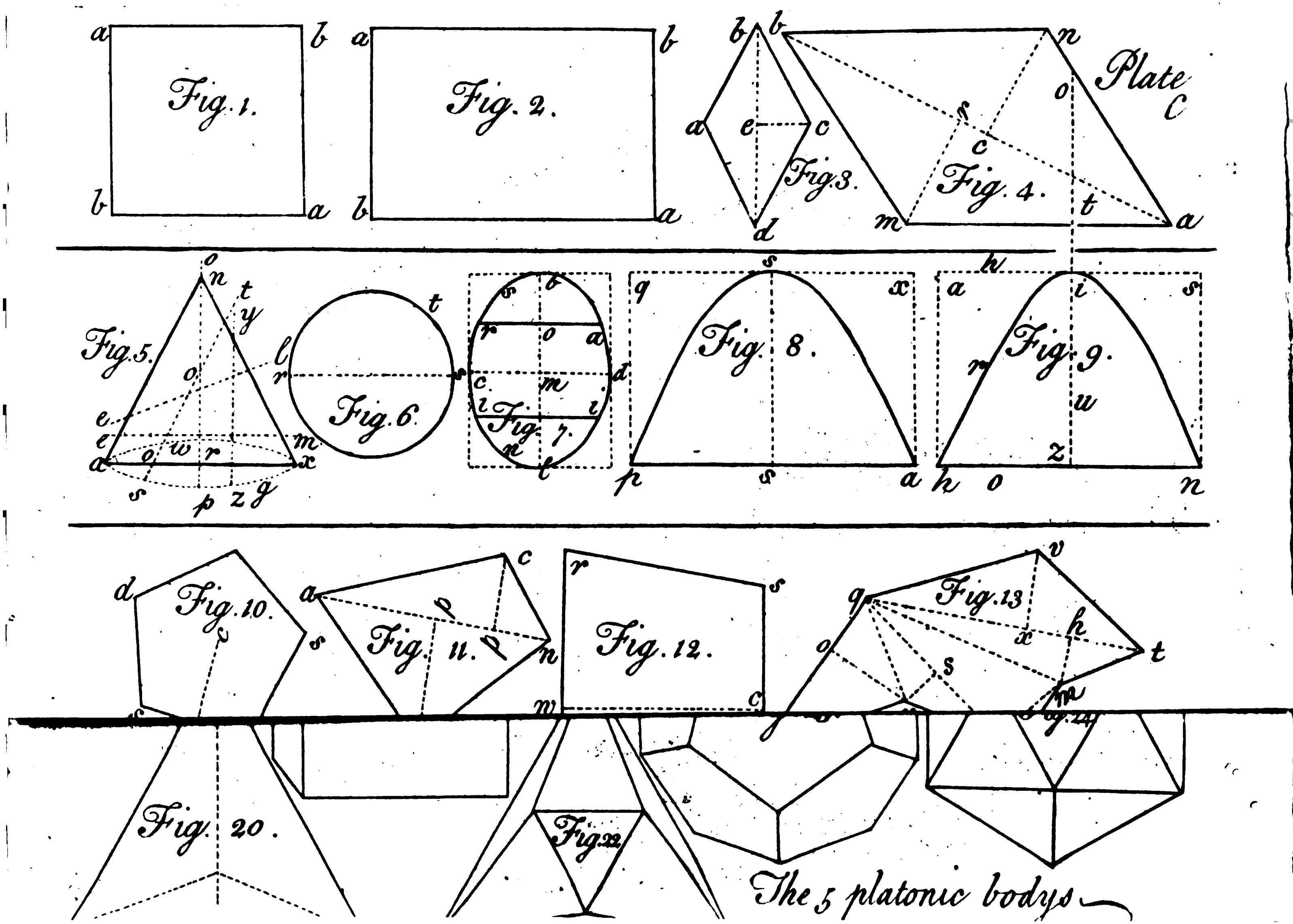


Plate C
Fig. 1.
Fig. 2.
Fig. 3
Fig. 4.
Fig. 5.
Fig. 6
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13
Fig. 20.
Fig. 22
The 5 platonic bodys